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SCALE SOLUTIONS AND COUPLING CONSTANT IN ELECTRODYNAMICS OF VECTOR PARTICLES

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<u>Abstract</u>

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A new approach in nonrenormalizable gauge theories is studied ~~in the work~~, the electrodynamics of vector particles being taken as an example. One- and two-loop approximations in Schwinger-Dyson set of equations are considered with account for conditions imposed by gauge invariance. It is shown, that solutions with scale asymptotics can occur in this case but only for a particular value of coupling constant. This value in solutions obtained is close to the value of the fine structure constant $\alpha = 1/137$.

<u>Аннотация</u>

Арбузов Б.А., Боос Э.Э., Куренной С.С.

Масштабные решения и константа связи в электродинамике векторных частиц. Серпухов, 1980.

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На примере электродинамики векторных частиц ~~в работе~~ исследуется новый подход к неренормируемым калибровочным теориям. Рассмотрены одно- и двухпетлевое приближения системы уравнений Швингера-Дайсона с учетом условий, накладываемых калибровочной инвариантностью. Показано, что при этом могут осуществляться решения с "масштабной" асимптотикой, однако, лишь при выделенном значении константы связи. Это значение в полученных решениях оказывается близким к величине постоянной тонкой структуры $\alpha = 1/137$.

1. INTRODUCTION

A success of gauge theories of electroweak[1] and strong (see for example review[2]) interactions causes great interest in quantum field theory of vector particle interaction. Nowadays a preference is given to renormalizable versions of theory, which need additional Higgs scalar fields in the case of broken symmetry. One of possibilities to consider vector models without Higgs scalars is to use massive vector fields ad hoc, which as is well-known leads to a non-renormalizable theory. The opinion is often expressed, that renormalizability is a physical principle in some sence, which selects possible versions of the theory. The reason for such a point of view is only that the well developed perturbative method exists in these theories. However let us note, that one cannot consider the inapplicability of non-renormalizable theories as an established fact. The investigation of several examples of non-renormalizable models (see[3-5] and subsequent papers) leads to the conclusion that the failure of the perturbation theory here is connected first of all with the nonanaliticity of Green's functions in the origin of the coupling constant g^2 complex plane, i.e. with the presence of the terms $\log g^2, \sqrt{g^2}$ etc. It is therefore evident, that non-renormalizable theories can be studied only beyond the framework of perturbation theory.

In the present work we consider a new approach[6] to investigation of non-renormalizable gauge theories, the electrodynamics of massive vector particles taken as an example. We use the Schwinger-Dyson set of equations as a tool to get beyond the framework of perturbation theory and look for the ultraviolet asymptotic behaviour of Green functions. We assume a special "scale" type of Green's function asymptotics and study the conditions under which the Schwinger-Dyson set of equations is satisfied. It occurs that the solutions with the scale asymptotics exist only for some particular values of the coupling constant.

Note that the emergence of "eigenvalue problem" for the charge is not at all unexpected just in non-renormalizable theory. Indeed the perturbative series in a renormalizable theory exists for an arbitrary small coupling constant (yet, seemingly, asymptotic). On the contrary in a non-renormalizable theory one can say nothing as far as an existence of a solution for an arbitrary value of the coupling constant due to failure of perturbation theory. However it is not excluded that the solution exists for some of its particular values.

The approach which is to some extent analogous to the one considered here was earlier developed in the papers by Johnson, Baker and Willey[7] on the "finite" spinor electrodynamics. It is well-known that their program has failed (see for example[8]) and this may be considered as an indication of the fact that such an approach works only in non-renormalizable theory, if it does at all.

2. FORMULATION OF THE PROBLEM

Let us consider the Lagrangian of electrodynamics of massive charged vector particles[9].

$$L = -\frac{1}{4}F_{\mu\nu} F_{\mu\nu} - \frac{1}{2}(D^{+}_{\mu}V^{+}_{\nu} - D^{+}_{\nu}V^{+}_{\mu}) \times \\ \times(D_{\mu}V_{\nu} - D_{\nu}V_{\mu}) + M^2 V^{+}_{\mu}V_{\mu} - ie(1 - \varkappa_0)F_{\mu\nu}V^{+}_{\mu}V_{\nu}; \tag{2.1}$$

where $F_{\mu\nu} = \delta_{\mu}A_{\nu} - \delta_{\nu}A_{\mu}$ is the tensor of electromagnetic field, $D_{\mu} = \delta_{\mu} - ieA_{\mu}$, M is the mass of the charged vector field V_{μ}, e is the bare charge and the parameter $(1 - \varkappa_0)$ defines anomalous dipole magnetic and quadrupole electric moments of the charged vector particle[9/*)]. Lagrangian (2.1) is invariant under gauge transformations

$$V_{\mu} \to e^{ie\alpha(x)}V_{\mu}, \quad V^{+}_{\mu} \to e^{-ie\alpha(x)}V^{+}_{\mu}, \quad A_{\mu} \to A_{\mu} + \partial_{\mu}\alpha \tag{2.2}$$

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*) Here and in what follows we do not distinguish co-and contravariant indices, bearing in mind that summations are performed in the Minkovski metrics.

