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CHIRAL BAG MODEL

M O S C O W 1 9 8 0

We suggest the chiral $SU(2) \times SU(2)$ generalization of the MIT bag model, where the role of Goldstone particles is played by pions which interact only with the surface of a quark "bag" and do not penetrate inside.

Investigation of the model has shown that in the case of a large bag ($R \sim 1 \text{ fm}$) the pion field is rather weak and goes to the linearized chiral bag model. Within that model we have calculated:

- a) baryons' mass spectrum,
- b) the axial constant of β -decay,
- c) magnetic moments of baryons,
- d) pion-baryon coupling constants and their form-factors.

We found that pion corrections are of importance, especially for the case of $N \Delta$ system.

The results are found to be in a reasonable agreement with the experimental data.

I n t r o d u c t i o n

In the Chew-Low model a nucleon is considered as a static source of the ρ -wave pions and has a structure (form-factor) and intrinsic degrees of freedom (spin and isospin). (For a review of model see [1]).

On the other hand, a pion-nucleon interaction at low energies manifests chiral $SU(2) \times SU(2)$ properties - PCAC (See, for instance, ref. [2]).

The Chew-Low model and PCAC in fact describe different sides of $\tilde{\eta}$ -mesons physics at the same energies and may be unified in the framework of more general model such as the chiral bag model (CBM). The CBM provides the synthesis of MIT bag model and the nonlinear σ model.

In the MIT bag model a hadron is considered to be filled by weak interacting quarks and gluons (QCD) with boundary conditions which provide the confinement of the "colour" and stability of the bag. The MIT bag model explains successfully static properties of hadrons. [3,4,5] It should be noted that the first version of the bag model [6] leads practically to the same values of the static properties of hadrons as the MIT bag model.

However there is no natural explanation of the lightness of the pion and the approximate chiral symmetry in Nature.

CBM suggests the chiral $(SU(2) \times SU(2), PCAC)$ generalization of the MIT bag model, where pions are Goldstone particles and interact only with the surface of a quark "bag" not penetrating inside.

In the CBM axial currents are conserved in the limit of massless quarks and pions. The CBM pions interact only with

light quarks (u and d). Immediately, one has zero coupling constants for pions with hadrons which built up out heavy quarks (s, c, \dots). This is nothing else but the well-known OZI rule.

So, the CBM contains information about main properties of hadrons and their interactions in the low energy region.

The role of chiral invariance in the bag model has been firstly pointed out in ref. [7,8].

Recent investigations [9,10] of the properties of the QCD vacuum have shown that Goldstone bosons live in the true vacuum. In another phase, inside hadrons, the vacuum is perturbative and no Goldstone bosons are expected there [10].

In particular, in ref. [10] it has been proposed that CBM can be based on the QCD that stimulated a number of publications [11-15].

The success of the MIT bag model naturally have led to the proposition that CBM pion field surrounding the bag is rather weak. Therefore, the main approximations for quark wave functions used are those from the MIT bag model and chiral corrections only weakly modify them. In this case one can use the spherical approximation for the bag filled by $N = 1/2$ quarks. Investigation of the CBM equations in ref. [11,12] and independently in ref. [13,14] has confirmed such a proposition.

The present paper is organized as follows. In Section 1 we investigate the CBM equations. The linearization of these equations is discussed in Section 2. In Section 3 we calculate the contribution of pions into the masses of hadrons and perform a fit of the mass spectrum of baryons (octet and decuplet) with the account of gluon contributions. In Section 4 the recalculation of the β -decay axial constant is presented for the case when quarks and pions have a mass. The magnetic moments of

baryons are considered in Section 5. Finally, in Section 6 the pion-baryon coupling constants and their form-factors are calculated.

§ I. Equations of the CBM

It is commonly believed that the bag is filled by coloured quarks and gluons (bagged QCD). The masses of u, d quarks and pions is supposed to be zero, so that the chiral symmetry is fulfilled. By means of pion's field we can formulate the chiral invariant boundary conditions for quarks. It is also considered that pion's fields do not penetrate inside the bag. Two last points lead immediately to the pion-quark interactions on the surface of the bag.

So, the lagrangian of such a system can be written as

$$\begin{aligned} \mathcal{L} = \Theta_R(x) & \left\{ \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - g \bar{\Psi} \lambda_i \gamma_\mu \Psi A_\mu^i - \right. \\ & \left. - \frac{1}{4} (F_{\mu\nu}^i)^2 - B + \partial_\mu [\xi_a A_\mu^a \bar{\Psi} (\sigma + i \vec{\tau} \cdot \vec{\pi}) \Psi] \right\} \\ & + (1 - \Theta_R(x)) \frac{1}{2} (\partial_\mu \vec{\Phi})^2 \end{aligned} \quad (I.1)$$

Here Ψ is a quark field, A_μ^i is gluon field, $F_{\mu\nu}^i$ is gluon strength tensor, B is vacuum pressure, ξ_a is Lagrange multiplier, $\vec{\Phi}$ is pion field, $\partial_\mu = (1 + \Phi^2)^{-1/2} \partial_\mu$, and λ_i is Gell-Mann matrix,

$$\Theta_R(x) = \begin{cases} 1 & \text{inside the bag,} \\ 0 & \text{outside,} \end{cases} \quad \partial_\mu \Theta_R(x) = -n_\mu \delta_S$$

δ_S is a surface δ function, $\vec{\Phi} = \vec{\pi}/\sqrt{2f}$, $f \equiv f_{\pi} = f_{\rho} = 90 \text{ MeV}$

$$\sigma = f \frac{\Phi^2 - 1}{\Phi^2 + 1}, \quad \vec{\pi} = 2f \vec{\Phi} (1 + \Phi^2)^{-1}$$

So, one has the equation and boundary conditions for quarks

$$\partial_2(x) (\gamma_5 \partial - g \lambda_i A_\mu^i \gamma_\mu) \psi = 0 \quad (1.2)$$

$$\frac{i}{2f} \gamma_5 n \psi|_S = \xi_a (n \cdot A^a) (\sigma + i \vec{c} \cdot \vec{\pi} \gamma_5) \psi|_S \quad (1.3)$$

