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DISLOCATIONS, THE ELASTIC ENERGY MOMENTUM TENSOR AND CRACK PROPAGATION *

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ABSTRACT

Based upon dislocation theory, some stress intensity factors can be calculated for practical cases. The results obtained by this method have been found to agree fairly well with the results obtained by the conventional fracture mechanics. The elastic energy momentum tensor has been used to calculate the force acting on the crack tip. A discussion on the kinetics of migration of impurities to the crack tip was given. It seems that the crack tip sometimes may be considered as a singularity in an elastic field and the fundamental law of classical field theory is applicable on the problems in fracture of materials.

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I. INTRODUCTION

Dislocation theory has been successfully used to explain the plastic deformation processes of solids [1], but here it is applied to a deeper understanding of the brittle fracture of materials. It seems that this problem interests many people since the progress in the understanding of brittle fracture.

There are two quite distinct roles of dislocation theory in fracture mechanics and the fracture process. First, the presence of the crystal dislocations themselves in crystalline material enables us to understand how a fracture may be initiated and how all the plastic relaxation phenomena associated with the presence of cracks and microscopic inhomogeneities take place. Second, we must also examine the use of the dislocation concept in macroscopic fracture mechanics. This paper reports the author's work on the latter problem, the future progress of which will depend largely on the interactions between solid state physics and fracture mechanics.

A macrocrack may be considered as arrays of dislocations. Based on this concept and the theory of continuous distribution of dislocations we may get a relationship between the stress field of dislocations and that in the neighbourhood of a crack tip. Furthermore, a macrocrack may also be considered as a big dislocation in a continuum medium. They correspond to each other according to their modes of deformation; for example, the mode I deformation in fracture corresponds to the climbing edge dislocations, the mode II deformation corresponds to the gliding of an edge dislocation, and mode III deformation corresponds to the gliding of screw dislocations. Based on this concept and the continuum theory of elastic singularity, we may analyse the crack opening force and the kinetics of migration of impurities to the crack tip.

II. ARRAYS OF DISLOCATIONS AND STRESS FIELD IN THE NEIGHBOURHOOD OF A CRACK TIP

There are many similarities between the mechanical behaviour of a macrocrack and dislocations piled up against a locked dislocation. Both have stress concentration effects. The Griffith formula for brittle fracture of solids may be derived from the fundamental properties of dislocations piled up against a locked dislocation. [2] But the physical concept of a crack dislocation is not the same as an ordinary dislocation [3]. The extra half atomic plane does not really exist, and crack dislocations are defined by discontinuities occurring naturally across unwelded cuts when a body is stressed. Perhaps the simplest way to regard a crack dislocation is as a type of imperfect dislocation whose associated sheet of bad crystal is a missing plane of atoms. The existence of the applied force is necessary for the existence of crack dislocations. It should be noted that crack dislocation densities cannot be any distribution other than that satisfying the condition of the stress-free surface on crack planes.

1. The fundamental properties of dislocation arrays

If the dislocation density is $\mathfrak{D}(x')$ and the stress applied at the point X on the crack plane is $\sigma^A_{yy}(x)$ (Fig.1); the dislocation density distribution function can be determined by the condition that the crack planes should be free surfaces; then

$$\sigma^A_{yy}(x) + A\int_{-a}^{a} \frac{\mathfrak{D}(x')\,dx'}{x - x'} = 0 \qquad (1)$$

$A = \frac{Gb}{2\pi(1-\nu)}$, G is the shear modulus, b is the Burgers vector. Solving Eq.(1), the dislocation density distribution function $\mathfrak{D}(x')$ can be determined.

The total stress $\sigma^A + \sigma^D$ can be determined at any point of the solid material and

$$\sigma^D_{ij}(\vec{r}) = b\int_{-a}^{a} \mathfrak{D}_y(x')\,\sigma^y_{ij}(x-x', y, z)\,dx' \qquad (2)$$

$\sigma^y_{ij}(\vec{r})$ is the stress at point $\vec{r}$ due to the dislocation, the core of which is at x' and the Burgers vector of which is [010].

Since σ^D is much larger than σ^A in the neighbourhood of the crack tip, we need to consider only σ^D.

