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ON THE SOLID STRESS IN A FLUIDIZED BED

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ABSTRACT:

The existence of solid stress in an incipiently gas-fluidized bed is shown by experimental measurement. This stress is shown to have two components: an isotropic pressure and an extra stress which depends on the relative velocity between fluid and solid.

Both the solid pressure and the solid extra stress component are found to be of the same order of magnitude as the fluid pressure. (author).

1. INTRODUCTION

In the Davidson theory (1) for the motion of the solid and fluid constituents in the particulate phase of a fluidized bed, a term for solid pressure appears in the solid momentum equation. Not only is there no a priori reason to ignore this term, but also its disappearance from the equations of motion would leave the system overdetermined. Furthermore, the theoretical solutions of Davidson for bubble motion in a fluidized bed, show clearly that the solid pressure is by no means zero or negligible everywhere in the flow field.

There is little direct experimental evidence for the magnitude of the solid pressure or whether it is just a pressure or a more complicated quantity. Measurements have been reported (2, 3) for freely bubbling fluidized beds; however, the presence of numerous bubbles in the bed tends to throw some doubt on the interpretation of these results.

In this work, an attempt is made to answer two questions concerning the solid stress in an incipiently fluidized bed: a) What magnitude does the solid stress have and b) is the solid stress dependent on the relative velocity between fluid and solid? We do this by measuring both the total and the fluid stresses in directions parallel and perpendicular to the relative velocity.

2. THEORY

Considering the particulate phase as a binary mixture of two interacting incompressible continuous media, the equation of mass and linear momentum balance may be written as

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\epsilon \underline{v}_f) = 0 \quad , \quad (1)$$

$$-\frac{\partial \epsilon}{\partial t} + \nabla \cdot [(1 - \epsilon)\underline{v}_s] = 0 \quad , \quad (2)$$

$$\rho_f \epsilon \left(\frac{\partial \underline{v}_f}{\partial t} + \nabla \underline{v}_f \cdot \underline{v}_f \right) = \nabla \cdot \underline{T}_f - \underline{m} + \rho_f \epsilon \underline{g} \quad , \quad (3)$$

$$\rho_s (1 - \epsilon) \left(\frac{\partial \underline{v}_s}{\partial t} + \nabla \underline{v}_s \cdot \underline{v}_s \right) = \nabla \cdot \underline{T}_s + \underline{m} + \rho_s (1 - \epsilon) \underline{g} \quad . \quad (4)$$

These equations can be reduced to Davidson's equation by assuming that the voidage is uniform and is at its incipient fluidization value, negligible fluid inertia and external body force, solid potential motion, linear interaction force in the relative velocity and that the fluid stress tensor consists of a pressure component only. We then obtain

$$\nabla^2 p_f = 0 \quad , \quad (5)$$

$$\nabla^2 \phi_s = 0 \quad , \quad (6)$$

and the solid stress tensor is determined from the solid momentum equation. Obviously, if we were to put $\underline{T}_s = 0$, then equation (4) would not necessarily be satisfied; i.e., the system (1) - (4) be-

comes over-determined. If instead, we write

$$\underline{T}_s = -p_s \underline{1} \quad (7)$$

then equation (4) could be used to determine p_s . This shows the theoretical necessity for not ignoring the solid stress term altogether in the solid momentum equation.

Since p_s is not relative velocity-dependent, the question arises whether the solid stress contains an additional component which depends on $(\underline{v}_f - \underline{v}_s)$. The possibility of this is shown if we recognize that equation (7) is a special case of a more general form

$$\begin{aligned} \underline{T}_s = & -p_s \underline{1} + \alpha_s (||\underline{v}_f - \underline{v}_s||) \underline{1} + \\ & + \left[\beta_s (||\underline{v}_f - \underline{v}_s||) \right] \left[(\underline{v}_f - \underline{v}_s) \otimes (\underline{v}_f - \underline{v}_s) \right]. \end{aligned} \quad (8)$$

As p_s can be redefined to incorporate the function α_s , we shall not be concerned with the latter. The question of interest is whether the β_s term is significant in an incipiently fluidized bed. We note that Silve Telles and Fernandes (4) determined this type of extra stress for Non-Newtonian fluid flow through a porous media.

