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A POSSIBLE REALIZATION OF EINSTEIN'S CAUSAL THEORY  
UNDERLYING QUANTUM MECHANICS \*

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ABSTRACT

It is shown that a new microscopic mechanics formulated earlier can be looked upon as a possible causal theory underlying quantum mechanics, which removes Einstein's famous objections against quantum theory. This approach is free from objections raised against Bohm's hidden variable theory and leads to a clear physical picture in terms of familiar concepts, if self interactions are held responsible for deviations from classical behaviour. The new level of physics unfolded by this approach may reveal novel frontiers in high-energy physics.

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I. INTRODUCTION

Theoretical physics is now confronted with a degree of confusion that is worse than a similar situation prevailing half a century ago before the formulation of quantum theory. All the novel physical hypotheses and mathematical techniques within classical theory had failed at that time because phenomena at the atomic level required a new level of physics, namely the quantum theory. Similarly, the current situation in high-energy physics and the vast amount of data perhaps indicate that yet another level of physics is needed in the theoretical front. This belief is supported by the failure of numerous attempts within the quantum picture incorporating strange postulates and sophisticated mathematical techniques. It is fair to say that phenomenology has achieved the most in the realm of particle physics. This has produced pessimists who think that perhaps one should give up the quest for fundamental particles and interactions.

The purpose of the present work is to provoke the optimists who are in search of a unified picture in theoretical physics. This is done by digging up some of the unanswered questions of the past and showing that a possibility exists for answering them within a framework of new but not too unfamiliar concepts. One of the most famous of such questions was raised by Einstein et al.<sup>1)</sup> regarding the quantum-mechanical description of the physical reality. It was implied therein that if microscopic systems can be governed by a deterministic theory, only then can one claim to have a satisfactory physical theory. Pursuing this goal, Bohm et al.<sup>2)</sup> tried to formulate a causal interpretation of quantum theory which is now well known as the theory of hidden variables. They postulated that an electron is a particle with well-defined trajectory accompanied by a physically real wave field  $\psi(\vec{r}, t)$ . Then they assumed that i)  $\psi(\vec{r}, t)$  satisfies Schrödinger's equation, ii) the momentum of the particle is proportional to the gradient of the phase of  $\psi(\vec{r}, t)$  and iii) the probability distribution of an ensemble of electrons with the same  $\psi(\vec{r}, t)$  is  $|\psi|^2$ . This last assumption was severely criticized as baseless by Pauli and others<sup>3)</sup>. Other attempts at a re-interpretation of quantum theory are too numerous to be cited here.

In a recent work<sup>4)</sup> (hereafter referred to as I), it has been shown that it is possible to have a deterministic mechanics that governs the microscopic systems and the average results of measurements on ensembles of such systems are described exactly by quantum mechanics. This work has a degree of similarity with Bohm's work in that there is an extra force on the moving electron but the physical pictures are widely different. Bohm had postulated a fluid with

irregular fluctuations that surround the electron and the model was essentially hydrodynamic. In addition, his three assumptions, described above, had to be made in connection with the resulting interpretation. On the other hand, in I, one derives all three assumptions of Bohm from the construction of  $\psi(\vec{r}, t)$  and thus the approach is free from the most serious objections of Pauli and others against Bohm's theory. Thus, the work reported in I can be looked upon as a possible realization of what Einstein was seeking as the physical reality in the microscopic systems.

In the following section, a description is given of this new mechanics which can possibly govern microscopic systems such that the statistical behaviour of an ensemble of such systems under the disturbance inseparably associated with measurements is correctly described by quantum theory. Thus the new physical picture does not contradict the quantum-mechanical predictions and goes over to the classical picture in the proper limit. The only assumption involved is a model expression for the radiation reaction force.

In the last section, a discussion is given of the new level of physics that is unfolded through such an approach. It gives rise to the possibility of examining afresh the bound states of stable fundamental particles using known fundamental interactions before making additional postulates. But a lot of formal development of this approach will be needed before handling such problems. The mathematical basis for this approach appears to be the relationship between linear differential equations and non-linear equations that have stable solutions (like solitons). One good reason for pursuing this approach is the simplicity of the physical picture where essentially a damping force appears in Newton's equation (see I). Thus, in this picture, many fundamental concepts of mechanics, electrodynamics and relativity may find a unified interpretation because the damping force is assumed to originate from self-interactions.