Displacing the origin of co-ordinates to the crack tip, let $s = x - a$, and be taken as small. After certain calculation, the expressions for $\sigma^D(s)$ and $\mathfrak{D}(s)$ may be obtained

$$\sigma^D(s) = \frac{K^D}{(2\pi)^{1/2}}\, s^{-1/2}, \qquad (3)$$

$$\mathfrak{D}(s) = \frac{K^D}{A\pi(2\pi)^{1/2}}\,(-s)^{-1/2}. \qquad (4)$$

In comparing formula (3) with that in conventional fracture mechanics, we see that they are similar in mathematical forms.

$$\sigma^c(s) = \frac{K^c}{(2\pi)^{1/2}}\, s^{-1/2}. \qquad (5)$$

Therefore

$$\frac{\sigma^c(s)}{\sigma^D(s)} = \frac{K^c}{K^D} = \alpha, \qquad (6)$$

We do not know whether they are similar or equivalent, because we do not know whether α is equal to unity or not. Although the general solution of K^D from Eq.(1) is the same as that of K^c in symposium ASTM, STP 381 in the central symmetrical crack case, we think, it is necessary to compare the calculated numerical results from Eqs.(3) and (4) with that from coventional fracture mechanics. According to the definition of the stress intensity factor,

$$K^D = (2\pi)^{1/2} \lim_{s\to 0} s^{1/2}\,\sigma^D(s) \qquad (7)$$

from Eq.(4),

$$K^2 = 2\pi^3 A^2 \lim_{x\to a} (a-x)\,\mathfrak{D}^2(x). \qquad (8)$$

Solving the integral Eq.(1), $\mathfrak{D}(x)$ is obtained and then K^D may be obtained from Eq.(8). Expressions (4) and (8) were described by Bilby and Eshelby [3]; but no paper reporting practical numerical results calculated with this method has been seen up to now. Perhaps there are some practical problems to be solved. For example, it is not so easy to solve the integral equation for $\mathfrak{D}(x')$ because of the variety of distribution of $\sigma^A_{yy}(x)$. Furthermore, some problems should be solved for edge cracks or bend specimens though it is easier for central symmetrical cracks (the total sum of the number of dislocations is zero).

In this paper two methods are used for obtaining practical results: 1) solving the integral equation with the Chebyshev polynomials, $\mathfrak{D}(x)$ is easily obtained; and using the semi-empirical method associated with the semi-theoretical method, any arbitrary distribution of stress may be expressed in polynomials; 2) the results for edge cracks calculated by conventional fracture mechanics was used in our calculation. Then, an analytical formula of K^D [4] was obtained and was applied to some practical cases.

2. Solving the integral equation with Chebyshev polynomials

$T_n(x)$ and $U_n(x)$ are a pair of normalized orthogonal polynomials between (-1, +1), the weight functions of which are $(1-x^2)^{-1/2}$ and $(1-x^2)^{1/2}$, respectively,

$$T_0(x)=1\ ,\quad T_1(x)=x,\quad T_2(x)=2x^2-1\ ,\ T_3(x)=4x^3-3x\ \cdots$$

$$U_0(x)=0\ ,\quad U_1(x)=1,\quad U_2(x)=2x\ ,\ U_3(x)=4x^2-1\ \cdots \tag{9}$$

The relation between $T_n(x)$ and $U_n(x)$ is

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f(y)\,dy}{y-x}=\frac{1}{\pi}\int_{-1}^{1}\frac{1}{y-x}\left[\frac{T_n(y)}{(1-y^2)^{1/2}}\right]dy=U_n(x)\ . \tag{10}$$

3. The stress intensity factor of a single edge crack under arbitrary distribution of applied stresses in an infinitely large plate

If

$$\sigma(x)=\sum_{n=0}^{n}a_n x^n=\sum_{n=0}^{n}(a_n a^n)\eta^n$$

$$=\sum_{n=1}^{n+1}C_n(a)U_n(\eta) \tag{11}$$

$\eta=\frac{x}{a}$, $U_n(\eta)$ has the form of (9). Comparing (9) with (1), we obtain

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\mathfrak{D}(\eta)\,d\eta}{\eta-\frac{x}{a}}=\sigma_{yy}^{A}\left(\frac{x}{a}\right)\ . \tag{12}$$