Since $(i \gamma_5 n)^2 = 1$ the equation (1.3) implies that

$$4f^2 (n \cdot A^a \xi_a)|_S^2 = 1, \quad \bar{\psi} (\sigma + i \vec{c} \cdot \vec{\pi} \gamma_5) \psi|_S = 0 \quad (1.4)$$

In order to have MIT boundary condition in the limit $f \rightarrow 0$ it is necessary to choose

$$n \cdot A^a \xi_a|_S = -(2f)^{-1}$$

so that the equation (1.3) becomes

$$i \gamma_5 n \psi|_S = -\frac{1}{f} (\sigma + i \vec{c} \cdot \vec{\pi} \gamma_5) \psi|_S \quad (1.3^1)$$

Variation over gluon fields leads to the following equation

$$\begin{aligned} \partial_2(x) (\delta_{ij} \partial_\mu - 2g f_{ijk} A_\mu^k) F_{\mu\nu}^j &= \\ &= \partial_2(x) \cdot g \bar{\psi} \lambda_i \gamma_\nu \psi, \end{aligned} \quad (1.5)$$

where f_{ijk} is the structure constant of $SU(3)$, and to the boundary condition

$$n_\mu F_{\mu\nu}^a|_S + \xi_a \bar{\psi} (\sigma + i \vec{c} \cdot \vec{\pi} \gamma_5) \psi|_S = 0 \quad (1.6)$$

The latter, with the equation (I.4) taken into account, can be transformed into (I.6'). It follows from eq. (I.5), (I.6) that the colour charge of the bag is equal to zero ($F_i = 0$)

$$n_\mu F_{\mu\nu}^a|_S = 0, \quad (I.6I)$$

$$F_i = \int d^3x Y_0^i(x) = - \oint ds n_\mu F_{0\mu}^i = 0$$

(here $Y_\nu^i = g \bar{\Psi} \lambda_i \gamma_\nu \Psi + 2g f_{ijk} A_\mu^k F_{\mu\nu}^j$).

The variation over the pion field $\vec{\Phi}$ provides the equation for pion field

$$(1 - \Theta_2(x)) \left[\partial^2 \phi_6 - 2 \frac{(\partial_\mu \vec{\Phi}^2)^2}{1 + \Phi^2} + 2 \frac{(\partial_\mu \vec{\Phi})^2 \phi_6}{1 + \Phi^2} \right] = 0 \quad (I.7)$$

and the boundary condition

$$4f^2 n \cdot \partial \phi_a|_S + \bar{\Psi} [i \bar{c}_a \gamma_5 + \frac{2\phi_a}{1 + \Phi^2} (1 - i \vec{c} \cdot \vec{\Phi} \gamma_5)] \Psi|_S = 0, \quad (I.8)$$

where it is taken into account that $n A^a \xi_a|_S = -(2f)^{-1}$.

The above equations and boundary conditions should be completed by the stability condition, which can be obtained either by variation of $\Theta_2(x)$ or from the requirement that the total pressure on the surface of the bag in equilibrium should be equal to

zero.

Taking the equation (I.4) into account, one gets

$$\left\{ -\frac{1}{4} (F_{\mu\nu})^2 - B - \frac{1}{2f} n \cdot \partial [\bar{\Psi} (\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5) \Psi] + \right. \\ \left. + 2f^2 (\partial_\mu \vec{\Phi})^2 \right\}_{1S} = 0. \quad (I.9)$$

Equations (I.1) - (I.9) describe very complicated nonlinear system.

When solving this task we shall proceed from the fact that the MIT bag model satisfactorily describes masses and static properties of hadrons, so that one can assume that MIT bag model wave functions provide quite a good zero approximation for the wave functions of quarks.

Let us transform Ψ in equations (I.2) - (I.9) to more convenient for the perturbation theory form Ψ'

$$\Psi = (1 + \Phi^2)^{-\frac{1}{2}} (1 + i\gamma_5 \vec{\tau} \cdot \vec{\Phi}') \Psi' \quad (I.10)$$

where $\vec{\Phi}'$ is a function arbitrary inside the bag and coinciding with $\vec{\Phi}$ on the surface and outside the bag. As a result the equations (I.2) and (I.5) take the form

$$\Theta_2(x) [i\gamma \cdot \partial - g\lambda_i A^i \gamma - \gamma_\mu \vec{\tau} (\vec{\Phi}' \times \partial_\mu \vec{\Phi}') - \\ - \gamma_\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\Phi}'] \Psi' = 0,$$

(I.11)

$$\begin{aligned}
 & (\epsilon_{2\nu}) [\delta_{ij} \partial_\mu - 2g f_{ijk} A_\mu^k] F_{\mu\nu}^j = \\
 & = (\epsilon_{2\nu}) \psi^\dagger \lambda_i \gamma_\nu \psi' \quad (1.12) \text{ respectively.}
 \end{aligned}$$

The equation (I,7) for pion field remains unchanged and the boundary conditions are transformed into

$$i \gamma_n \psi'_{1s} = \psi'_{1s} , \quad (1.13)$$

$$4f^2 n \cdot \vec{x} \phi_{a1s} + \bar{\psi}' i \gamma_5 \tau_a \psi'_{1s} = 0 ,$$

$$-n \cdot \partial (\bar{\psi}' \psi') = 2B + \quad (1.14)$$

$$+ \left[\frac{1}{2} (F_{\mu\nu}^a)^2 - 4f^2 (\partial_\rho \vec{\phi})^2 \right]_{1s} . \quad (1.15)$$

As the boundary condition (I,13) for quarks has now the form coinciding with the MIT bag model condition, one can use for the equation (I,II) standard methods of the perturbation theory, where the zero approximation is provided by the functions of MIT bag model.

§ 2. Linearization of pion field equations

In order to clarify the possibility of linearization of the equation (I,7) let us ignore for the moment problems related with the "colour" and consider the pion field as a classic field of a quark bag. As a starting point we shall consider the field of a single quark.

