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QUANTUM GRAVITY REMOVES CLASSICAL SINGULARITIES AND SHORTENS THE LIFE OF BLACK HOLES

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INTERNATIONAL ATOMIC ENERGY AGENCY

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International Atomic Energy Agency
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QUANTUM GRAVITY REMOVES CLASSICAL SINGULARITIES AND SHORTENS THE LIFE OF BLACK HOLES *

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ABSTRACT

The problem of the gravitational collapse is considered in the framework of the quantum gravity effective action. It is shown that quantum gravity removes classical singularity and possibly shortens the lifetime of the black hole.

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1. Introduction

One of the fundamental problems in classical General Relativity is what should be done with singularities which inevitably arise in the theoretical description of the massive body (or total Universe) collapse. Although the singularities arising as a result of the gravitational collapse are believed to be hidden under event horizons and thus are not visible to an external observer, their very existence means the crisis of the classical gravitational physics. It is generally believed that the proper account of quantum effects may cure this desease.

The aim of the present work is to show that it really happens, and quantum gravity does remove classical singularities.

2. The effective Lagrangian

The revision of the problem of classical singularities is one of the final goals of quantum gravity, but the genuine achievement of this goal requires the "completion" of the quantum gravitational theory.

Indeed, to decide on the problem of singularities one needs the asymptotic behaviour of the effective gravitational Lagrangian *) at small distances. However, because of the dimensionality of the gravitational coupling constant, each order of perturbation theory is larger at small distances than the previous order. Therefore in order to obtain the small-distance behaviour one must sum up the perturbation series. This task being

*) The effective Lagrangian generates equations of motion for the vacuum average of a quantized field in the presence of external sources.

diff... itself requires the solution of another problem first - the problem of ultraviolet divergences. Neither of these problems has yet been solved.

Nevertheless, as we explain below, there is a possibility of arriving at definite conclusions even at the present state of knowledge.

Let us consider the effective Lagrangian of the gravitational field, arising at the one-loop level.*) It is of the following structure:

$$\frac{\mathcal{L}_{eff}}{\sqrt{-g}} = -R + (\ln\Lambda^2)\left[a R_{\mu\nu}R^{\mu\nu} + b R^2 + c \overset{*}{R}_{\alpha\beta\gamma\delta}\overset{*}{R}{}^{\alpha\beta\gamma\delta}\right] + const\cdot R_{\mu\nu}R^{\mu\nu} + const\cdot R^2 + const\cdot \overset{*}{R}_{\alpha\beta\gamma\delta}\overset{*}{R}{}^{\alpha\beta\gamma\delta} + R\ldots\frac{1}{\Box}R^2\ldots + R\ldots\frac{1}{\Box}R\ldots\frac{1}{\Box}R^2\ldots + \ldots - \left[a R_{\mu\nu}(\ln\Box)R^{\mu\nu} + b R(\ln\Box)R + c\overset{*}{R}_{\alpha\beta\gamma\delta}(\ln\Box)\overset{*}{R}{}^{\alpha\beta\gamma\delta}\right] + O(\hbar^2), \tag{1}$$

where $R_{\ldots\ldots}$ and $\overset{*}{R}_{\ldots\ldots}$ denote the Riemann and dual Riemann tensors, and Λ is the regularization parameter.

*) If particles are created, there are two inequivalent vacua: |in, vac> and |out, vac> [1]. The effective Lagrangian is the Lagrangian of the real macroscopic field: <in, vac| g |in,vac> [2]. As shown in Ref. 2, this Lagrangian describes both the vacuum polarization and the back-reaction of created particles on the metric.

The first line of expression (1) is the tree Lagrangian. The second line is the logarithmically divergent part of the one-loop contribution. The next two lines symbolize all possible finite terms (both local and non-local) containing four derivatives. Finally, the last line contains terms generating the so-called non-local trace anomalies [3]. Since there are logarithmic divergences, the presence of the latter terms is required simply by dimension.

