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HOW IS THE CHARMONIUM SPLITTING IN QCD?
PREDICTIONS FROM EXPONENTIAL MOMENTS AS LIMIT OF POWER MOMENTS

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Abstract

Using the SVZ moment procedure to predict resonance masses within QCD we have calculated exponential moments as a limit of the QCD formulae given by Reinders, Rubinstein and Yazaki. Applied to charmonium we reproduce their results (besides $^3P_0$) very well.

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Shifman, Vainshtein and Zakharov[1),2)] (SVZ hereafter) have introduced an original method, the moment procedure, to dicuss resonance properties within QCD. In introducing power moments, derivatives of the vacuum polarization function $\pi(Q^2)$

$$M_n(Q^2) = \frac{1}{n!} \left(-\frac{d}{dQ^2}\right)^n \pi(Q^2) = \frac{1}{\pi} \int \frac{\mathrm{Im}\,\pi(s)\, ds}{(s+Q^2)^{n+1}} , \qquad (1)$$

the could relate the physical Im $\pi(s)$, the resonance structure to the vacuum properties of QCD. Their novel idea in proceeding from short to long distances was to include nonperturbative terms, gluonic vacuum fluctuations. Applied to charmonium SVZ have estimated (at $Q^2 = 0$) the charmed quarkmass m_c, the strong coupling constant α_s and the gluon condensate $\langle\frac{\alpha_s}{\pi} GG\rangle$.

In a series of papers Reinders, Rubinstein and Yazaki[3),4),5)] (RRY hereafter) generalized the calculations of SVZ to several types of heavy quark currents at arbitrary Q^2. In fitting a few parameters only the lowest lying charmonium states have been reproduced extremely well.

Whereas SVZ and RRY use the power moments to analyze heavy quark states we work with exponential moments. They appear to be an improvement within a nonrelativistic approach[6)]. In this paper we want to present results given by a limit of the QCD formulae of RRY[5)] which provides the exponential moments.

According to SVZ the power moments are calculated in the following way

$$M_n(Q^2) = A_n(Q^2) \, [1 + \alpha_s a_n(Q^2) + \Phi \, b_n(Q^2)] , \qquad (2)$$

where

A_n is the free quark contribution,

$\alpha_s a_n$ represents the perturbative gluon corrections,

$\Phi \, b_n$ originates from the nonperturbative gluonic vacuum fluctuations.

The gluon condensate parameter Φ is conventionally defined by

$$\Phi = \Phi_1/(4m^2)^2 \qquad (3)$$

with

$$\Phi_1 = \frac{4\pi^2}{9} \langle \frac{\alpha_s}{\pi} GG \rangle \quad . \tag{4}$$

We regard Φ_1 to be the flavour independent (mass normalization independent) quantity (see Ref. 1).

In an elaborate work RRY have calculated the functions A_n, a_n, b_n for a great variety of currents, for the pseudoscalar $^1S_0$ and scalar $^3P_0$, for the vector $^3S_1$, axial vector $^3P_1$ and $^1P_1$, and for the tensor current $^3P_2$. On their formulae listed on Table 1 of Ref. 5 we are going to rely on.

We take now the limit

$$\begin{aligned} n &\to \infty \\ Q^2 &\to \infty \end{aligned} \tag{5}$$

with

$$\frac{n}{Q^2} = \sigma \quad \text{fixed} ,$$

$$\lim\, (Q^2)^{n+1}\, \pi\, M_n(Q^2) = M(\sigma) , \tag{6}$$

and obtain the exponential moments

$$M(\sigma) = \int ds\, e^{-\sigma s}\, \mathrm{Im}\, \pi(s) \;. \tag{7}$$

Splitting the moments into

$$M(\sigma) = e^{-4m^2\sigma}\, \pi\, A(\sigma)\, [1 + \alpha_s\, a(\sigma) + \Phi\, b(\sigma)] \tag{8}$$

we can calculate for above currents the free quark contribution $\pi A(\sigma)$, the perturbative $\alpha_s a(\sigma)$ and nonperturbative correction $\Phi b(\sigma)$ from the RRY formulae by taking the above limit. For the $^3S_1$ current this has been demonstrated explicitly by J.S. Bell and the author[7]. For all the other currents the calculations proceed very similar, but are somewhat lengthy and will be presented in detail somewhere else[8].

To obtain the mass of the ground state we apply the logarithmic derivative to $M(\sigma)$ yielding a ratio of moments[9]

$$R(\sigma) = -\frac{d}{d\sigma}\log M(\sigma)\ . \tag{9}$$

Whereas the exact ratio in the limit

$$R(\sigma) \xrightarrow[\sigma\to\infty]{} M^2 \tag{10}$$

just approaches the mass of the ground state, we regard the minimum of the approximated $R(\sigma)$

$$\min_{\sigma} R(\sigma) = M^2 \tag{11}$$

to be an approximation to the ground state.