3. EXPERIMENTAL

The experiment set-up consists of a perspex bed through which air is metered by rotameters. All experiments performed at

room temperature and pressure. Sand was used as the bed material. Its characteristics are summarized in Table I.

TABLE I - SAND MATERIAL PROPERTIES

Density (g/cm ³)	2.65
Mean Particle Size (μm)	512
Incipient Fluidization Velocity (cm/s)	13.47

In order to measure the total stress in the direction parallel to the relative velocity, the weight of a thin cylinder immersed in the bed is determined by suspending it from one arm of a pan balance. The total stress in the direction perpendicular to the relative velocity was measured by a strain gauge aligned parallel to the flow. The perturbing effect of the cylinder on the flow was evaluated by inserting an identical hollow cylinder with its lower end covered by a porous membrane and whose interior is connected to a micromanometer for fluid pressure measurement. Similarly, a porous disk of the same size and geometry as the strain gauge probe was used to determine the fluid pressure. These two values of the fluid pressure showed little difference from the value determined at the wall fluid pressure tap, thus showing that the cylinder and strain gauge probe did not disturb the flow in the bed to any significant extent.

4. RESULTS

The solid stress values parallel and perpendicular to the flow are determined by subtraction of the fluid pressure from the

total stress in these directions respectively; i.e.,

$$(T_s)_{xx} = (T_T)_{yy} - P_f \quad (9)$$

$$(T_s)_{yy} = (T_T)_{yy} - P_f \quad (10)$$

Further, upon using equation (8), we have

$$(T_s)_{xx} - (T_s)_{yy} = B_2 (u) u^2 \quad (11)$$

The values of fluid pressure and solid stress parallel and perpendicular to the flow are shown in Table II at two bed heights, 22.5 cm and 17.5 cm.

TABLE II - FLUID PRESSURE AND SOLID STRESS

Height (cm)	P_f (cm H ₂ O)	$(T_s)_{xx}$ (cm H ₂ O)	$(T_s)_{yy}$ (cm H ₂ O)
22.5	3.00	0.02	0.51
17.5	16.60	0.17	1.79

The variation of the extra solid stress values with height can only be due to changes in the flow conditions throughout the bed. Local voidage measurements using γ -ray attenuation has shown that there was such a variation, from 0.341 at 22.5 cm to 0.330 at 17.5 cm.

5. CONCLUSIONS

The following conclusions have been reached:

(1) Solid stresses in an incipiently fluidized bed are by no means negligible with respect to fluid pressure. In particular, the solid stress component parallel to the flow is of the same order of magnitude as the fluid pressure.

(2) There exists difference between the solid stress components parallel and perpendicular to the flow. This difference is also of the same order of magnitude as the fluid pressure. This fact suggests the existence of an extra solid stress, which is dependent on the relative velocity.

We suggest that the variation of the extra solid stress with bed height should be examined by means of more experiments in which local measurements are performed. Also, in order to determine the function β_s , measurements have to be made at velocities less than the incipient value. In such a case, it is necessary to devise another method for the determination of the total stress parallel to the flow, since the weighing technique fails, for obvious reasons, in the fixed bed range.

NOTATION

Symbols

ϵ - voidage

\underline{v} - velocity vector

ρ - density

\underline{T} - stress tensor

\underline{m} - fluid-solid interaction force vector

\underline{g} - gravitational acceleration

- p - pressure
- ϕ - velocity potential
- I - identity tensor
- α, β - material constitutive functions
- u - incipient fluidization velocity

Subscripts

- f - fluid
- s - solid
- T - total
- x - direction parallel to flow
- y - direction perpendicular to flow

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