Using the relation of (10), we may obtain $\mathfrak{D}(\eta)$ and then the expression for K^D. The stress intensity factor is multiplied by 1.12 for the edge crack case (we did not suppose $\mathfrak{D}(0)=0$)

$$K^D=a_0 F(a)\sqrt{\pi a}\ \ . \tag{13}$$

$$F(a)=1.12\left(1+\frac{a_1}{2a_0}a+\frac{a_2}{2a_0}a^2+\frac{3a_3}{8a_0}a^3\right)\ . \tag{14}$$

Comparing (14) with that in the handbook edited by Sih [5] (Fig.2),

$$\sigma(x)=\sigma\sum_{n=0}^{n}C_n\left(\frac{x}{a}\right)^n \tag{15}$$

then (14) changes its form to

$$F(a)=1.12+0.56\left(\frac{C_1}{C_0}\right)+0.56\left(\frac{C_2}{C_0}\right)+0.42\left(\frac{C_3}{C_0}\right) \tag{16}$$

in the handbook [5]

$$F(a)=1.12+0.677\left(\frac{C_1}{C_0}\right)+0.52\left(\frac{C_2}{C_0}\right)+0.438\left(\frac{C_3}{C_0}\right) \tag{17}$$

(16) and (17) are similar.

After certain steps of calculation, formula (14) was applied to a rotary plate (steam turbine wheel) with an edge crack at the central hole. The result obtained by this method was compared with that of the finite element method. The differences in percentage are about 2-15%. The calculation based on the dislocation theory is simpler (Fig.3).

Chebyshev polynomials may be applied to more general cases. Any function $\sigma(x)$ may be considered as a "vector" in the Hilbert space. The orthogonal fundamental function sets (such as $U_n(x)$ or $T_n(x)$) are the base vectors in Hilbert space. If the orthogonality and weight functions were known, the components of every base vector (the coefficient of certain $U_n(x)$ terms in the expansion of the function) can be calculated by the expansion of $\sigma(x)$. So that, in principle, any stress distribution function $\sigma(x)$ can be expressed in terms of $U_n(x)$.

The errors in the above calculation were not too large, though some approximations were introduced during calculations. From the above, we may say that the fractural behaviour of the dislocation model gives approximately the same results as that calculated by other methods of fracture mechanics. Over and above, the author has used this method for calculating stress intensity factors for bending specimens [6].

III. ELASTIC ENERGY MOMENTUM TENSOR AND FORCE ACTING ON THE CRACK TIP

A macrocrack may be considered as a big dislocation in a continuum medium. In general, point defects, dislocations, crack tips, etc. and all other defects may be considered as singularities in an elastic field. Their mechanical behaviour obeys the law of the general elastic field theory. Their differences reflect their specificities. Eshelby has discussed the interaction between the applied stress and the defects [7].

1. Elastic energy momentum tensor

The Lagrangian density for the free elastic field

$$L = \frac{1}{2}\rho \dot{u}^2 - W(u_{i,j}) \tag{18}$$

with an external force density f_i is not taken into account in the Lagrangian. The equation of motion is

$$\frac{\partial}{\partial x_i}\frac{\partial L}{\partial u_{i,j}} - \frac{\partial L}{\partial u} = 0 \tag{19}$$

The methods of field theory enable us to derive an energy momentum tensor,

$$T_{\eta\lambda} = \left(\partial L / \partial u_{i,\eta}\right) u_{i,\lambda} - L\,\delta_{\eta\lambda} \tag{20}$$

where $\eta,\lambda = 1,2,3,4$, $x_4 = t$, $u_4 = 0$ and their components are

$$T_{jl} = P_{jl} - \frac{1}{2}\rho \dot{u}^2 \delta_{jl}\,, \qquad T_{44} = W + \frac{1}{2}\rho \dot{u}^2\,,$$

$$s_j = T_{j4} = -\sigma_{ij}\dot{u}_i\,, \qquad g_l = T_{4l} = \rho \dot{u}_i u_{i,l} \tag{21}$$

and $P_{j\ell} = W\delta_{j\ell} - \sigma_{ij} u_{i,\ell}$ and the relation $\sigma_{ij} = \frac{\partial W}{\partial u_{ij}}$ was used.

The physical significance of $T_{j\ell}$ is the ℓ component of momentum flowing through the unit area which is perpendicular to the ℓ axis in unit time [8]; They are tensors. For the static case in the absence of a body force, only $P_{j\ell}$ and W of $T_{\eta\lambda}$ come into existence.