As a zero-order approximation, ψ_0 to wave functions of quarks let us choose solutions of the MIT bag model with minimal energy E_0 in the static spherical bag approximation [3]

$$\Psi_0 = \begin{pmatrix} j_0(\epsilon_0 r) \varrho_0 \\ j_1(\epsilon_0 r) i \vec{\sigma} \vec{n} \varrho_0 \end{pmatrix}, \quad \varrho_0^+ \varrho_0 = \frac{k_0^2}{R^3 [\sqrt{\pi} (1 - j_0^2(k_0))]},$$

$$k_0 \equiv \epsilon_0 R = 2.04,$$

$$i \gamma_5 \partial (\Psi_0 e^{-i \epsilon_0 t}) = 0 \quad (r < R), \quad -i \vec{\gamma} \vec{n} \Psi_0 = \Psi_0 \quad (r = R)$$

Then we can write

$$\Psi' = (\Psi_0 + \Psi_n) e^{-i \epsilon t}, \quad \epsilon = \epsilon_0 + \epsilon_n, \quad (2.1)$$

where ϵ_n and Ψ_n are respectively the energy and wave function shifts required. The pion field can also be decomposed into two parts

$$\vec{\Phi} = \vec{\Phi}_0 + \vec{\Phi}_n,$$

where $\vec{\Phi}_0$ is a solution of the equation (I,7) under the boundary condition

$$4f^2 n \cdot \partial \vec{\Phi}_0 + \vec{\Phi}_0 i \gamma_5 \vec{\tau} \Psi_0 = 0 \quad (r = R) \quad \text{or}$$

$$4f^2 (1 + \Phi_0^2)^{-1} \frac{\partial \vec{\Phi}_0}{\partial r} = 2f_0^2 j_0^2(k_0) \varrho_0^+ (\vec{\sigma} \vec{n}) \vec{\tau} \varrho_0 \quad (r = R) \quad (2.2)$$

and $\vec{\Phi}_n$ is the corresponding shift.

The boundary condition (2.2) allows one to represent $\vec{\Phi}_0$ in the form $\vec{\Phi}_0 = \vec{t} \operatorname{tg} \chi_0$ where \vec{t} - constant and $\vec{t}^2 = 1$. As a result, equation (I,7) and (2,2) take the form

$$\partial^2 \chi_0 = 0 \quad (r > R), \quad (2.3)$$

$$\vec{t} \frac{\partial \chi_0}{\partial r} = \frac{1}{2f} 2f_0^2 j_0^2(k_0) \varrho_0^+ (\vec{\sigma} \vec{n}) \vec{\tau} \varrho_0 \quad (r = R). \quad (2.2')$$

The solution of equations (2,3) and (2,2) has a form

$$\vec{t} \psi_0 = \frac{1}{4f^2 R^2} \frac{\kappa_0}{2(\kappa_0 - 1)} \left(-\frac{R^3}{r^2}\right) \frac{1}{4\pi} (\vec{\sigma} \vec{n}) \vec{t} \quad (2.4)$$

Let us now turn to calculation of ψ_n and E_n . It follows from equations (I,II) and (2.1) that in the first approximation

$$\gamma_0 E_n \psi_0 + (E_0 \gamma_0 + i \vec{\gamma} \vec{\partial}) \psi_n - V_0 \psi_0 = 0 \quad (r < R), \quad (2.5)$$

where we designated

$$\vec{\Phi}' \equiv \vec{t} t \gamma \psi', \quad \gamma_m \gamma_5 \vec{t} \vec{t} \vec{\sigma}_m \psi' \equiv \gamma_0.$$

It is clear that one may suppose that $\psi_n = \sum_{n \geq 1} C_n \psi_n$ where ψ_n are wave functions of the excited states of quarks in a "bag". As a result we have

$$C_n = (E_n - E_0)^{-1} \left[-i R^2 \int_{r=R} d^3 r \bar{\psi}_n \gamma_5 \vec{\sigma} \psi_0 \vec{t} \gamma' + i (E_n - E_0) \int_{r < R} d^3 r \psi_n^+ \gamma_5 \vec{t} \psi_0 \vec{t} \gamma' \right] \quad (2.6)$$

Let us discuss firstly the contribution of radial excitations:

$$\psi_{2k} = \begin{pmatrix} j_0(E_{2k} r) \psi_{2k} \\ j_1(E_{2k} r) i \vec{\sigma} \vec{n} \psi_{2k} \end{pmatrix}, \quad E_{2k} R = K_{2k}$$

under the boundary conditions $j_0'(K_{2k}) = j_1'(K_{2k})$

$$\psi_{2k+1} = \begin{pmatrix} \int_0^1 (E_{2k+1}(z)) i \vec{\sigma} \cdot \vec{n} \psi_{2k+1} \\ \int_0^1 (E_{2k+1}(z)) \psi_{2k+1} \end{pmatrix},$$

$$E_{2k+1} R \equiv X_{2k+1}, \quad \int_0^1 (X_{2k+1}) = - \int_1^0 (X_{2k+1}),$$

where, for instance, $X_1 = 3,81$; $X_2 = 5,4$; $X_3 = 7$; $X_4 = 8,55$.

It is easy to infer that $C_{2k+1} = 0$ and

$$C_{2k} = \frac{-4\pi R^3}{(E_{2k} - E_0) 6f} \int_0^1 (X_{2k}) \int_0^1 (X_0) \psi_0^+ \sigma_i \tau_j \psi_0 \psi_{2k}^+ \sigma_i \tau_j \psi_0 \quad (2.7)$$

It turns out that numerically coefficients C_{2k} are small quantities, for example

$$C_2 = 0,011 (s_i a_j)_{00} (s_i a_j)_{20},$$

$$C_4 = -0,006 (s_i a_j)_{00} (s_i a_j)_{40},$$

where we have designated

$$(s_i a_j)_{n0} = \frac{\psi_n^+ \sigma_i \tau_j \psi_0}{(\psi_n^+ \psi_n)^{1/2} (\psi_0^+ \psi_0)^{1/2}},$$

so that we can neglect the contribution of the radial excitations into ψ' .