Note that for $\varkappa_o = 0$ the Lagrangian coincides with the electromagnetic part of Salam-Weinberg Lagrangian [1].

Free propagators and vertices have the form in momentum representation (the notations are clear from Fig. 1).

The propagator of the vector particle

$$G^{o}_{\mu\nu}(p) = \frac{g_{\mu\nu} - p_\mu p_\nu/M^2}{p^2 - M^2 + i\varepsilon} . \qquad (2.3)$$

The photon propagator

$$D^{o}_{\mu\nu}(p) = \frac{g_{\mu\nu} - dp_\mu p_\nu/p^2}{p^2 + i\varepsilon} \qquad (2.4)$$

where d is a well-known gauge parameter.

The free triple and quartic vertices are

$$\Gamma^{o}_{\mu\nu\alpha}(p,q;k) = ie[g_{\mu\nu}(p - q)_\alpha + g_{\nu\alpha}(q - k)_\mu + g_{\mu\alpha}(k - p)_\nu + \varkappa_o(g_{\nu\alpha}k_\mu - g_{\mu\alpha}k_\nu)] , \qquad (2.5)$$

$$\Pi^{o}_{\mu\nu\alpha\beta}(p,q;t,k) = e^2[2g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}].$$

Using the standard functional technique [10,11] we obtain the Schwinger-Dyson equations which are grafically presented in terms of diagrams in Fig. 2. The invariance under the gauge transformations (2.2) leads to Ward-Takahashi identities

$$(p + q)_\beta \Gamma_{\mu\nu\beta}(p,q;-p-q) = ie[G^{-1}_{\mu\nu}(p) - G^{-1}_{\mu\nu}(q)] \qquad (2.6)$$

$$k_\beta \Pi_{\mu\nu\alpha\beta}(p,q;t,k) = ie[\Gamma_{\mu\nu\alpha}(p,q+k;t) - \Gamma_{\mu\nu\alpha}(p+k,q;t)] . \qquad (2.7)$$

The main point of our approach consists in the assumption that the Green's functions have scale asymptotics in the ultraviolet region of momentum space $|p^2|, |q^2|, \ldots >> M^2$.

$$G_{\mu\nu}(p) \to \frac{A(g_{\mu\nu} - y\cdot p_\mu p_\nu/p^2)}{p^2}, \qquad (2.8)$$

$$D_{\mu\nu}(p) \to \frac{C(g_{\mu\nu} - z\cdot p_\mu p_\nu/p^2)}{p^2};$$

$$\Gamma_{\mu\nu\alpha}(p,q;k) \to ie\{A' g_{\mu\nu}(p-q)_\alpha + B'[g_{\nu\alpha}(q-k)_\mu + g_{\mu\alpha}(k-p)]_\nu + \varkappa'(g_{\mu\alpha}k_\nu - g_{\nu\alpha}k_\mu)\}, \qquad (2.9)$$

$$\Pi_{\mu\nu\alpha\beta}(p,q;t,k) \to e^2\{A'' g_{\mu\nu}g_{\alpha\beta} - B''(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\} \qquad (2.10)$$

where A, C, y, z, A', B', ,... are some constants.

Identities (2.6), (2.7) impose the following constraints on the parameters

$$A' = \frac{1}{A}, \quad A'' = \frac{2}{A}, \qquad (2.11)$$

$$B' = B'' = \frac{1}{A}\cdot\frac{1}{y-1}.$$

Note that in expressions (2.9), (2.10) we write down only the simplest tensor structures having corresponding momentum dimensions. We shall see below the account for other (transverse) structures will be made in the course of subsequent approximations to the solution of Schwinger-Dyson equations.

Our basic assumption indicates in particular finiteness of renormalization constants and from such a point of view we look for "finite" vector electrodynamics.

Substituting expressions(2.8) - (2.10) into Schwinger-Dyson equations we see, by simple counting of momentum powers, that these asymptotics can satisfy the equations. However the divergences in the integrals arising destroy completely the equality between the corresponding left- and right-hand sides. That is why it is necessary to impose the conditions on the parameters, which quarantee cancellation of divergent terms. Other conditions arise from the equality of the finite parts in the corresponding equations.

So our task is to solve a problem wherether the scale asymptotics proposed agrees with the Schwinger-Dyson equations. We start with the simplest one-loop approximation.