The effective Lagrangian cannot be computed exactly even at the one-loop level, but a, b and c are calculable constants. The behaviour of the field at small distances is determined by terms containing the maximal number of derivatives. Therefore the leading part of the Lagrangian (1) is that containing $\ln\Box$, and the coefficients entering this part are known:

$$\frac{\mathcal{L}_{eff}}{\sqrt{-g}} = -\left[a R_{\mu\nu}(\ln\Box)R^{\mu\nu} + b R(\ln\Box)R + c\overset{*}{R}_{\alpha\beta\gamma\delta}(\ln\Box)\overset{*}{R}{}^{\alpha\beta\gamma\delta}\right] + O(\nabla^4) + O(\hbar^2); \tag{2}$$

$O(\nabla^4)$ denotes the omitted weaker terms, containing exactly four derivatives.*)

*) When deriving the field equations from the Lagrangian (2), the variations $\delta(\ln\Box)\delta g_{\mu\nu}$ can be omitted, because they lead to terms of order $O(\nabla^4)$. Since $\overset{*}{R}_{\ldots}\overset{*}{R}{}^{\ldots}\sqrt{-g}$ is the total derivative, the term $\overset{*}{R}_{\ldots}(\ln\Box)\overset{*}{R}{}^{\ldots}$ of (2) will not contribute to the linear part of field equations. If non-linearities are not essential (as is the case), this term can be omitted altogether.

It is important, that the leading terms (2) do not depend on the renormalization (or subtraction) arbitrariness. (This arbitrariness concerns terms on the second line of Eq. (1).)

It was shown in Ref. 4 that the radiative corrections in the source-free Einstein gravity theory can be computed in a manifestly conformally invariant way. The one-loop effective Lagrangian found in this way reads (only leading terms are retained):

$$\frac{\mathcal{L}^{conf}_{eff}(x)}{\sqrt{-g}} = -\frac{a}{2} C_{\alpha\beta\gamma\delta} \ln(\Box+\dots) C^{\alpha\beta\gamma\delta}(x) + \frac{a}{2} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta}(x) \ln \Phi^2(g|x) + O(\nabla^4) + O(\hbar^2), \tag{3}$$

where $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor, and $\Phi(g|x)$ is the following non-local functional of $g_{\mu\nu}$:

$$\Phi(g|x) = 1 - \frac{1}{6}\left(\Box + \frac{1}{6}R\right)^{-1} \cdot R. \tag{4}$$

The dots denote curvature additions (possibly non-local) making the operator $(\Box+\dots)$ conformally covariant [3]. These curvature additions are not known, but their particular form is irrelevant for small distances, since their contribution can be included in $O(\nabla^4)$. The second term of Eq. (3) cancels the trace anomaly produced by the first term [4].

According to Ref. 4, the effective Lagrangians (2) and (3)

generate one and the same graviton S- matrix. These Lagrangians are equivalent also off the mass shell if the sources of the gravitational field (including external ones) are conformally invariant.

In the present work we shall consider mainly the traceless sources. Therefore we may adopt the Lagrangian (3). This will give us invaluable technical simplifications.

The defect of the Lagrangian (3) (or (2)) is the term $O(\hbar^2)$. However, there are indications [5,6], that if the gravity theory is renormalizable *) in any thinkable sense, then the one-loop result (3) correctly reproduces the true ultraviolet asymptotic behaviour of the gravitational field.**)

Independent of considerations in Refs. 5 and 6:

1. The term (3) is present in the true effective Lagrangian.

2. Possible additional terms $\sim O(\hbar^2)$ may only be higher order in derivatives. Therefore the true effective Lagrangian can only be better than (3).

Consequently, if the revision of the problem of classical singularities on the basis of the Lagrangian (3) gives negative results, then one has no statement, but still has a hope for

*) Since we are interested only in the gravitational sector here, it makes no difference in what system of fields the renormalizability will be realized (gravity, supergravity etc.).