Eshelby used an elastic singularity motion model to derive the following expression:

$$F_\ell = \int_\Sigma P_{j\ell}\, dS_j \tag{22}$$

where F_ℓ is the ℓ-component of the forces acting on all the sources of internal stresses as well as elastic inhomogeneities in region I (Fig.4). These forces are caused by sources of internal stresses and elastic inhomogeneities in region II and by the image effects associated with boundary conditions.

2. Force acting on the crack tip

Suppose there is only one crack tip in region I, F_ℓ may be taken as the force acting on the crack tip. The formula (22) is the form in three dimensions. If we use the form in two dimensions, it is similar to the J integral in fracture mechanics.

Let $\ell = 1$,

$$\begin{aligned} F_1 &= \oint (W\delta_{1j} - \sigma_{ij} u_{i,1})\, ds_j \\ &= \oint_s \left[W dx_2 - \vec{T}\cdot\left(\frac{\partial \vec{u}}{\partial x_1}\right) ds \right] \end{aligned} \tag{23}$$

$ds_1 = dx_2$, $T_i = \sigma_{ij} n_j$ and ds_j denotes the line element.

1) $F_1 = G$ [3] for mode I deformation,

2) mode II and combined mode deformation [9].

The crack extends along the x_1 direction under pure mode I deformation, i.e. the direction of the crack extension coincides with F_1. If mode II deformation is mixed with mode I, the direction of $F_{\ell 0}$ would not coincide with the x_1 direction. Suppose it has an angle α with x_1 (Fig.5),

$$F_\ell = \oint W ds_1 - \oint \sigma_{ij} u_{i,\ell} ds_j$$

$$= \oint W r d[\sin(\theta-\alpha)] - \oint \sigma_{ij}[u_{i,1}(\frac{\partial x_1}{\partial \ell}) + u_{i,2}(\frac{\partial x_2}{\partial \ell})] ds_j$$

$$= F_1 \cos\alpha + F_2 \sin\alpha \quad (24)$$

with

$$F_1 = \oint_s [W dx_2 - \vec{T}\cdot(\frac{\partial \vec{u}}{\partial x_1}) ds]$$

$$F_2 = \oint_s [W dx_1 - \vec{T}\cdot(\frac{\partial \vec{u}}{\partial x_2}) ds] \quad (25)$$

for maximum F_ℓ,

$$\frac{\partial F_\ell}{\partial \alpha} = 0 \;, \qquad \alpha_0 = \operatorname{arc\,tg}(F_2/F_1) \quad (26)$$

because $\left(\frac{\partial^2 F_\ell}{\partial \alpha^2}\right) < 0$, so that α_0 denotes the direction of maximum $F_{\ell 0}$,

$$F_{\ell 0} = \sqrt{F_1^2 + F_2^2} = F_1 \sec\alpha_0 \;. \quad (27)$$

Suppose $F_{\ell 0}(K,K_2) \geqslant F_{\ell 0c}$, the crack extends; this is the criterion of fracture under combined mode deformation. That is to say, F_ℓ is the effective crack extension force of an elastic field acting on the crack along the ℓ direction, $F_{\ell 0}$ is the maximum value of F_ℓ and $F_{\ell 0c}$ is the critical value of $F_{\ell 0}$, which is a material constant.

The author's results obtained by this method agreed with Hellen et al. though the steps of calculation and numerical results were a little different. In fact, α_0 in formula (27) is small, and the errors of calculation may be smaller, and its effects on σ_F (its dimension is $[F_{\ell 0}]^{1/2}$) would be much smaller.

In comparison with the methods of maximum, strain energy density factor $S, G = \frac{1}{E'}(K_1^2 + K_2^2)$ and his stress projection method, Wang [11] found that their differences are quite small and all are good from the engineering point of view. He pointed out that the fracture stress of the singularity motion model, G value and his stress projection method are much simpler (Fig.6).