## II. MICROSCOPIC MECHANICS

The statement that every real particle in nature will be surrounded by its own field, may not belong to the category of postulates because a totally non-interacting particle is completely unobservable. Then there are self-interactions which give rise to renormalizations in quantum field theory and damping forces on the particle in classical electrodynamics. The equation of motion of an electron of mass  $m$ , momentum  $\vec{p}$ , with a self-damping force  $\vec{f}$  and an external force  $\vec{F}_{ext}$  may be written as

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} - \vec{f} \quad (1)$$

The sum of kinetic and potential energies of the particle will not be conserved, but if  $E_r$ , the energy lost due to radiation is added to the sum, the total conserved energy  $E$  is given by

$$E = \frac{p^2}{2m} + V(\vec{r}) + E_r \quad (2)$$

where  $V(\vec{r})$  is the potential energy due to the external force  $\vec{F}_{ext}$ .

The exact form of the radiation reaction or self-damping force  $\vec{f}$  is one of the unresolved problems <sup>5)</sup> of classical electrodynamics and none of the existing forms are completely satisfactory. The most reliable form is the Abraham-Lorentz force, which corresponds to the Larmor formula for radiated energy. But apart from other problems, it is incorrect when a charged particle starts from rest because initially it would give radiated energy far in excess of energy gained by the particle. Now, in I, a model expression for  $f$  is chosen that goes over to the Abraham-Lorentz form in the proper limit and may serve as a possible model expression for the radiation reaction force in non-relativistic limit. It must be emphasized that there is no claim here as to the model expression being the exact expression for the radiation reaction force but the model expression is the single crucial assumption for connecting the resulting mechanics with the Schrödinger theory.

For motions confined to take place in one dimension, one has

$$f = \left(1 - \frac{p^2}{m^2 c^2}\right)^{1/2} \left(\frac{m^2 \hbar^2}{8p}\right) \frac{d}{dt} \left[ \frac{5 \left(\frac{dp}{dt}\right)^2}{p^4} - \frac{2 \left(\frac{d^2 p}{dt^2}\right)^2}{p^3} \right] \quad (3)$$

Setting the relativistic factor to unity for non-relativistic motions, the equation of motion integrates out to give

$$E = \frac{p^2}{2m} + V(x) + \frac{\hbar^2}{2m} \left[ \frac{d^2 A}{dx^2} - \left(\frac{dA}{dx}\right)^2 \right] \quad (4)$$

with

$$\frac{dA}{dx} = \frac{dp}{dx} / (2p) \quad (5)$$

Define  $p = \frac{dW}{dx}$  and

$$\psi = L^{-1/2} \exp[-A(x) + iW(x)/\hbar] \quad (6)$$

where  $L$  is a constant. Then one gets the corresponding Schrödinger equation

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)]\psi = 0. \quad (7)$$

Note that the same  $E$  and  $V(x)$  appear in Eq.(2) and Eq.(7). Also integrating Eq.(5) and using Eq.(6), one gets  $\psi^* \psi$  proportional to  $p^{-1}$  whence it is the probability density. Thus, the three assumptions of Bohm et al. regarding  $\psi$  are actually results in the present framework.

The model  $f$  is negligible not only for  $\hbar \rightarrow 0$  but also for large mass and large momenta; and then one has Newtonian mechanics. However, Eq.(1) admits of a new class of bound states not present in Newtonian mechanics where  $\vec{F}_{\text{ext}}$  equals the damping force in magnitude. This happens for  $p \rightarrow 0$  together with all its derivatives and it is the regime where classical electrodynamics has no satisfactory expression for the damping force. Now the new class of bound states have discrete energy levels that are also given by the Schrödinger equation. Even the  $\psi$  derived from the new equation of motion through Eq.(5) and Eq.(6) agrees, as it should <sup>6)</sup>, with the  $\psi$  from the Schrödinger equation.