**) The Lagrangian (3) (or (2)) already implies the summation of the infinite perturbation series in such a way that $(\ln \Lambda^2)$ in Eq. (1) converts into $(\ln \hbar)$. The mechanism of this summation is described in Ref. 5. The Lagrangian (3) (or (2)) is of order $\hbar \ln \hbar$, rather than purely one-loop.

future improvement. However, if the results are positive, no future development will disprove them. Now, the fact is that the results are positive.

Let us summarize our model Lagrangian. In order to have the correct behaviour of the field at large distances we must retain also the classical term of (1). The effective Lagrangian valid for large and small distances is of the form [5] :

$$\frac{\mathcal{L}(x)}{\sqrt{-g}} = R - \frac{a}{2} C_{\alpha\beta\gamma\sigma} \ln(\Box + \ldots) C^{\alpha\beta\gamma\sigma}(x) + \frac{a}{2} C_{\alpha\beta\gamma\sigma} C^{\alpha\beta\gamma\sigma}(x) \ln \Phi^2(g|x), \tag{5}$$

with sign conventions:

$$\operatorname{sign} g = -2, \quad R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R_{\mu\sigma\nu}{}^{\sigma} = g^{\mu\nu} (\partial_\sigma \Gamma^\sigma_{\mu\nu} - \ldots),$$

and $\Phi(g|x)$ given by Eq. (4). The second term of (5) is of order ∇^4. Therefore its contribution can be neglected in all equations of motion except one: the trace equation. This is because the trace equation does not contain $\nabla^4 \ln \Box$. Since quantum corrections in the Lagrangian (5) are exactly conformally invariant, the trace equation is exactly:

$$R = 0.$$

The constant a is the sum of individual contributions of all quantized fields and is positive:

$$a = \frac{1}{320\pi^2} \left(7 n_2 + 2 n_1 + \frac{1}{2} n_{1/2} + \ldots \right) > 0, \tag{6}$$

where n_s denotes the number of species of particles with spin s existing in nature. (The weight coefficients in Eq. (6) are given for the case of massless particles.)

The numerical value of a is unimportant for us. It is only important that a is of the order of (Planck length)^2 and is positive. The positivity of a is the fundamental fact following from the positivity of energy of physical quantized fields [7,8]. The positivity of a makes quantum gravity asymptotically free [5].

The asymptotic freedom and the negative energy of the vacuum polarization, due to higher derivatives in the effective Lagrangian, work and remove classical singularities.

Already in the linear approximation we see that the propagator changes as

$$G \sim \frac{1}{p^2} \quad \rightarrow \quad G \sim \frac{1}{p^2 + \alpha p^4 + \beta p^4 \ln p^2}.$$

This corresponds to the following modification of the Newton law at small distance:

$$F \sim \frac{GM^2}{r^2} \quad \rightarrow \quad F \sim \frac{GM^2}{\alpha + \beta \ln(\frac{1}{r})}.$$

The force remains finite (if $\beta=0$) or even tends to zero (if $\beta\neq 0$ - asymptotic freedom !). We shall show that in the full non-linear theory the situation is qualitatively the same: the gravitational attractive force does not increase infinitely during the gravitational collapse and the singularity does not arise.

In the present work we confine ourselves to considering the spherically symmetric collapse.

3. Preliminary analysis

To begin with, we analysed all static spherically symmetric solutions of our Lagrangian. Their full list will be given in an extended version of this work. It turned out that there are many solutions: the 3 - parameter family (in contrast to the 1-parameter Schwarzschild solution in the classical theory). <u>This general solution with three parameters turns out to be singular</u>, and the degree of singularity is the same as that in the Schwarzschild solution. <u>However, there is also a 1-parameter family of regular static solutions</u>.