3. ONE-LOOP APPROXIMATION

In this approximation we consider only one-loop terms in the Schwinger-Dyson equations for the full propagators $G_{\mu\mu'}$ and $D_{\mu\mu'}$ (see Fig. 2a,b). As a result we obtain the equations

$$G^{-1}_{\mu\mu}(p) = g_{\mu\mu'}(p^2-M^2) - p_\mu p_{\mu'} + \frac{1}{i(2\pi)^4}\int d^4q\,\Gamma^o_{\mu\rho\nu}(p,-q;q-p)D_{\nu\nu'}(p-q)G_{\rho\rho'}(q)\Gamma_{\rho'\mu'\nu'}(g,-p;p-q) \quad (3.1)$$

$$D^{-1}_{\mu\mu'}(p) = g_{\mu\mu'}p^2 - \frac{d}{d-1}p_\mu p_{\mu'} + \frac{1}{i(2\pi)^4}\int d^4q\,\Gamma^o_{\rho\nu\mu}(q-p,-q;p)G_{\nu\nu'}(q)\Gamma_{\nu'\rho'\mu'}(q,p-q;-p)G_{\rho\rho'}(p-q). \quad (3.2)$$

Taking expression (2.5) for the free vertex and expressions (2.8), (2.9) under the conditions (2.11) for the full propagators and vertices we obtain a set of equations on the introduced parameters.

While evaluating the integrals we use the dimensional regularization that ensures the validity of the gauge invariance and makes it possible to calculate the next approximations. In this in the integrals of equations (3.1), (3.2) there arise terms proportional to $1/\varepsilon$ ($n = 4 - 2\varepsilon$ is the space-time dimension). According to our program it is necessary to impose the conditions for all the coefficients afore $\frac{1}{\varepsilon}$ in each independent tensor structure to vanish. Both equations (3.1), (3.2) have two independent structures $g_{\mu\mu'}p^2$ and $p_\mu p_{\mu'}$. Since in equation (3.2) for the photon propagator the integral term has transverse structure $(g_{\mu\mu'}p^2 - p_\mu p_{\mu'})$ due to the gauge invariance, we obtain from (3.2) only one condition. Equation (3.1) gives us two independent conditions. Other three equations follow from the equality of the finite parts (here also one obtains only one condition from (3.2) because the gauge parameter d is arbitrary). Thus we obtain six equations

$$\frac{25}{12} - \frac{1}{12}y + \frac{13}{12}\frac{y}{y-1} + \frac{7}{12}y\frac{y}{y-1} + z\cdot\frac{3}{4} + \varkappa\left(-\frac{1}{12} - \frac{3}{4}y\right) +$$

$$+ \varkappa_0\left(-\frac{1}{6}y - \frac{1}{12}\frac{y}{y-1} - \frac{7}{12}y\frac{y}{y-1} - \frac{1}{6}\varkappa + \frac{1}{2}\varkappa y\right) = 0. \quad (3.3a)$$

$$-\frac{7}{3} - \frac{5}{12}y - \frac{4}{3}\frac{y}{y-1} - \frac{1}{12}y\frac{y}{y-1} + z\left(\frac{1}{4}y - \frac{1}{2}\frac{y}{y-1} - \frac{1}{4}y\frac{y}{y-1}\right) + \varkappa\left(\frac{1}{3} + \frac{3}{4}y\right) +$$

$$+ \varkappa_0\left(\frac{2}{3}y + \frac{1}{3}\frac{y}{y-1} + \frac{1}{12}y\frac{y}{y-1} + \frac{2}{3}\varkappa - \frac{1}{2}\varkappa y\right) = 0 \quad (3.3b)$$

$$- 1 - \frac{1}{3}y + \frac{25}{6}\frac{y}{y-1} + \frac{4}{3}y\frac{y}{y-1} + \frac{1}{2}y^2 - \frac{1}{2}y^2\frac{y}{y-1} + \varkappa\left(-3 + \frac{1}{2}y\right) +$$

$$+ \varkappa_0\left(-3\frac{y}{y-1} + \frac{1}{2}y\frac{y}{y-1} - \frac{1}{2}y^2 + \frac{1}{2}y^2\frac{y}{y-1} + 2\varkappa - \varkappa y\right) = 0. \quad (3.3c)$$

$$\frac{1}{A} - 1 + \lambda C\{\frac{17}{6} - \frac{1}{6}y + \frac{3}{2}\frac{y}{y-1} - \frac{7}{6}y\frac{y}{y-1} + z(-1 + \frac{1}{2}y) + \varkappa(-\frac{1}{6}) +$$