The contamination of orbital excitations in ψ' may be present if the bag is nonspherical. The shape of the bag is condi-

tioned by the stability condition (1.9), that can now be re-written as

$$-n\bar{v}(\vec{\psi}'\psi')_{15} = 2B - 4\pi^2 (\vec{\xi}, \vec{\psi}')_{15}^2 \quad (2.8)$$

Obviously, nonsphericity of the bag in the lowest approximation is related with the contribution of the pion field $\vec{\phi}$ into the requirement (2.8). However, the numerical estimates show that maximal nonsphericity is rather small [13] $R(\frac{\pi}{2}) - R(0) \approx 0,01 \bar{R}$ ($\bar{R} \approx 1 \text{ fm}$), so that the contribution of orbital excitations into $\vec{\phi}$ also is negligible.

Let us now estimate from the formula (2.4) the magnitude of the pion field on the surface of the bag where it is maximal

$$|\gamma| = \frac{1}{4\pi^2 R^2} \frac{\chi_0}{2(\chi_0 - 1)} \frac{1}{4\pi} |\langle (\vec{\sigma} \vec{n}) \vec{\tau} \rangle| \quad (2.9)$$

If $R \sim 1 \text{ fm}$, then $|\gamma| \sim 0,08 |\langle (\vec{\sigma} \vec{n}) \vec{\tau} \rangle|$

Therefore on the surface of a nucleon, when $R \approx 1 \text{ fm}$ one has

$$|\vec{\psi}'_{1/2}| = |\gamma| \leq 0,16 (\langle N | (\vec{\sigma} \vec{n}) \vec{\tau} | N \rangle = \frac{5}{3} (\vec{\sigma} \vec{n}) \vec{\tau}_N)$$

It is obvious in this case, when the radius of a bag of the same order of magnitude as electromagnetic radius of a proton, that one can linearize the pion field equations, since

$$|\vec{\psi}'_{1/2}| \leq 0,16.$$

Note, that for $R \sim 0,3 \text{ fm}$ (a small bag) $|\gamma| \leq 1$, and

linearization is impossible because the effects of pion-pion interaction become of great importance. In this case the pions give a very sizeable contribution to the mean squared isovector charge radius of a nucleon.

On the other hand, the smaller the bag radius is, the smaller the isoscalar part of the charge radius of a nucleon will be. But it is well-known experimentally that the isoscalar charge radius, r_1 , approximately equals the charge radius of the proton $r_p = 0,88 \text{ fm}$,
 $(r_p = 0,78 R_N)$, that is compatible with the nucleon bag radius $R_N = 1,2 \text{ fm}$.

With such a value of R it is possible to linearize the equations for pion fields in the form

$$(\partial^2 + \mu^2) \vec{\varphi} = 0 \quad (z > R) \quad (2.10)$$

$$2f n \cdot \partial \vec{\varphi}|_S = -\bar{\psi}_0 i \gamma_5 \vec{\tau} \psi_0|_S \quad (z = R) \quad (2.11)$$

where we the pion mass μ is introduced explicitly. In this case the axial current becomes to be partially conserved (PCAC).

It should be noted, that by introducing the mass of a pion we hiddenly introduced also the nonzero masses, m for light quarks (since $\mu^2 \sim m$), the wave functions of which are slightly different from those provided by equation (2.1) and have now the form

$$\psi = \frac{N(x)}{(4\pi)^{1/2}} \left(\begin{array}{c} \left(\frac{E+m}{E}\right)^{1/2} j_0\left(x \frac{r}{R}\right) \\ \left(\frac{E-m}{E}\right)^{1/2} j_1\left(x \frac{r}{R}\right) (\vec{\sigma} \vec{n}) \end{array} \right)$$

$$N^{-2} = R^3 \int_0^R dx \left[2E(E-R^{-1}) + mR^{-1} \right] \left[E(E-m) \right]^{-1}, \quad (2.12)$$

$$\text{where } E = R^{-1} (x^2 + m^2 R^2)^{1/2},$$

$$X = X(mR),$$

E is the energy of a quark with the mass m and the value of X can be obtained from the equation

$$\text{tg } X = X \left[1 - mR - (x^2 + m^2 R^2)^{1/2} \right]^{-1}.$$

Finally, one has for the pion field

$$\Phi_a = \left(\frac{1}{\mu R} + \frac{1}{\mu^2 R^2} \right) e^{\mu(R-r)} \left(-\frac{\mu^2}{4f^2} \right) \frac{1}{4\pi} \cdot \frac{1}{1 + \mu R + 0.5 \mu^2 R^2} \cdot \frac{E^2 - m^2}{2E(E-R^{-1}) + mR^{-1}} \sum (\vec{\sigma} \vec{n}) \tau_a \quad (2.13)$$

§ 3. The mass spectrum of baryons in the spherical approximation.

The chiral correction to masses of hadrons in the MIT bag model is composed of the pion field energy E_ϕ and the quark energy shift E_η , caused due to interactions with the pion

field.

In order to calculate \hat{E}_n let us use the equation (2.5) from which it follows that

$$E_n = -iR^2 \int_{z=R} d^3z (\bar{\Psi}_0 \gamma_5 \vec{E} \Psi_0 \vec{t} \gamma_0) \quad (3.1)$$

and we find

$$E_n = -\frac{2\pi}{3f^2 R^3} \left[\frac{j_0^2(x_0) \cdot x_0^2}{4\pi(1-j_0^2(x_0))} \sum_i \sigma_i E_j \right]^2 \quad (3.2)$$

The energy of a pion field can be found from the expression

$$E_\phi = 2f^2 \int_{z>R} d^3z (\partial_m \vec{\Phi}_0)^2 = \frac{i}{2} R^2 \int_{z=R} d^3z (\bar{\Psi}_0 \gamma_5 \vec{E} \Psi_0 \vec{t} \gamma_0) \quad (3.3)$$

Thus, the chiral correction to the mass of an hadron, composed of N quarks is [14, 15]

$$\hat{M}_\phi = \hat{E}_n + E_\phi = -E_\phi = -\left(\frac{2\pi}{3f^2 R^3}\right)^{-1} \left(\frac{x_0}{2(1-x_0)}\right)^2 \left(\sum_{n=1}^N \sigma_n \tau_n\right)^2 \quad (3.4)$$

It should be noted, that the expression for E_ϕ above coincides with the result obtained in ref [16].