On the problem of fracture angle, different authors have different opinions [12]. The author of this paper thinks that α_0 calculated from formula (26) denotes the direction of $F_{\ell 0}$ (a vector) but it does not denote the direction of the fractural plane (which should be determined by some tensors, such as σ_{ij}). This reflects the contradiction between the variety of modes of applied stress (e.g. mode I, mode II, mode III or the complex mode etc.) and the single model of actual fracture mechanism (only mode I). This also reflects the fact that fracture processes are closely connected with the properties of materials. In fact, hardly anyone saw a sliding mechanism during fracture. To answer the question why this contradiction exists is not a pure fracture mechanical problem. It would be a problem of solid state physics or metal physics. All are doubtful whether to take the argument of vector $\bar{J} = J_1 - iJ_2$ as a fracture angle as in (10) and (12) or to take the direction of $F_{\ell 0}$ as that of crack extension as in (9). α_0 and fracture angle are not the same thing but it would seem that there exists a relation between them. In Ref.9, the author suggested a relation

$$\theta_0 = \operatorname{arc\,tg}(K_1/K_2) + \alpha_0 - \frac{\pi}{2} \;. \quad (28)$$

The calculated results of θ_0 agree qualitatively with the experiments. Further work is needed.

3. Elastic energy momentum tensor and experimental measurement of the J integral

1) In the integral expression of the force F_ℓ, the condition $C_{ijk\ell,m} = 0$ is required, the path of the integral may envelop inhomogeneous

regions as well as sources of internal stresses, but must not run across them [7]. In any case, the condition $C_{ijk\ell,m} \neq 0$ would make the measurement of the J integral lose physical significance.

2) If there are many sources of internal stress and inhomogeneities within $\sum$, $C_{ijk\ell,m} \neq 0$. F_ℓ gives the combined force on them; there is no way of separating the two contributions [7].

It seems that no attention has been paid to the two points in the experimental measurement of the J integral. Perhaps these effects would be cancelled out automatically if the formula for the J integral calculation in Ref.15 were used.

IV. KINETICS OF MIGRATION OF IMPURITIES TO CRACK TIP AND HYDROGEN DELAYED FRACTURE

It is well known that delayed fracture of ultra-high strength steel is closely connected with the diffusion process of hydrogen atoms. Under a certain amount of load, hydrogen atoms in steel will diffuse to the stress region near the crack tip. It will be brittle as the concentration of the hydrogen atom reaches a certain critical value. Every step of fracture may only make a leap of a finite distance, the crack propagation will be arrested as the crack tip approaches the region of low concentration of hydrogen atoms. Then, the hydrogen atoms diffuse to the new crack tip again, the process is repeated again and again, the crack propagates discontinuously.

The crack tip may be considered as big dislocations in a continuum medium as above, we may use the method of kinetics of migration of impurities to the crack tip to analyse this process, but the singularity of the crack tip has its own characteristics.

1. Interaction energy between point defects and stress field near a crack tip

Let P be the pressure at a point near a crack tip, and V the expansion due to a point defect. The interaction energy

$$E = pv \tag{29}$$

and

$$p = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \tag{30}$$

under the plain strain condition,

$$\sigma_z = \nu(\sigma_x + \sigma_y) \tag{31}$$

ν is the Poisson ratio, then

$$E = -\frac{1+\nu}{3}(\sigma_x + \sigma_y)v \tag{32}$$

For mode I deformation,

$$\sigma_x = \frac{K_1}{(2\pi r)^{1/2}}\cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K_1}{(2\pi r)^{1/2}}\cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \tag{33}$$

Then

$$E = -\frac{1+\nu}{3}\,\frac{2K_1 v}{(2\pi r)^{1/2}}\cos\frac{\theta}{2} \tag{34}$$

2. The number of point defects arrived at the crack tip during time t and the time interval between two steps of crack extension

In line with the general formula of Bullough [1], if

$$E = -\frac{A}{r^n} \tag{35}$$

the number of point defects arrived at the lattice defect is

$$N(t) = c_0\pi\left[\frac{ADn(n+2)}{kT}\right]^{\frac{2}{n+2}} t^{\frac{2}{n+2}} \tag{36}$$

In our case $n = \frac{1}{2}$, $A = \frac{1+\nu}{3}\cdot\frac{2K_1 v}{(2\pi)^{\frac{1}{2}}}$ and the angle-dependent factor may be neglected, as Bullough has pointed out, since it has little effect on this analysis,

$$N(t) = \left(\frac{5}{4}\right)^{\frac{4}{5}} \pi c_0 \left[\frac{AD}{kT}\right]^{\frac{4}{5}} t^{\frac{4}{5}} \quad . \tag{37}$$