$$+ \varkappa_o(-\frac{1}{6}y - \frac{1}{6}\frac{y}{y-1} + \frac{1}{6}y\frac{y}{y-1} - \frac{1}{6}\varkappa + \frac{1}{6}\varkappa y)\} = 0. \tag{3.3d}$$

$$\frac{1}{A}\frac{y}{y-1} - 1 - \lambda C\{-\frac{10}{3} - \frac{1}{3}y - 2\frac{y}{y-1} + \frac{5}{3}y\frac{y}{y-1} + z(2 - \frac{1}{6}y - \frac{2}{3}\frac{y}{y-1} - \frac{1}{3}y\frac{y}{y-1}) + \varkappa\frac{2}{3} +$$

$$+ \varkappa_o(\frac{2}{3}y + \frac{2}{3}\frac{y}{y-1} - \frac{2}{3}y\frac{y}{y-1} + \frac{2}{3}\varkappa - \frac{1}{6}\varkappa y)\} = 0. \tag{3.3e}$$

$$\frac{1}{C} - 1 + \lambda A\{-\frac{4}{3} - \frac{1}{3}y + \frac{17}{3}\frac{y}{y-1} - \frac{7}{3}y\frac{y}{y-1} + \frac{1}{6}y^2 + \frac{1}{3}y^2\frac{y}{y-1} + \varkappa(-4 + \frac{8}{3}y - \frac{1}{2}y^2) +$$

$$+ \varkappa_o(-4\frac{y}{y-1} + \frac{8}{3}y\frac{y}{y-1} - \frac{1}{6}y^2 - \frac{1}{3}y^2\frac{y}{y-1} + \frac{8}{3}\varkappa - \frac{7}{3}\varkappa y + \frac{1}{2}\varkappa y^2)\} = 0, \tag{3.3f}$$

where $\lambda = e^2/16\pi^2$, $\varkappa = \varkappa' A$.

We see that seven parameters enter set (3.3)

$$A, C, y, Z, \quad \lambda = e^2/16\pi^2, \quad \varkappa_o, \quad \varkappa .$$

The parameters $\varkappa$ and $\varkappa_o$ are in general not independent but they have to be connected between each other by the vertex equation (see Fig. 2c.). Hence we have in principle six equations for six parameters including the charge. We do not take into account the vertex equation in this approximation, so let us consider the most natural case $\varkappa_o = 0$. In this case as it was noted earlier the Lagrangian (2.1) coincides with the electromagnetic part of the Salam-Weinberg Lagrangian. Note, that other versions have also been studied by us, for example the case $\varkappa = 0$, $\varkappa_o \neq 0$. We have found that the solution of the set changes insignificantly in comparison with the solution which is written down below.

Now for $\varkappa_0 = 0$ from equations (3.3a,b,c) we obtain the simple equation for y:

$$3y^2 + 16y - 44 = 0 \tag{3.4}$$

which has two roots: $y_1 = -22/3$, $y_2 = 2$. Accordingly our set has two solutions

$$y = -\frac{22}{3}, \quad z = \frac{4}{9}, \quad \varkappa = -\frac{1}{25}, \tag{3.5}$$

$$A = 3.74, \quad C = 13.12, \lambda = 5.1 \cdot 10^{-3};$$

$$y = 2, \quad z = 2, \quad \varkappa = 5, \tag{3.6}$$

$$A = 1, \quad C = 1, \; \lambda = -\frac{1}{3}.$$

The second solution gives negative coupling constant squared therefore it is physically inadmissible. By the way the obvious constraints to be satisfied by the solution are: all the parameters have to be real, $\lambda > 0$, $C \geqslant 1$. The first solution satisfies all these conditions and moreover gives small values for the parameters λ and $\varkappa$. The smallness of λ justifies to some extent our one-loop approximation because the terms neglected are proportional to λ^2. The smallness of $\varkappa' = \varkappa/A$ agrees with the initial assumption $\varkappa_0 = 0$, since one can expect $\varkappa' - \varkappa_0 \sim \lambda \ln\lambda$ /3/ by the order of magnitude.

So we conclude that in the given approximation the desired scale solution exists only for one chosen value of the bare coupling constant. The well-known Lehmann theorem /12/ helps us to get the renormalized coupling constant. Namely this theorem being applied to the photon propagator yields the relation between asymptotics of the renormalized propagator in the structure $g_{\mu\mu'}$ and the charge renormalization constant z_3. In our case the requirement for absence of divergences quaranties that the residue of $D_{\mu\mu'}(k^2)$ at $k^2 = 0$ in the necessary structure is equal to 1 (one should bear in mind, that the charged particle is massive), i.e. the obtained propagator is the renormalized one. Therefore with the Lehmann theorem we have a relation $z_3^{-1} = C$. Hence the renormalized coupling constant is expressed in the following way

$$\alpha_r = \frac{e_r^2}{4\pi} = \frac{e^2}{4\pi C} = \frac{4\pi\lambda}{C} . \tag{3.7}$$

Inserting here the numbers from (3.5) we have

$$\alpha_r = 4.93 \cdot 10^{-3} \tag{3.8}$$

So we have unique physically acceptable scale solution in the one-loop approximation. Note that the obtained value of the renormalized coupling constant is close to the physical value of electromagnetic constant $\alpha = 1/137 = 7.30 \cdot 10^{-3}$.