The situation is in fact very familiar. Indeed, let us consider for example the linear equation with higher derivatives:

$$(\ell^2\Delta+1)\Delta\varphi=0.$$

The solution decreasing at infinity

$$\varphi=M\frac{1}{r}+A\frac{e^{-\frac{r}{\ell}}}{r}$$

contains two parameters and behaves at $r=0$ as

$$\varphi=A_1\frac{1}{r}+A_2+\ldots \quad .$$

Among the solutions there are those of the classical equation $\Delta\varphi=0$ and the new regular ones. <u>The general solution behaves like the classical one</u>.

However, we know that φ is the distribution and in fact does not satisfy the homogeneous equation, but rather the equation with the local source:

$$(\ell^2\Delta+1)\Delta\varphi=-4\pi\left[M\delta(\vec{r})+(M+A)\ell^2\Delta\delta(\vec{r})\right],$$

which immediately shows that only the regular solution

$$\varphi_{phys}=M\frac{1-e^{-\frac{r}{\ell}}}{r}$$

describes the field of a positive-energy source M .

<u>Lesson</u>: in order to single out the needed solution of higher-derivative equations one must take into account the properties of the source creating the field. In the considered example:

higher derivatives + the positive energy of the source unambiguously single out the regular solution.

However, in our non-linear gravitational equations there are specific difficulties with a static source. In particular, the static source is unphysical in the T region. Therefore we prefer to deal with realistic dynamic models of a collapsing body. At the same time, since the Birkhoff theorem is not valid in our theory, we have to consider general non-static solutions.

4. Spherically-symmetric collapse in effective equations of quantum gravity

For simplicity we consider models of thin massive collapsing shells with the traceless energy-momentum tensor, but as will be seen, our main conclusions are model-independent.

There can be two thin traceless shells with equations of state: $p=\varepsilon/2$ or $p=0$ respectively. We shall consider the null shell first: $p=0$. Its energy-momentum tensor reads:

$$T_{\mu\nu}=\frac{M\sqrt{2}}{4\pi r^2}\,\delta(v)\,\ell_\mu\ell_\nu\,, \qquad (7)$$

where $v=0$ is the equation of a null surface describing the evolution of the shell, and

$$\ell_\mu=\partial_\mu v\ ,\ \ell_\mu\ell^\mu=0.$$

At the flat infinity $v=\frac{r-t}{\sqrt{2}}$, and M = const is the mass of the shell.

The main advantage of thin shells is that all the information about the source can be formulated in the form of jump conditions for some invariants on the shell surface.

So, we must carry out the following programme:

i) Find the general solution of <u>source-free</u> equations regular at $r=0$ <u>for the region inside the shell</u>.

ii) Find the general asymptotically flat solution of <u>source-free</u> equations <u>for the region outside the shell</u>.

iii) Using the known form of $T_{\mu\nu}$, find the jump conditions from our equations.

iv) Sew the two solutions across the shell using jump conditions.

The realization of this programme in Einstein equations gives the Schwarzschild singularity as the final stage of the collapse process.

<u>Our main result</u> is that we carried out this programme for effective field equations of quantum gravity and showed that no singularity arises when the shell crosses $r=0$. The space-time remains regular, <u>and the shell, forced by field equations, begins expanding</u>.

At the particular stages of the above programme the results are the following:

i) The solution of our equations near $r=0$ are governed by the quantum part of the effective action:

$$\int C_{\alpha\beta\gamma\delta}\ln(\Box+\ldots)\,C^{\alpha\beta\gamma\delta}\sqrt{-g}\,dx \qquad (8)$$

plus the trace equation:

$$R=0\,. \qquad (9)$$

The main technical achievement is that we found <u>all exact</u> spherically symmetric solutions for the action:

$$\int C_{\alpha\beta\gamma\delta}\,C^{\alpha\beta\gamma\delta}\sqrt{-g}\,dx \qquad (10)$$

and proved the conformal version of the Birkhoff theorem for this action. We found the invariant parametrization of the field in which the field equations of this action become linear. This enables us to find asymptotic solutions also for the theory (8) with the $(\ln \Box)$ insertion.