If N(t) reaches a critical value, the crack extends; then t_F (the time interval between steps of crack extension) is

$$t_F = B\left(\frac{N_F}{C_0}\right)^{\frac{5}{4}} \frac{kT}{K_1 D_0} e^{U/kT} \tag{38}$$

and

$$B = \frac{2^{3/2}}{5} \pi^{-\frac{3}{4}} \left(\frac{3}{1+\gamma}\right) \frac{1}{\nu} \quad , \qquad D = D_0 e^{-U/kT} \quad .$$

3. The relation between the macroscopic process of delayed fracture and the microscopic diffusion process of hydrogen atoms

Zhurkov and others [13] have measured the time of fracture under varied stress of many kinds of solid materials, such as metals, alloys, glasses, high polymers. They found a general empirical formula

$$\tau = \tau_0 \exp\left[(U_0 - \gamma\sigma)/kT\right] \tag{39}$$

τ is the fracture time, U_0 is the cohesive energy of atoms or molecules, ν is a parameter depending on structure, k is the Boltzmann constant and τ_0 is a certain constant which is the fracture time as $U_0 = \nu_0$.

In formula (39), when σ approaches zero, we have

$$\tau = \tau_0 \exp(U_0/kT) \tag{40}$$

and, on the other hand, when the variation of T can be neglected as compared with exp(U/kT) in a certain temperature range, formula (38) approaches

$$t_F \approx t_0 \exp(U/kT) \tag{41}$$

then (40) and (41) are similar. It means that the results calculated from the analysis of the microscopic process under certain conditions are similar to the empirical relation of the macroscopic fracture process. Therefore,

$$U \approx U_0 \quad , \tag{42}$$

that is, the activation energy of fracture is nearly equal to the activation energy of diffusion of hydrogen atoms. Johnson and Willner [14] obtained the activation energy of fracture (9,000 cal/g-atom) to be equal to the activation energy of diffusion of hydrogen atoms, from measuring $\frac{da}{dt}$ ($\propto t_F^{-1}$) at varied temperatures.

4. Microscopic mechanism of a delayed fracture

One thinks of using (39) as a general method to determine the microscopic mechanism as formula (42) in the course of nature. But, the author of this paper thinks, this should be done with care.

1) It is known from (39) that the activation energy in appearance (U_0-$\gamma\sigma$) approaches U_0 only if σ approaches zero.

2) After comparing (38) with (40), the microscopic mechanism approximately represents the macroscopic phenomenon only if it is under certain conditions.

As a consequence of the above, it would seem that the experimental values of U_0-$\gamma\sigma$ are a function of σ and U_0 might be obtained as σ approaches zero. We would rather compare U_0 with that of a microscopic process. In fact, only the experimental activation energy of fracture due to low stresses corresponds to the activation energy of a diffusion process of hydrogen atoms [14]. Perhaps a simple exponential form such as (40) would be doubtful under low loads. This calls for further experimental work.

V. FUNDAMENTAL PROPERTIES OF AN ELASTIC FIELD

All the point defects, dislocations and crack tips obey the fundamental law of the elastic field theory, although they have different lattice structures and properties of their own.

In Sec.I, Eq.(1) is fundamental. The most fundamental physical quantity is the force acting on dislocations and it may be derived from fundamental properties of an elastic field [7]. The interaction energy between point defects and crack tips may also be calculated from fundamental properties of the elastic field.

In Sec.II, it has been shown that the elastic energy momentum tensor is a fundamental physical quantity of the J integral in fracture mechanics.

In consequence, to consider the crack tip as a concrete special singularity in an elastic field and to apply the fundamental law of the field theory to crack extension is an important problem in fracture of materials.

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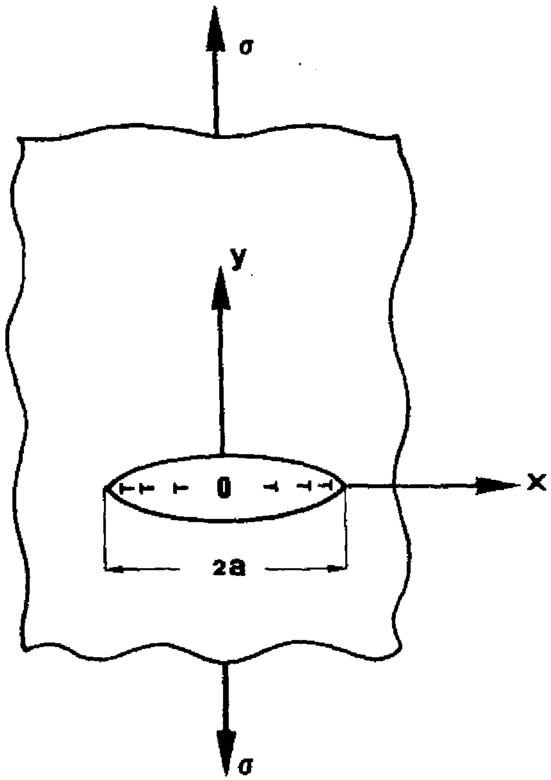

Fig.1 The crack dislocation model.