4. TWO-LOOP APPROXIMATION

The natural step in the further developement of the problem is a simultaneous account of two-loop terms in the equations for propagators (see Fig. 2a,b) and of one-loop terms in the vertex equation (Fig. 2c). In doing this we insert the vertex equation into the propagator equations, keeping only one- and two-loop terms.

As a result we get a set of equations, presented in Fig.3.The insertion of the vertex equations allows us to take into account the transverse structures in the vertex. Therefore we in particular omit in the full vertex (2.9) the $\mathcal{X}$ term, which serves for the effective account of transverse terms. Moreover we again consider the case $\mathcal{X}_0 = 0$ according to the reasons mentioned above.

Now we put expressions (2.3)-(2.5), (2.8)-(2.11) into the set of equations in Fig. 3 and use again the dimensional regularization method to calculate the integrals. The typical two-loop integrals are of the form

$$\int \frac{d^n q \; d^n k}{(q^2)^\alpha (k^2)^\beta [(p-q)^2]^\gamma \; [(p-k)^2]^\delta [(q-k)^2]^\sigma} . \tag{4.1}$$

These integrals are calculated by means of the technique using the expansion in Gegenbauer polynomials in x-space [13]. Very tedious evaluations arising in this approximation were made with the computer program of analytic calculations SCHOONSCHIP in JINR (see review [14]). The set of equations for our parameters, which is obtained by setting to zero the coefficients afore $1/\varepsilon$ and of the equality of finite parts in the equations of the set (Fig. 3) has the following form

$$A_1^{(1)} - \lambda C \cdot A_1^{(2)} = 0, \tag{4.2a}$$

$$B_1^{(1)} - \lambda C \cdot B_1^{(2)} = 0, \tag{4.2b}$$

$$C_1^{(1)} - \lambda C\cdot C_1^{(2)} = 0, \tag{4.2c}$$

$$\frac{1}{A} - 1 + \lambda AC\cdot A_o^{(1)} - \lambda^2 AC^2\cdot A_o^{(2)} - \lambda^2 AC^2\delta A_1^{(2)} = 0, \tag{4.2d}$$

$$\frac{1}{A}\frac{y}{y-1} - 1 - \lambda ACB_o^{(1)} - \lambda^2 AC^2 B_o^{(2)} - \lambda^2 AC^2\delta B_1^{(2)} = 0, \tag{4.2e}$$

$$\frac{1}{C} - 1 + \lambda A^2 C_o^{(1)} - \lambda^2 A^2 CC_o^{(2)} - \lambda^2 A^2 C\delta C_1^{(2)} = 0, \tag{4.2f}$$

where the symbols $A_n^{(m)}$, $B_n^{(m)}$, $C_n^{(m)}$ denote the coefficients by the poles $\frac{1}{\varepsilon^n}$ in order λ^m. In the Appendix we give the one-loop terms (m=1) and one example of two-loop coefficient $C_o^{(2)}$.

The new parameter appears in set (4.2)

$$\delta = \ln(\frac{\mu^2}{p^2}) - \gamma - \ln\pi \tag{4.3}$$

where $\gamma = 0.577$ is the Euler constant and μ is the regularization parameter introduced in the dimensional regularization by t'Hooft/15/. The parameter δ arises because of the calculations of divergent integrals are ambiguous in any regularization scheme in particular in the dimensional one. According the prescription of paper/15/ one should set $\mu^2 \sim p^2$ while studying the ultraviolet asymptotics so δ becomes some new numerical parameter. Note that an analoguous situation occurs in evaluations of Green's function asymptotics by renormalization group methods/16/. It should be noted also that the dependence on δ is connected with truncation of the perturbation series in usual methods and in our case it is connected with the subsequent approximation method used. This dependence should be absent in exact solutions. Therefore it is reasonable to consider weak dependence on δ of a result as a criterion of applicability of approximation.

The set of equations (4.2) is a complicated non-linear algebraic one imposed on the parameters $A, C, y, z, \lambda, \delta$. That is why we had to use the Monte Carlo method to find the numerical solution. The following approximate solution is obtained

$$
\begin{aligned}
&y = -3.504, \qquad z = -3.794, \qquad A = 1.010, \\
&C = 1.014, \qquad \delta = -4.920, \qquad a_r = 6.67 \cdot 10^{-3}.
\end{aligned}
\tag{4.4}
$$

Here we write down the value of renormalized coupling constant $a_r = e_r^2/4\pi$, instead of λ using relation (3.7).