The matrix element of eq. (3.4) can be evaluated straightforwardly, since spin-isospin wave functions of hadrons may be taken to correspond to those of the nonrelativistic quark model. Then, we get

$$\mu_{\phi} = - (48\pi f^2 R^3)^{-1} \left(\frac{1}{2(R-1)} \right)^2 (9N + \langle \Omega \rangle)$$

(3.5)

where $\Omega = \sum_{n \neq m} (\sigma_i^n \sigma_j^n) (\sigma_i^m \sigma_j^m)$

The account of the pion mass lead only to an additional factor in (3.4) and (3.5) [16]

$$G = \frac{1 + \mu R}{1 + \mu R + 0,5 \mu^2 R^2}$$

for $R = 1 f_m$ $G = 0,84$, that has a small influence on the results. Note, that the magnitude of G insignificantly changes for the values in the ranges $0,8 f_m < R < 1,5 f_m$.

In order to estimate the contribution of chiral (pionic) correction into hadrons masses, let us calculate the mass split of Δ isobars and nucleons due to the quantity (3.4), where we substitute $R_{\Delta} \approx R_N = 1,2 f_m$.

Calculating matrix elements of (3.4) over the corresponding wave functions we have

$$(\mu_{\Delta} - \mu_N)_{\phi} = 110 \mu e \nu \quad ((\mu_{\Delta} - \mu_N)_{exp} = 292 \mu e \nu)$$

It is seen that the chiral correction in this case is of the same order of magnitude as gluonic contribution to mass difference.

Gluonic corrections to hadron masses have been calculated in a number of papers [3,4]. We follow the calculation method of ref [4] where it is suggested to treat the interaction of a

quark inside a bag by a consistent method based on the quantum mechanical perturbation theory for emission and absorption of gluons by quarks with the perturbation

$$V = g \bar{\Psi} \gamma_{\mu} \lambda_a \Psi A_{\mu}^a$$

Let us represent M_g as $M_g = \frac{N \Sigma}{R} + \Delta M_g$ where ΔM_g is the contribution of quark-quark interaction due to the gluon exchange, and $\frac{\Sigma}{R}$ is the gluonic selfenergy of a quark. Here we use the results of the paper [4] in which it has been found that

$$\Delta M_g = -\alpha_c \omega \sum_{i \neq j} \frac{\lambda_a^i \lambda_a^j}{4} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{4} I(m_i R) I(m_j R),$$

$$I(0) = 0,716, \quad (3.6)$$

where $\omega = \frac{2,743}{R}$ is the ground state of the gluons with quantum numbers $Y^P = 1^+$, m_i is the quark mass and $\alpha_c = g^2/4\pi$. Following the ref [4] we shall consider to be that the quantity Σ is a free parameter including also the translation (centre of mass) corrections [12]. The mass of a hadron consisting of N quarks now would be equal to

$$M = \frac{4}{3} \pi R^3 B + \frac{NX}{R} + M_g + M_{\phi} \quad (3.7)$$

It has been shown earlier [3] that in the spherical approximation the stability condition (I.15) (without pions contribution) is equivalent to the requirement that the equilibrium radius corresponds to the minimum of the mass (3.7) (without M_{ϕ}) as a function of the radius.

We shall demonstrate that the pions contribution into (I.15)

and (3.7) does not change this conclusion if the shape of the bag is assumed to be close to the spherical one.

Let us consider in detail the stability condition (I.15) and the expression for the mass (3.7) ignoring for the sake of simplicity the gluonic contribution. Then

$$\frac{dM}{dR} = 4\pi R^2 B - \frac{N x_0}{R^2} + \frac{1}{16\pi f^2 R^4} \left(\frac{x_0}{2(x_0-1)} \right)^2 \left(\sum \sigma_a \tau_a \right)^2 \quad (3.8)$$

$$\frac{d^2 M}{dR^2} = 8\pi R B + \frac{2N x_0}{R^3} - \frac{1}{4\pi f^2 R^5} \left(\frac{x_0}{2(x_0-1)} \right)^2 \left(\sum \sigma_a \tau_a \right)^2 \quad (3.9)$$

On the other hand, substituting into (I.15) the quark wave functions ψ_0 and the expression for the pion field (2.4) we obtain

$$\frac{N x_0}{4\pi R^4} = 2B + (64\pi^2 f^2 R^6)^{-1} \sum_{n,m} (\sigma_e \tau_a^n) (\sigma_c \tau_a^m) (f_{ec} + 3n_2 n_2) \quad (3.10)$$

Averaging eq (3) over the angles (the spherical approximation) it is easy to infer that it is equivalent to

$$dM_i/dR |_{R=R_i} = 0 \quad (3.11)$$

The expression for the baryon mass spectrum contains four parameters B, d_c, Σ and the strange quark mass m_s . By fixing these parameters through the masses of protons, Δ -isobars, Σ -particle and the mean squared charge radius of proton $\langle r_p^2 \rangle^{1/2}$, we obtain predictions for masses of other baryons ($\Lambda, \Sigma, \Xi, \Sigma^*, \Xi^*$) and for radii of quark bags. The results of our calculations are listed in Table I. The values of parameters

used were $\beta^{14} = 0,1166 \text{ GeV}$, $\alpha_2 = 0,56$, $\Sigma = 0,840$, $m_3 = 0,206 \text{ GeV}$
 Baryon's mass spectrum in CBM

Table I

Particle	Mass		Bag's radius GeV^{-1}
	Theor.	Exp.	
P	0,938	0,938	6
Λ	1,110	1,116	6,5
Σ	1,180	1,192	6,5
Ξ	1,320	1,320	6,5
Δ	1,232	1,232	7,3
Σ^*	1,410	1,385	7,4
Ξ^*	1,540	1,530	7,4
Σ	1,672	1,672	7,4

§ 4 . The β -decay axial constant

The β -decay axial constant has been calculated earlier in papers [11,12,15,17]. In the present paper we will take into account effects of chiral invariance breakdown by calculating of g_A at nonzero pion and quark masses.

In the spirit of previous paper [15] the axial current can be represented as

$$A_\mu^a(x) = \frac{1}{2} \bar{\Psi}_0 \tau_a \gamma_\mu \gamma_5 \Psi_0 \Theta(r-L) - 2f^2 \partial_\mu \Phi_a \Theta(r-L), \quad (4.1)$$

where we use the wave functions of quarks from (2.12) and the pion field of eq. (2.13).