There exists a regular solution of the theory (8) and it coincides asymptotically with the regular solution of (10). This solution is of the following form:

$$
\begin{aligned}
&\sqrt{C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta}} \equiv \mathbb{C} = F(t)\frac{r^2}{a^2} + \dots , \\
&(\nabla r)^2 \equiv g = 1 + \frac{5}{\sqrt{3}} F(t)\frac{r^2}{a^2} + \dots , \\
&\sqrt{R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}} \equiv \mathbb{R} = \frac{12}{a} F(t) + \dots .
\end{aligned}
\tag{11}
$$

There are also singular solutions of actions (8) and (10). The singularity of solutions of (8) is always weaker by $(\ln \Box)$ than that of (10). Only for regular solutions $(\ln \Box)$ is unimportant. Taking this into consideration, we used the following strategy: we first considered the gravitational collapse in the theory without $(\ln \Box)$ and then verified that $(\ln \Box)$ does not change the result. The reason is that the spacetime turns out to be regular already in the C^2- theory.

ii) The general solution of the theory (9)-(10) is parametrized by two arbitrary functions. The requirement of asymptotical flatness at $r = \infty$ does not reduce the functional arbitrariness, but only imposes certain restrictions upon the class of these functions. This means that almost all non-static solutions are asymptotically flat.

iii) It is convenient to represent the general spherically symmetric line element as:

$$
ds^2 = r^2[d\gamma^2 \underset{\text{direct sum}}{+} d\Omega^2] , \tag{12}
$$

where $d\Omega^2$ is the metric on a unit sphere, and

$$
d\gamma^2 = \gamma_{AB}\, dx^A dx^B , \quad A, B = 0, 1 , \tag{13}
$$

is some 2-dimensional metric. We are looking for the geometry γ of this 2-dimensional space and r as a function on it. Let k denote the Gaussian curvature of γ and

$$
K \equiv 1 + k \equiv \frac{\sqrt{3}}{2} r^2 \mathbb{C} . \tag{14}
$$

Then our equations are of the following form:

$$
\begin{aligned}
&[-2r\nabla_A\nabla_B r + 4\nabla_A r \nabla_B r + \gamma_{AB}(2r\Box r - \nabla r \nabla r - r^2)] + \\
&\qquad (\text{"classical part"}) \\
&+\frac{2a}{3}[\nabla_A\nabla_B K - \gamma_{AB}(\Box K - \tfrac{1}{2}K^2 + K)] = 8\pi r^2 T_{AB} ; \\
&\qquad (\text{"quantum part"})
\end{aligned}
\tag{15}
$$

$$
\Box r - \frac{1}{3} K r = 0 . \quad (\text{"trace equation"}) . \tag{16}
$$

These equations are of the second order for r and K. Therefore the invariant jump conditions are:

$a=0$; Einstein equations	$a\neq 0$; Effective equations of quantum gravity
$\Delta[r]=0$;	$\Delta[r]=0$;
$\Delta[(\nabla r)^2]=-\frac{2M}{r}$;	$\Delta[(\nabla r)^2]=0$;
$\Delta[K]=\frac{3M}{r}$;	$\Delta[K]=0$;
	$\Delta[(\nabla K)^2]=\frac{6\sqrt{2}}{a}M\ell^A\nabla_A K$;
	$\Delta[(\nabla K\cdot\nabla r)]=\frac{3\sqrt{2}}{a}M\ell^A\nabla_A r$.

The invariant parametrization of the field and the general covariance of equations made it possible to obtain this jumps, because equations proved to be linear in highest-order derivatives.

Here the main point comes to light, which conditions the appearance of the singularity in the classical theory and the regularity of the same problem in the higher-derivative theory. The regularity conditions at $r=0$ are:

$$(\nabla r)^2\big|_{r=0}=1\,,\quad K\big|_{r=0}=O(r^2).$$

We see that if the internal solution is regular, then the external solution cannot be regular in the Einstein theory, because $(\nabla r)^2$ and K jump, and their jumps grow infinitely when $r\to 0$.