The influence of the variation of δ on the solution was analyzed when investigating the set (4.2). It occurs that while δ is varied in wide ranges ($|\delta| \leqslant 100$) other parameters change slightly. This serves as an indication to applicability of the approximation as we have mentioned above.

Let us emphasize another important circumstance. In two-loop integrals there are terms proportional to $1/\varepsilon^2$ as well as $1/\varepsilon$ poles. The coefficients afore $1/\varepsilon^2$ $A_2^{(2)}$, $B_2^{(2)}$, $C_2^{(2)}$ should vanish for the values of the parameters corresponding to the solution of the system (4.2) if the given approximation is completely self-consistent. The substitution of (4.4) into the expressions for these coefficients does not give exact cancellation, though it leads to small values of $A_2^{(2)}$, $B_2^{(2)}$, $C_2^{(2)}$ being by two-three orders of magnitude less than other two-loop coefficients $A_{1,0}^{(2)}$, $B_{1,0}^{(2)}$, $C_{1,0}^{(2)}$. Here one should bear in mind that solution (4.4) was approximated. Furthermore we might expect complete consistency only for an exact solution, i.e. if we take into account the succession of subsequent approximations as a whole (all n-loop contributions). In any case, the smallness of these coefficients may indicate to the given approximation being reasonable.

Comparing the solution (4.4) with the solution of one-loop approximation (3.5), (3.8) we see that the value of renormalized coupling constant α_r changes slightly, whereas the other parameters do change significantly. However the very existence of the solution and its properties discussed above serve as an argument for the validity of the proposed approach.

5. ON POSSIBILITY TO INTRODUCE FERMIONS

As we mentioned earlier attempts to find the scale solutions in renormalizable theories, for example, in the spinor electrodynamics[7] had failed. However any realistic model has to include also spinor fields. Therefore let us consider the question how spinor contributions, taken into account very roughly, influence the solution in the framework of our approach.

As the first step let us add N spinor loops with free vertices and propagators to equation (3.2) for the photon propagator in the set of one-loop approximation. This corresponds to the simplest account of N fermions with charges e, or equally to other combinations of spinors, the sum of their charges squared in units of e being equal to N. As a result in the set (3.3) (where $\varkappa_0 = 0$) the term $-\frac{4}{3}\frac{N}{A}$ is added to Eq. (3.3c) and $-\lambda N(\frac{20}{9} - \frac{4}{3}\ln 2)$ to Eq(3.3f), and the other equations do not change. Equation (3.4) which defines y, transforms into the following one

$$(3y^2 + 16y - 44) + 8N\frac{(y-1)(3y+2)}{3y^2 - 6y - 16} = 0 . \tag{5.1}$$

The dependence of the solution of (5.1) on N was studied. It has turned out that for $N \geqslant 9$ the equation has no real solutions. The solution of the set for N = 1

$$\begin{aligned} &y = -7.08, \quad z = 0.55, \quad \varkappa = -0{,}08, \quad A = 3.65, \\ &C = 13.29, \quad \alpha_r = \frac{4\pi\lambda}{C} = 5.1 \cdot 10^{-3} \end{aligned} \tag{5.2}$$

differs only slightly from the solutions (3.5), (3.8) for N = 0. While N increases from 1 to 8 the parameters change monotonously and for N = 8 the solution looks like

$$\begin{aligned} &y = -4.38, \quad z = 1.78, \quad \varkappa = -0{,}81, \quad A = 2.10, \\ &C = 12.95, \quad \alpha_r = 11.5 \cdot 10^{-3}. \end{aligned} \tag{5.3}$$

Thus there exists some boundary number N_0 ($N_0 = 9$ in this approximation) so that for $N < N_0$ the solution exists and remains in the region of small coupling constants α_r, and for $N \geqslant N_0$ the solution does not exist at all.