In fact we now perturb this ratio

$$R(\sigma) = F(\sigma)\ [1 + \alpha_s\ P(\sigma) + \Phi\ Q(\sigma)]\ , \tag{12}$$

where

$$F(\sigma) = 4m^2 - \frac{\pi A'(\sigma)}{\pi A(\sigma)}$$

$$P(\sigma) = -\frac{a'(\sigma)}{F(\sigma)} \tag{13}$$

$$Q(\sigma) = -\frac{b'(\sigma)}{F(\sigma)}\ .$$

We argue that the ratio $R(\sigma)$ is the more stable quantity. Whereas the corrections $\alpha_s a(\sigma)$, $\Phi b(\sigma)$ blow up for increasing σ, their derivatives $-\alpha_s a'(\sigma)$, $-\Phi b'(\sigma)$ stay reasonably small.

In order to lower the gluonic corrections SVZ and RRY have introduced an off shell mass

$$m = m(p^2 = -m^2) \tag{14}$$

providing in $\alpha_s a_n$ an extra term proportional to

$$\alpha_s \frac{4 \ln 2}{\pi} \tag{15}$$

which reduces the net correction in M_n.

As we already discussed in Refs. 7 and 9 this is not the case in the ratio $R(\sigma)$. There the mass correction (15) just adds to the quarkmass (14). Therefore we work with an on shell mass

$$\bar{m} = m(p^2 = m^2) \tag{16}$$

which is related to the euclidean mass by

$$\bar{m}^2 = m^2(1 + \alpha_s \frac{4 \ln 2}{\pi}) \ . \tag{17}$$

Now, in order to compare our results for charmonium very directly with the ones of RRY or SVZ we have calculated both versions:

i) using an off shell mass m and retaining terms proportional to (15) in the corrections $\alpha_s a(\sigma)$ and $- \alpha_s a'(\sigma)$,

ii) using an on shell mass $\bar{m}$ and dropping terms proportional to (15).

The results of both calculations differ only by a few MeV!

Before we present our results we want to mention an important feature of RRY's approach in comparison to ours. RRY have chosen Q^2 quite differently for s- and p-states thus yielding a significant change in m_c and α_s when going from s- to p-states. In our approach, however, by using exponential moments as limit of power moments the Q^2 dependence disappeared and quarkmass and coupling constant remain fixed in moving from s- to p-states. For the parameters m_c (or $\bar{m}_c$), α_s, Φ_1 we did not try to find a best fit. For ease of comparison we used quarkmasses and coupling constants as given by SVZ[1], RRY[5] and Miller-Olsson[10]. The condensate parameter Φ_1 was fixed to yield the correct J/ψ ($^3S_1$). Again, as Bell and myself conjectured before (Refs. 6,7,9) Φ_1 is larger than 0.05 GeV^4. This is now confirmed by Bradley, Langensiepen and Shaw[11], and by Miller and Olsson[10].

We present our results for several parameter sets on Table 1. The best set we have collected in column 1. These are parameter values advocated by RRY[5] for $Q^2 = 0$. We reproduce their results for charmonium splitting very well, except for the scalar $^3P_0$ which lies about 60 MeV too high. For parameters of SVZ[1] (column 4), and of Miller-Olsson[10] the p-states emerge too high. The reason may be that we cannot vary m_c and α_s in moving from s- to p-states. We regard the first parameter set of Table 1 as significant to produce a charmonium splitting in agreement with experiment.

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Table Caption

Table 1 Charmonium results from the exponential moment ratio $R(\sigma)$, equ. (12), for several parameter sets. In the first 3 columns we list results for α_s and m_c values as used by RRY[5], in the fourth column for values claimed by SVZ[1], and in the fifth column for values given by Miller and Olsson[10]. For comparison we collect the predictions of RRY[5] in column 6 and experimental data[12] in column 7.

Table 1

all mass dimensions in GeV

α_s	0.3	0.27	0.27	0.2	0.2		
m_c	1.28	1.28	1.26	1.26	1.24	RRY[5]	experiment[12]
$\bar{m}_c$	1.44	1.42	1.40	1.37	1.34		
Φ_1	0.07	0.08	0.11	0.14	0.19		
$^3S_1$	3.10	3.10	3.10	3.10	3.10	3.10±.01	3.10
$^1S_0$	3.04	3.03	3.02	3.00	2.98	3.01±.02	2.98
$^3P_0$	3.48	3.49	3.54	3.58	3.63	3.40±.01	3.41
$^3P_1$	3.52	3.53	3.59	3.63	3.68	3.50±.01	3.51
$^3P_2$	3.56	3.57	3.64	3.68	3.75	3.56±.01	3.55
$^1P_1$	3.51	3.52	3.57	3.61	3.65	3.51±.01	