In our effective theory the derivatives of the curvature

jump, but the curvature K and $[(\nabla r)^2-1]$ remain continuous. This means that they remain small, when the shell crosses $r=0$! Since the space-time remains regular, it is not surprising that, solving our equations, we found that the shell crosses $r=0$ without any trouble and begins expanding! Since it goes away, the space-time at $r=0$ remains regular always.

<u>This is how higher derivatives remove singularities</u>.

iv) The solution of equations (8)-(9) near $r=0$ is of the following form (see also Fig. 1):

$$ds^2=-A\,dt^2+\frac{dr^2}{g}+r^2\,d\Omega^2\,,$$

$$g\equiv(\nabla r)^2=1+\frac{M}{a}\theta(r^2-t^2)\frac{r^2-t^2}{r}\ln\left[\frac{\gamma}{2}(r^2-t^2)^{1/2}a^{-1/2}\right]+$$

$$+\frac{5}{\sqrt{3}}F(t)\frac{r^2}{a}+\dots\,, \qquad (17)$$

$$A=g^{-1}\exp\left[\frac{1}{\sqrt{3}}\int_0^r\frac{\mathbb{C}\cdot r}{g}\,dr\right],$$

$$\mathbb{C}\equiv\sqrt{C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}}=\frac{\sqrt{3}M}{a}\;\theta(r^2-t^2)\frac{r^2-t^2}{r^3}+F(t)\frac{r^2}{a^2}+\dots\,,$$

where $F(t)$ is an arbitrary function. We have

$$\mathbb{C}\big|_{r=0}=0\,,\quad \mathbb{R}\big|_{r=0}=12\,\frac{F(t)}{a}\,. \qquad (18)$$

As seen from (17), the curvature becomes infinite only at one space-time point: ($r=0$, $t=0$), but this is simply the defect of models of thin shells: the dimension of a shell becomes zero, while its total mass remains finite. The spreading of a shell to a finite thickness would remove this defect.

We have considered the collapse of the null shell. What if the shell were timelike but still traceless? The result of the consideration of this case is that such a shell would not reach $r=0$ at all, but would begin expanding even earlier. The line $r=0$ always remains inside the shell and is always regular.

Finally, there comes the question about shells with $T^{\mu}_{\mu}\neq 0$. In this case there would be the delta-function type source in the trace equation (16). This would lead to jumps of $(\nabla r)^2$, because our trace equation $R=0$ does not contain higher derivatives. The jump of $(\nabla r)^2$ would immediately lead to the rise of a singularity. But in fact all this does not happen, because the conformal version (3) of the effective Lagrangian cannot be used for sources with $T^{\mu}_{\mu}\neq 0$. For such sources one must use the Lagrangian (2), containing trace anomalies. The trace anomalies save the situation, and the final conclusion is the same: no singularities arise.

Clearly, possible multi-loop additions to the effective Lagrangian cannot change this conclusion either (just like ($\ln\Box$) insertions do not change it).

Resume: Quantum gravity does remove the singularity in the spherically symmetric collapse, and this conclusion is model-

independent and stable.

However, the next question immediately arises: if the shell begins expanding and returns to infinity, then what happens with the black hole??

To answer this question we must know the global behaviour of the above solution. We do not have this behaviour at present, but we obtained the exact solution of approximated equations, which is of the following form:

$$g = 1 + \frac{2M}{r}\,\theta(r^2-t^2)\left[\sqrt{\frac{r^2-t^2}{a}}\,K_1\!\left(\sqrt{\frac{r^2-t^2}{a}}\right)-1\right],$$

$$A = \frac{1}{g}\exp\left[\frac{2}{3}\int_0^r \frac{K}{g\cdot r}\,dr\right], \qquad (19)$$

$$K = \frac{3M}{r}\left[1-\frac{r^2-t^2}{2a}\,K_2\!\left(\sqrt{\frac{r^2-t^2}{a}}\right)\right]\theta(r^2-t^2),$$

where K_ν is the known (Mac-Donald) cylindric function.