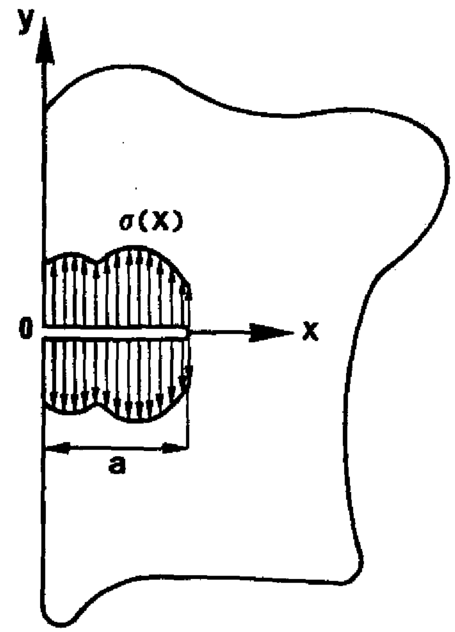

Fig.2 A single edge crack under arbitrary distribution of applied stresses in a semi-infinitely large plate.

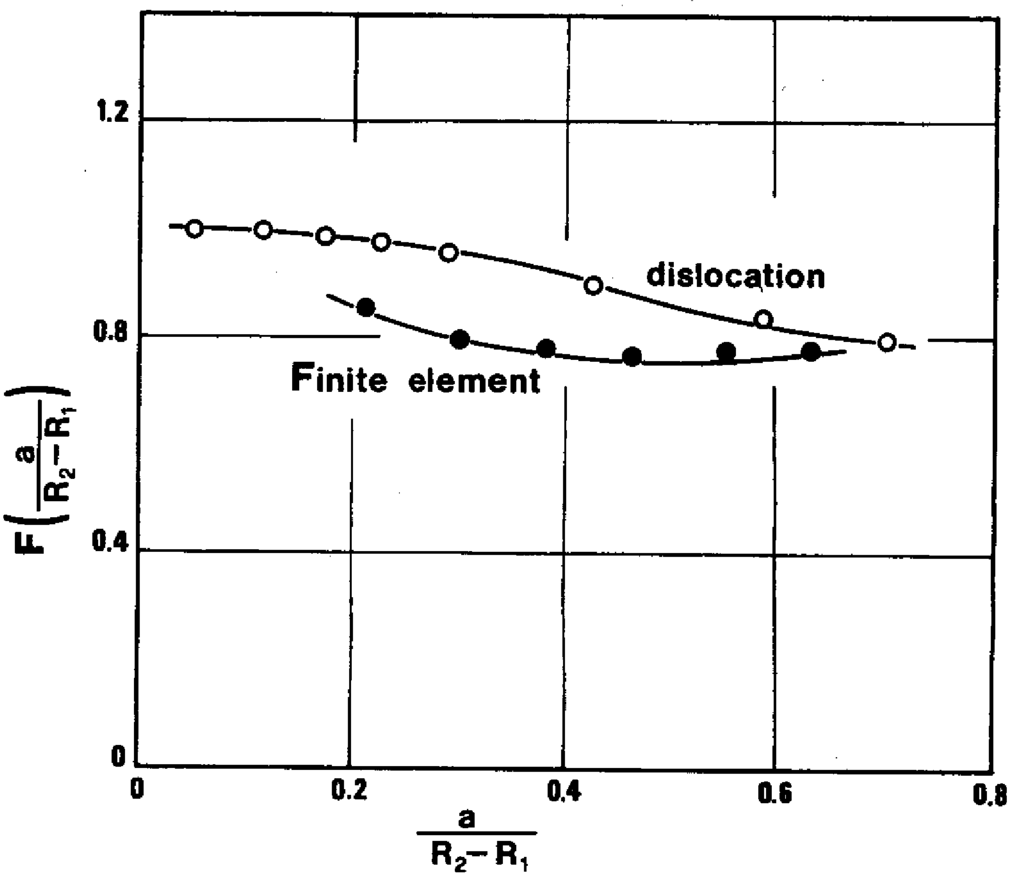

Fig.3 The stress intensity factors of a rotary plate with an edge crack at the central hole.