One may propose a more consistent model including fermions. Namely let us consider alongside with the Schwinger-Dyson equations for vector particles corresponding equations for charged spinors. The set of equations is given in Fig. 4. Assuming the scale form of Green's function asymptotics for spinors too $S(p) \to F\,\hat{p}/p^2$, $\Gamma_\mu \to$ $\to ie\gamma_\mu/F$, one can easily get convinced by simple calculations that the necessary condition for the existence of the solution is the transversality of the photon propagator in asymptotics, i.e. $z = 1$ (the asymptotic "Landau gauge"). As it is well-known, in this case there is no divergences in the equation for the fermion propagator (Fig. 4a), and the equation for F reads

$$\frac{1}{F} = 1 - \frac{e^2}{16\pi^2}C\,. \tag{5.4}$$

Thus the set of equations (fig. 4) reduces to the one considered in the first part of this section where $N \to N_{eff} = NF$ and under the condition $z = 1$. Considering N_{eff} as a parameter, we come to the following solution

$$\begin{gathered} y = -6,\ z = 1,\ \varkappa = -0.29,\ A = 3.05, \\ C = 13.22,\ F = 1.10,\ N_{eff} = 4.57, \\ N = N_{eff}/F = 4.14,\ \alpha_r = 6.75\cdot 10^{-3}. \end{gathered} \tag{5.5}$$

One should not be surprised with the fact that the parameter N occurs to be not integer, because the considered scheme was obviously approximate. The existence of the unique solution having the desired properties and admissible value of renormalized coupling constant are the positive results of this work.

6. CONCLUSION

The results obtained here allow us to make the conclusion, that the existence of scale asymptotic solutions in the vector electrodynamics does not contradict the Schwinger-Dyson equations. It is important that such solutions occur only for the particular value of coupling constant. We would like to draw attention to the fact that the obtained value of renormalized coupling constant lies in the region close to the value of the well-known electromagnetic constant $\alpha = 1/137 = 7.30 \cdot 10^{-3}$. This is really quite natural because we are dealing with the electrodynamics itself. Namely in the one-loop approximation of the vector electrodynamics (section 3)

$$\alpha_r = 4.93 \cdot 10^{-3}. \qquad (6.1)$$

The two-loop approximation (section 4) gives

$$\alpha_r = 6.67 \cdot 10^{-3}. \qquad (6.2)$$

The one-loop approximation for the electrodynamics of vector and spinor particles leads to the value

$$\alpha_r = 6.75 \cdot 10^{-3}. \qquad (6.3)$$

It is interesting that both taking into account of two-loop terms and the introduction of charged spinors lead to the change of the value (6.1) towards the value α.

Note also that the requirement of the scale asymptotics to occur imposes restriction on the possible number of charged spinor particles introduced in the scheme.

One cannot think of course that the approach is free of difficulties which have been discussed in particular in section 4. Maybe we shall succeed in a formulation of a more adequate approximation in the future.

Moreover it is an important problem in the framework of our approach to construct the regular method which makes it possible to calculate not only the Green's function asymptotics but to find their behaviour in the whole region of variation of momentum variables also in particular near the mass shell. This is obviously necessary for the calculation of physical amplitudes in the scheme under consideration.

Search for possibilities to apply the similar method in Salam--Weinberg type models seem to be of great interest. If one succeded in this, it would give a possibility to get rid of Higgs scalars.

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Appendix

We quote here one-loop coefficients of the set (4.2) and write down one of two-loop coefficients. It seems impossible to show all 9 two-loop coefficients as they are rather cumbrous.