This is the particular solution in which the spacetime inside the null shell is flat. We know for sure that this solution is correct in a certain region shown in Fig.2, and we presume that it is qualitatively correct everywhere. The corresponding space-time picture is shown in Fig.2.

If one believes in this solution globally, then:

1. The shell returns to our space infinity.
2. There are no event horizons and, strictly speaking, no black holes.
3. Nevertheless all observable properties of classical black holes are preserved, and the correspondence principle between quantum and classical theories is valid.

As seen from Fig. 2, the apparent horizon deviates from $r=2M$ negligibly, but becomes non-static. Yet near $r \approx 2M$ it is almost static and thus almost null. Therefore there will be the gigantic but still finite delay of out-going light signals. This (proper) time delay can be computed from the above solution and turns out to be:

$$\tau_{delay} = T \sim M \exp\left(M/m_{Planck}\right). \tag{20}$$

The "black hole" lives the finite time T, which tends to infinity when $m_{Planck}/M \to 0$. This is how the correspondence principle works.

For black holes of masses much larger than the Planckian mass the time delay (20) is much larger than the time of the Hawking evaporation. Therefore the discussed modification will become essential only at the final stage of the evaporation process when the remaining mass becomes compatible with the Planckian mass. Speaking about the final product of the Hawking evaporation, our results teach us that at least the naked singularity is excluded. Other possibilities discussed in Ref. 9 remain.

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Figure captions

Fig. 1 Asymptotic behaviour of the solution near $r=0$.

Fig. 2 Space-time picture of the solution (19). The region outside the dashed line is that where the solution (19) is certainly correct. For $M \gg m_{Planck}$:

$$r_0 \sim \left(\frac{m_{Planck}}{M}\right) \ell_{Planck}\,, \quad K(r_0) \sim 1\,;$$

$$r_1 \sim \ell_{Planck}\,, \quad K(r_1) \sim \frac{M}{m_{Planck}}\,;$$

$$r_2 \sim 2M\left[1 - \left(\frac{\pi M}{m_{Planck}}\right)^{1/2} e^{-\frac{2M}{m_{Planck}}}\right];\ K(r_2) \sim 1.$$