There are other interesting aspects of this approach. One can try to do a Hamilton-Jacobi type of theory and end up with an extended Hamilton-Jacobi equation, which is Eq.(4). This couples (through  $p = \frac{dW}{dx}$ ) the phase and amplitude of  $\psi$ , which get decoupled in the eikonal approximation. Also the connection between Eqs.(2) and (7) is a connection between the non-linear equation and a linear partial differential equation on the mathematical level. The Hamilton-Jacobi equation is a manifestation of a relationship <sup>7)</sup> of this kind at the level of first-order differential equations. Here it is a more involved connection of the same kind with a second-order differential equation, which happens to be the Schrödinger equation in the present case.

Thus the microscopic mechanics postulated here is a deterministic theory which can govern individual microscopic systems. When measurements are made on ensembles of such microscopic systems and the process of measurements disturbs the system in such a way that the disturbance cannot be accounted for in isolation, then one gets the results predicted by quantum theory. The uncertainty principle concerns this process of measurement and remains valid. But if an isolated system free from disturbances including those due to measurements is envisaged, then the Einstein criterion of physical reality applies to each microscopic system.

### III. DISCUSSION

The present approach has the virtue of simplicity in unfolding a new level of physics using rather familiar concepts. One can use them to answer many unanswered (or not to be answered) questions of the Schrödinger theory. For example, duality is there because both a particle and its surrounding (wave) fields are present. For negligible  $f$ , particle behaviour dominates, and otherwise wave behaviour due to the associated fields dominates, but both cannot appear at the same time. Similarly, during transition from one non-radiating discrete state (with damping force equal to external force) to another, i.e. the quantum jump, the electron undergoes acceleration and emits radiation. A photon is not "born" but comes out of the existing electromagnetic field.

The simplicity of the present approach is revealed also through the picture it provides for the success and inadequacy of quantum theory. Classical physics worked when we either neglected or used crude estimates for self-interactions. It did not work in atomic systems where the radiation reaction effects are large and more energy, angular momentum, etc. resided in wave fields surrounding the particles rather than particles themselves. The next step in this story is to realize that complicated non-linear equations were to be solved in conjunction with an account of the disturbance due to measurements to get the experimental results. Even the existence of discrete energy states was to be obtained through a similar process, but by chance the Schrödinger equation appeared, which was nothing but the second-order partial differential equation corresponding to the non-linear equation of motion including proper radiation effects. It also contained the probability amplitude for measurements on the ensemble of microscopic systems.

According to the present approach, the Schrödinger theory has correctly predicted, as it should, the statistical results of measurements. But to proceed further, one could examine the equation of motion (Eq.(1)) for inclusion of all the external forces and all the self forces. The result may be a complicated equation <sup>6)</sup> but the physical picture is rather familiar as compared with the case of mysterious physical postulates within the framework of the quantum picture. In the present approach, quantum mechanics has a role similar (but not the same) to the role of statistical mechanics in classical theory. Therefore one cannot hope to discover fundamental interactions within the statistical theory of measurements which is quantum mechanics, just as in the classical theory, statistical mechanics cannot yield the equations of motion.

Finally, it is necessary to comment on the status of the present approach at the present stage. Assuming the validity of the model expression for the damping force, one gets the microscopic mechanics which properly covers both the classical and quantum mechanics. The next stage of investigation should show the origin of the model expression and, as indicated in I, there are reasons to believe that an extended electron model with a (relativistically) variable size may correspond to the model expression. This may have many other implications which are yet to be investigated. For example, an extended electron inside a wave field which has angular momentum might "spin" in a balanced state (discrete values of spin). Another implication of such a model for the electron may be a rational explanation of the appearance of advanced solutions in classical electrodynamics for distances of the order of classical electron radius. It is hoped that such investigations might yield relationships between charge, mass, velocity of light and Planck's constant and hence tie together the basic concepts of mechanics, relativity and electrodynamics.

#### ACKNOWLEDGMENTS

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