Let us define the quantity

$$A_m^a = \lim_{q \rightarrow 0} \int e^{i\vec{q} \cdot \vec{x}} A_m^a(x) d^3x,$$

$$A_m^a = A_m^a(q) + A_m^a(\phi).$$

After some straightforward calculations, we obtain the following expression for the contribution of quarks

$$A_m^q(q) = -\frac{1}{6} \left\{ -1 + 2x^2 [2ER(ER-1) + mR]^{-1} [1 - (1 - mR - ER)(ER + mR)k^{-2}] \right\} \sum \sigma_m \tau_a \quad (4.2)$$

and pions

$$A_m^a(\phi) = -\frac{1}{6} \frac{1 + \mu R}{1 + \mu R + 0,5 \mu^2 R^2} \frac{x^2}{2ER(ER-1) + mR} \sum \sigma_m \tau_a. \quad (4.3)$$

For the case of $mR \ll 1$ eq. (4.2) and (4.3) can be reduced, respectively, to

$$A_m^a(q) = -\frac{1}{6} \frac{x_0}{x_0 - 1} \sum \sigma_m \tau_a, \quad (4.4)$$

$$A_m^a(\phi) = -\frac{1}{12} \frac{1 + \mu R}{1 + \mu R + 0,5 \mu^2 R^2} \frac{x_0}{x_0 - 1} \sum \sigma_m \tau_a,$$

and we find

$$g_A = 1,42 \quad g_A(q) = 1,1.$$

Here we take $R_N = 6 \text{ GeV}^{-1}$, $x_0 = 2,04$.

The account of quark masses slightly change this result. So, for $n=3$ we have $g_A = 0.93 g_A(9)$.

It is very important to take into account the gluonic corrections to the quark wave functions which lead to the result that $g_A(9) = 0.93$ [5]. As a result we have $g_A^0 = 1, 3$.

§ 5. Magnetic moments of baryons

Let us define the operator of the magnetic moment

$$\vec{\mu} = \frac{1}{2} \int d^3r \vec{r} \times \vec{j}^{\text{em}} \quad (5.1)$$

The contribution of quark into (5.1) can be calculated from the eq. [3]

$$\vec{\mu}(q) = \frac{R}{12} \sum_n \frac{4\alpha_n + 2\lambda_n - 3}{\alpha_n(\alpha_n - 1) + 0,5\lambda_n} Q_n \vec{\sigma}^n,$$

$$\lambda = mR, \quad \alpha = (x^2 + \lambda^2)^{1/2}. \quad (5.2)$$

Here Q is the charge of a quark, $\alpha \cdot R^{-1}$ is its energy.

The contribution of pions into (5.1) can be found from the formula

$$\mu_e(\phi) = -2f^2 \int_{z \gg R} d^3r \varepsilon_{esm} r_s (\vec{\phi} \times \partial_m \vec{\phi})_3 \quad (5.3)$$

Substituting the expression (2.13) for the pion field into the eq. (5.3) we obtain

$$\vec{\mu}(\phi) = (96\pi f^2 R)^{-1} \frac{1 + \mu R}{1 + \mu R + 0,5\mu^2 R^2}.$$

$$\left(\frac{\kappa_0}{2(\kappa_0-1)}\right)^2 \sum_{n,m} (\vec{\sigma}^n \times \vec{\sigma}^m) (\vec{\tau}^n \times \vec{\tau}^m)_3 \quad (5.4)$$

Note, that the results calculated in accordance with eq.(5.4) is two times less than that following from the formulae of ref. [17].

By making use the radii of bags from Table I one can estimate from eq. (5.2) and (5.4) the magnetic moments of baryons. The results are presented in Table 2, where the last column contains the experimental values of magnetic moments. The only

Baryon's magnetic moments in CBM.

Table 2

Particle	Bag's radius: from ref [15]:	Magnetic moment, $2 m_p \mu$		
		Pion's contr.:	Total	Exp.
p	6	0,53	2,82	2,793
n	6	-0,53	-2,06	-1,913
Λ	6,54	0	-0,64	-0,6138 \pm 0,047
Σ^+	6,54	0,08	2,5	2,370,14 [28]
Σ^0	6,54	0	0,76	
$\Sigma^0 \Lambda$	6,54	0,277	1,71	
Σ^-	6,54	-0,08	-0,97	-1,48 \pm 0,37
Ξ^0	6,54	-0,04	-1,44	-1,270,06 [29]
Ξ^-	6,54	0,04	-0,54	-1,85 \pm 0,75
Δ^{++}	7,3	0,284	5,85	3,6 \pm 2 [26]
Δ^+	7,3	0,09	2,87	
Δ^0	7,3	-0,09	-0,09	

serious disagreement observed is that for the magnetic moment of Ξ^- particle. The magnetic moment of Δ^{++} has been obtained in the framework of the laobar model [26] from the experimental data [27] on the reaction $\pi^+ p \rightarrow \pi^+ p \gamma$.

It can be noted also that in the framework of $SU(3)$ scheme

$$\mu_{\Sigma^+} = -\frac{\sqrt{3}}{2} \mu_n \quad \text{that leads to } 2 m_p \mu_{\Sigma^+} \approx 1.7.$$

§ 6. Pion-baryon coupling constants and form-factors

The formula (2.9) leads at $R \sim 1 \text{ fm}$ to the requirement $|\varphi|/25 \leq 0.16$ (φ is a classic field), that allows one to linearize the model equations (I.7), (I.8) over the pion field. As a result we get the lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$

$$\begin{aligned} \mathcal{L}_0 = & \theta_2(x) \left[\frac{i}{2} \bar{\Psi} \gamma_0 \vec{\partial} \Psi - \bar{\Psi} m \Psi - B \right] + \\ & + \frac{1}{2} \delta_3 \bar{\Psi} \Psi + (1 - \theta_2(x)) \frac{1}{2} \left[(\vec{\partial}_\mu \vec{\varphi})^2 - \mu^2 \vec{\varphi}^2 \right], \end{aligned} \quad (6.1)$$

$$\mathcal{L}_{int} = -\delta_3 \bar{\Psi} \left[\frac{\vec{\varphi}^2}{4f^2} + \frac{1}{2f} i(\vec{\sigma} \vec{\varphi}) \gamma_5 \right] \Psi$$

(6.2)

that corresponds to the linearized version of the CBM.