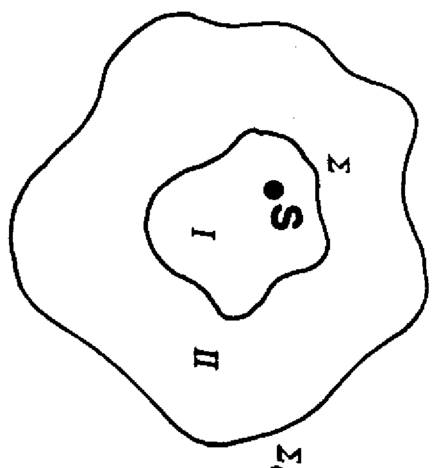

Fig.4 The internal stress source in an elastic continuum medium.

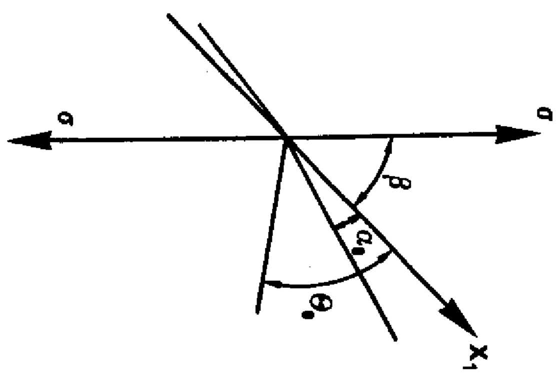

Fig.5 The direction of $F_{\theta 0}$ and the direction of the fractural plane.

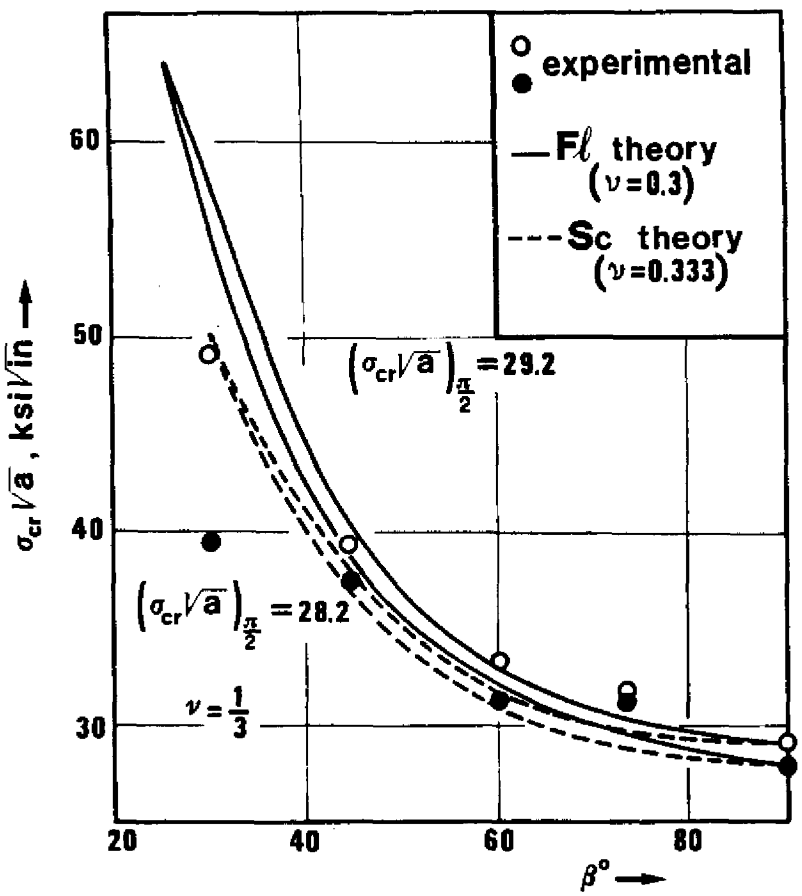

Fig.6 The relationship between $(\sigma_{cr}\sqrt{a})$ and β.

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