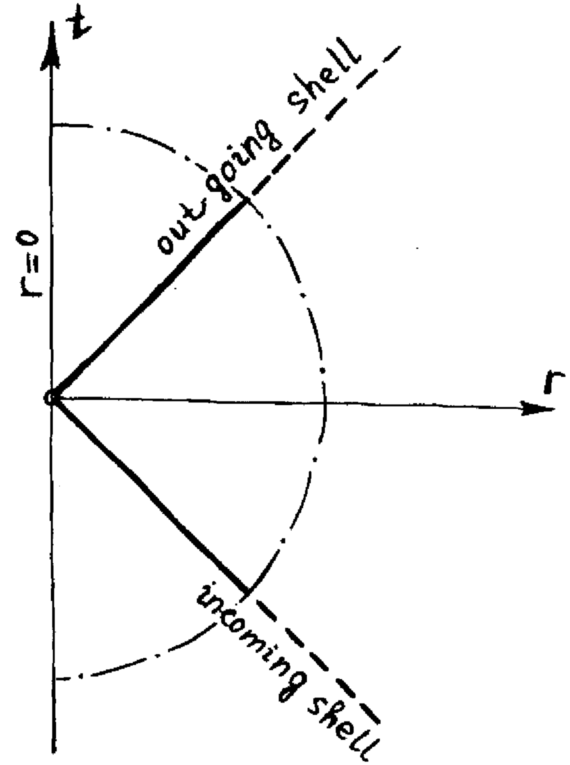

Fig.1

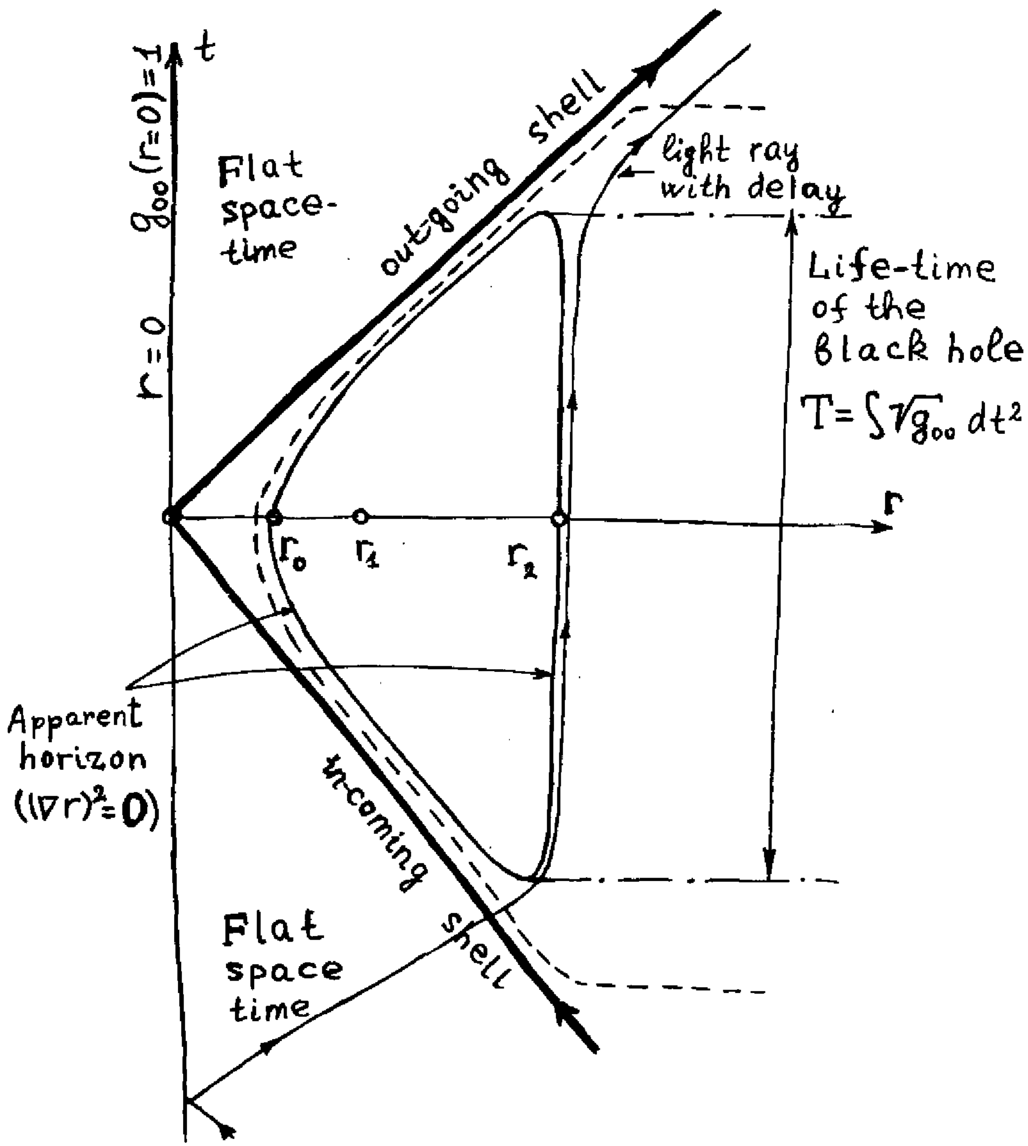

Fig.2

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