The quark field operator Ψ and pion field operator φ can be defined in terms of the complete system of solutions of the following equations and boundary conditions

$$i \gamma_0 \partial \Psi - m \Psi = 0 \quad (z < z_0), \quad (6.3)$$

$$-i \vec{\gamma} \vec{\pi} \Psi = \varphi \quad (z = z_0), \quad (6.4)$$

$$(\nabla^2 + \mu^2)\psi = 0 \quad (z > R), \quad (6.5)$$

$$\frac{\partial \psi}{\partial z} = 0 \quad (z = R). \quad (6.6)$$

The eigenstates of the eq. (6.3) and (6.4) are well known and discussed in many papers on the MIT bag model. For instance, the quark wavefunctions in the lowest on energy state and with the spin $J = 1/2$ have the form (2.12).

Now, the quark field operator can be represented as

$$\psi(\vec{x}, t) = b_0 \psi_0(\vec{x}) e^{-i\varepsilon_0 t} + \text{E.C.} + \dots \quad (6.7)$$

where b_0 is an annihilation operator for quarks. The solutions of equations (6.5) and (6.6) can easily be found in the angular momentum representation. For us it is sufficient to consider J and P waves.

In this case one has

$$\begin{aligned} \psi_i(\vec{z}, t) = & \sum_k [J_{k,i} \psi_{1,k}(z) Y_{00}(\hat{z}) e^{-i\omega_k t} + \\ & + P_{k,m,i} \psi_{p,k}(z) Y_{1m}(\hat{z}) e^{-i\omega_k t} + \text{E.C.}] + \dots \end{aligned} \quad (6.8)$$

Here $J_{k,i}$ and $P_{k,m,i}$ are the annihilation operators for J and P -wave pions, respectively, and

$$\psi_{1,k}(z) = (2\pi)^{-3/2} (2\omega_k)^{-1/2} \frac{i\pi^{1/2}}{kz} \psi_0(kz),$$

$$\psi_0(kz) = \exp(-ikz) - S_0 \exp(ikz),$$

$$U_{pk}^{-1} = (2\pi)^{-3/2} (2\omega_k)^{-1/2} \frac{(3\pi)^{1/2}}{k^2} \psi_1(kz).$$

where $\psi_1(kz) = e^{-ikz} \left(\frac{1}{kz} + i\right) - \int_1 \left(\frac{1}{kz} - i\right) e^{ikz}$.

From the boundary conditions (6.6) we find that

$$S_0 = \exp(-2ikR) (1+ikR) (1-ikR)^{-1}$$

$$S_1 = e^{-2ikR} \frac{1 - 0,5k^2R^2 + ikR}{1 - 0,5k^2R^2 - ikR}$$

As a result, we obtain for the interaction operator of

P-wave pions with quarks

$$i) \int_{L_{int}} d^4x = i 2\pi \delta(E_i - E_f) \frac{1}{2f} \frac{\kappa_c}{2(\kappa_0 - 1)}$$

$$\left[b_0^+ (\vec{\sigma} \vec{\kappa}) \tau_i b_0 \cdot \frac{P_{3q} i U_1(kR)}{(2\pi)^{3/2} (2\omega_k)^{1/2}} + E.C. \right] + \dots$$

(6.9)

The universal form-factor $U_1(kR)$ contained in eq. (6.9) has the form ^{*)}

$$U_1(y) = \frac{i}{y^2} e^{-iy} \left[\frac{1}{y} + i - \frac{1 - 0,5y^2 + iy}{1 - 0,5y^2 - iy} \left(\frac{1}{y} - i \right) \right], \quad |U_1(y)|^2 = \left(1 + \frac{y^4}{4} \right)^{-1}$$

(6.10)

^{*)} In ref [14] the form-factor $U_1(y) = \int_0^1 (y)$ has been obtained, which is right only for the case $y \ll 1$

Let us define the coupling constants in the system $\pi N \Delta$ by the equations

$$M_{NN\pi} = \frac{g_N}{2m_p} \varphi^+ (\vec{\sigma} \vec{k}) (\vec{\tau} \vec{\varphi}_\pi) \varphi, \quad (6.11)$$

$$M_{N\Delta\pi} = \frac{g_{N\Delta}}{2m_p} \varphi_{i,m}^+ \varphi_{k,m} \varphi_{\pi i}, \quad (6.12)$$

$$M_{\Delta\Delta\pi} = \frac{g_\Delta}{2m_p} \varphi_{i,m}^+ (\vec{\sigma} \vec{k}) (\vec{\tau} \vec{\varphi}_\pi) \varphi_{i,m}. \quad (6.13)$$

A straightforward calculations [30] of matrix elements of the operator (6.9) give:

$$g_N = 17 \varrho_2(kR) \quad (6.14)$$

without gluonic corrections, and

$$g_N = 14,4 \varrho_2(kR) \quad (6.14^b)$$

with gluonic corrections [5] to the quark wave function taken into account,

$$g_{\Delta N} = 29 \varrho_1(kR) \quad (6.15)$$

$$g_\Delta = 30,7 \varrho_1(kR) \quad (6.16)$$

The width Γ_Δ of the Δ isobars can be calculated from eq. (6.15) ($k = 228 \text{ meV}$)

$$\Gamma_\Delta = 150 \text{ meV} |\varrho_1(kR)|^2 = 80 \text{ meV} \quad (6.15^c)$$

Now we consider interactions of f -wave pions with nucleons. Through the transformation (1.10) of quark wave functions we get the interaction langangian

$$\mathcal{L}'_{int} = -\theta_2(x) \left[\frac{1}{4f^2} \bar{\Psi}' \gamma_\mu \vec{\tau} \Psi' (\vec{\Phi}' \times \partial_\mu \vec{\Phi}') + \frac{1}{2f} \bar{\Psi}' \gamma_\mu \gamma_5 \vec{\tau} \Psi' \partial_\mu \vec{\Phi}' \right]$$

(6.17)

Let us calculate the contribution of the second term (6.17) in the f -wave pion scattering amplitude (6.17).

The arbitrary inside the bag function $\vec{\Phi}'$ can be taken as

$$\Phi' = \Phi_3(R) Y_{00} e^{-i\omega_k t} \equiv \mathcal{U}_0(kR) e^{-i\omega_k t},$$

where

$$\mathcal{U}_0(y) = \frac{i}{2y} e^{-iy} \left(1 - \frac{1+iy}{1-iy} \right), \quad |\mathcal{U}_0(y)|^2 = (1+y^2)^{-1}$$

In the lowest second order of the perturbation theory the intermediate quark may be only in the excited state with $E_n \neq E_0$, because of

$$\bar{\Psi}_0 \gamma_0 \gamma_5 \vec{\tau} \Psi_0 = 0$$

For the case of $\omega_k \ll E_n - E_0$ the main contribution into the f -wave scattering of pions from the first term of eq. (6.17)

$$(E_1 = 3.81/R, E_0 = 2.04/R, \omega_k R \ll 1, 8)$$

Therefore the f -wave scattering operator for $\omega_k R \ll 1, 8$ can be represented as

$$i) \int_{int} L'(\mathbf{x}) d^4x = i 2\pi \delta(E_i - E_f) \left(-\frac{i}{4f^2}\right) (\omega_k + \omega_{k'})$$

$$[\beta_0^+ \tau_i \beta_0 \varepsilon_{ije} g_{k,j}^+ g_{k',e} \cdot (2\pi)^{-3} (4\omega_k \omega_{k'})^{-\frac{1}{2}} +$$

$$+ E.c.] + \dots \quad (6.18)$$

Here we take into account that $\beta_0(kR) \approx 1$ for $\omega_k R \ll 1, 8$. It is easy to infer that the matrix element of (6.18) over the nucleon wave functions coincides with the result of the current algebras [2].

Concluding remarks

The results of our calculations concerning the mass spectrum of baryons, the axial constant of β -decay, magnetic moments of baryons, pion-baryons coupling constants and so on, provide the basic for the hope that the version of CBM considered correctly reproduces main features of interactions of pions in the low energy range. The chiral (pionic) corrections to the results of the MIT bag model as a rule improve the agreement between the theoretical predictions and experimental data.

Since pions interact with the surface of the quark bag, one can hope that the detailed investigation of interactions of pions with nucleons and few-nucleon systems would allow to study the sizes and shapes of quark bags in baryons.

In the framework of the CBM without gluonic corrections the axial constant of β -decay and correspondingly pion-nucleon coupling constant (they are interrelated through the Goldberger - Treiman relation) are 20% larger than the experimental

values. It turns out of importance to take into account the gluonic corrections to the quark wave functions that improves the agreement with the experimental data. The effects of such kind are under detailed investigation and the results will be reported in a separate investigation.

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R E F E R E N C E S

1. D.V.Shirkov, V.V.Serebryakov, V.A.Mecheryakov. Dispersion theory of a strong interaction at low energy, "Nauka", Moscow, 1967.
2. V.De Alfaro, S.Fubini, G.Furlan, C.Rossetti, Currents in Hadron Physics, North Holland, New York, 1973.
3. T.A.De Grand, R.L.Jaffe, K.Johnson, Y.Riskis, Phys. Rev. D12, 2060, 1975.
4. I.Y.Kobzarev, B.V.Martemyanov, M.G.Shchepkin, Yadernaja Fisica 29, 1620, 1979.
5. I.Y.Kobzarev, B.V.Martemyanov, M.G.Shchepkin, Yadernaja Fisica 30, 504, 1979.
6. P.N.Bogolubov, Ann.Inst. Henri Poincare 8, 163, 1968.
7. A.Chodos, C.B.Thorn, Phys.Rev. D12, 2733, 1975.
8. T.Inoue, T.Maskawa Prog.Theor.Phys. 54, 1833, 1975.
9. M.Shifman, A.Vainshtein, V.Zakharov, Phys.Rev.Lett. 42, 297, 1979.
10. C.G.Callan, R.F.Dashen, G.J.Gross, Phys.Rev. D19, 1826, 1979.
11. R.L.Jaffe MIT report MIT CTP 814, 1979.
12. V.Vento, M.Rho, E.M.Nyman, J.H.Jun, G.E.Brown, Preprint Saclay DPh - T(80) 33, 1980.
13. M.M.Musakhanov Inv. contr. 9 Intern.Conf. Few Body Problem Oregon, 1980.
14. M.M.Musakhanov, Preprint ITF-80-53 P, Kiev, 1980.
15. M.M.Musakhanov, Preprint FTI 3-80-FVE, Tashkent, 1980.
16. M.V.Barnhill, W.K.Cheng, A.Halprin, Phys.Rev., D20, 727, 1979.
17. M.V.Barnhill, A.Halprin, Phys.Rev, D21, 1916, 1980.

18. G.E.Brown, M.Rho, Phys.Lett. 82B, 177, 1979.
19. G.E.Brown, M.Rho, M.Vento, Phys.Lett. 84B, 383, 1979.
20. R.L.Jaffe, Phys.Rev. D21, 3215, 1980.
21. G.E.Brown, M.Rho, V.Vento, Preprint Saclay DPh-T(80) II6, 1980.
22. M.Rho Inv. Talk Int.Conf.Nucl.Phys., Berkeley, USA, 1980.
23. A.Szymacha, S.Tatur Preprint IFT(6)80, Warszawa, 1980.
24. Y.Bartelski, A.Szymacha, S.Tatur Preprint IFT(16) 80, Warszawa, 1980.
25. G.A.Miller, A.W.Thomas, S.Théberge Preprint TRI-PP-79-16, TRIUMF, 1979.
26. M.M.Musakhanov, Yadernaia Fisica 19, 630, 1974.
27. M.Arman et al Phys.Rev.Lett. 29, 962, 1972.
28. R.Settles et al Preprint MPI-PAE/Exp. E1. 78, Munchen, 1979.
29. G.Bunche et al Bull. Am.Phys.Soc. 24, 46, 1979.
30. A.B.Govorkov Preprint JINR P2-I2803, Dubna, 1979.

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