$$A_o^{(1)} = \frac{1}{6}(26 - 8y + 3yz - 8z)$$

$$B_o^{(1)} = \frac{1}{6}(-32 + 8y - 3yz - 8z)$$

$$C_o^{(1)} = \frac{1}{6}(32 - 16y + 3y^2)$$

$$A_1^{(1)} = \frac{1}{6}(19 + 3y + 3z)$$

$$B_1^{(1)} = -\frac{1}{6}(22 + 3y + 3z)$$

$$C_1^{(1)} = \frac{1}{6}(19 + 6y)$$

$$\begin{aligned}
C_o^{(2)} = & -\frac{667}{192} + \frac{13387}{1152}y + \frac{48751}{1151}y\frac{y}{y-1} - \frac{51}{128}\frac{y^2}{y-1}z + \\
& + 4\frac{y^2}{y-1}z\zeta(3) - \frac{13}{2}\frac{y^2}{y-1}\zeta(3) + \frac{25211}{576}\frac{y^3}{(y-1)^2} + \\
& + \frac{191}{32}\frac{y^3}{(y-1)^2}z + 6\frac{y^3}{(y-1)^2}z\zeta(3) + \frac{25}{2}\frac{y^3}{(y-1)^2}\zeta(3) - \frac{443}{384}y\cdot z + \\
& + y\zeta(3) - \frac{12401}{1152}y^2 - \frac{3739}{576}\frac{y^3}{y-1} - \frac{25}{1152}\frac{y^3}{y-1}z - 6\frac{y^3}{y-1}z\zeta(3) - \\
& - 5\frac{y^3}{y-1}\zeta(3) + \frac{1007}{1152}\frac{y^4}{(y-1)^2} + \frac{407}{1152}\frac{y^4}{(y-1)^2}\cdot z + 4\frac{y^4}{(y-1)^2}z\zeta(3) + \\
& + \frac{9}{2}\frac{y^4}{(y-1)^2}\zeta(3) + \frac{841}{576}y^2z + 2y^2z\zeta(3) + 2y^2\zeta(3) - \\
& - \frac{145}{8}\frac{y^4}{(y-1)^2} + \frac{343}{64}y^3 - \frac{689}{128}\frac{y^4}{y-1} + \frac{619}{288}\frac{y^4}{y-1}z + \frac{15}{2}\frac{y^4}{y-1}z\zeta(3) + \\
& + \frac{2057}{384}\frac{y^5}{(y-1)^2} - \frac{1187}{1152}\frac{y^5}{(y-1)^2}z - \frac{9}{2}\frac{y^5}{(y-1)^2}\zeta(3) - \frac{1289}{1152}y^3z - \\
& - 3y^3\zeta(3) - \frac{7}{48}y^4 - \frac{1}{8}\frac{y^5}{y-1} + \frac{7}{48}\frac{y^6}{(y-1)^2} - \frac{19193}{576}\frac{y}{y-1} + \\
& + \frac{215}{16}\frac{y}{y-1}z + 38\frac{y}{y-1}\zeta(3) - \frac{3065}{48}\frac{y^2}{(y-1)^2} - \frac{127}{24}\frac{y^2}{(y-1)^2}z + \\
& + 2\frac{y^2}{(y-1)^2}z\zeta(3) - \frac{49}{2}\frac{y^2}{(y-1)^2}\zeta(3) - \frac{135}{16}z - 10\zeta(3);
\end{aligned}$$

where $\zeta(3) \approx 1.202.$

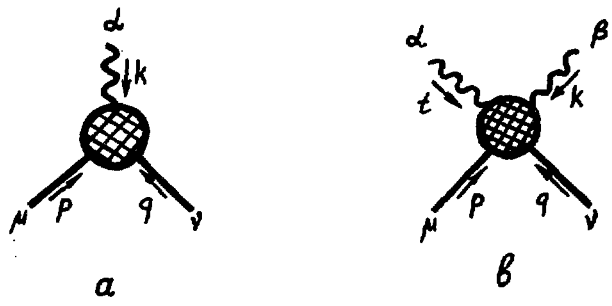

Fig. 1. a) one-particle irreducible triple vertex $i\Gamma_{\mu\nu\alpha}$ (p, q; k); b) one-particle irreducible quartic vertex $i^2\Pi_{\mu\nu\alpha\beta}$ (p, q; t, k); the wave line corresponds to the photon, the solid one to the charged vector particle.

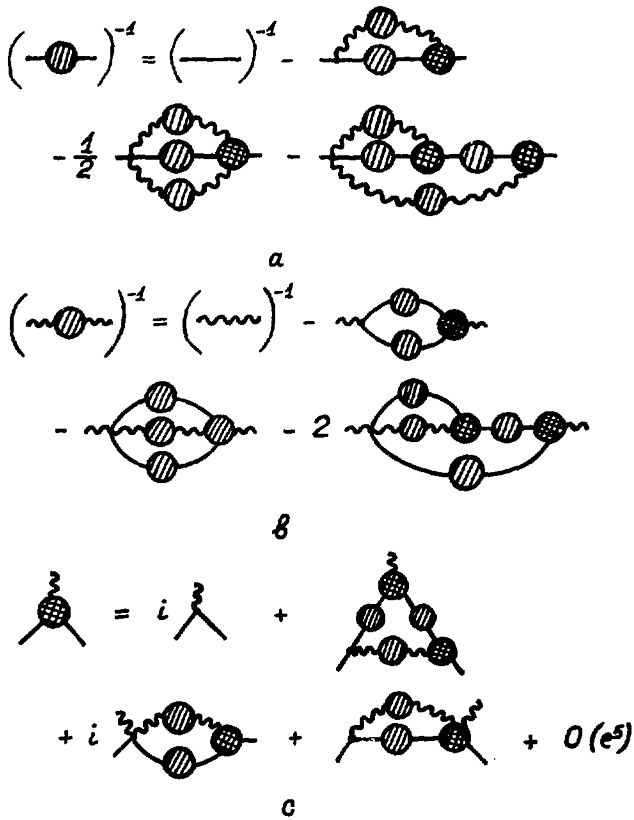

Fig. 2. a) the equation for full propagator $G_{\mu\nu}$ of the charged particle; b) the equation for full propagator $D_{\mu\nu}$ of the photon; c) the equation for one-particle irreducible vertex ($O(e^5)$ denotes the contribution of two-loop terms). "Tadpole" terms are not shown because we use in the following the dimensional regularization where they vanish.

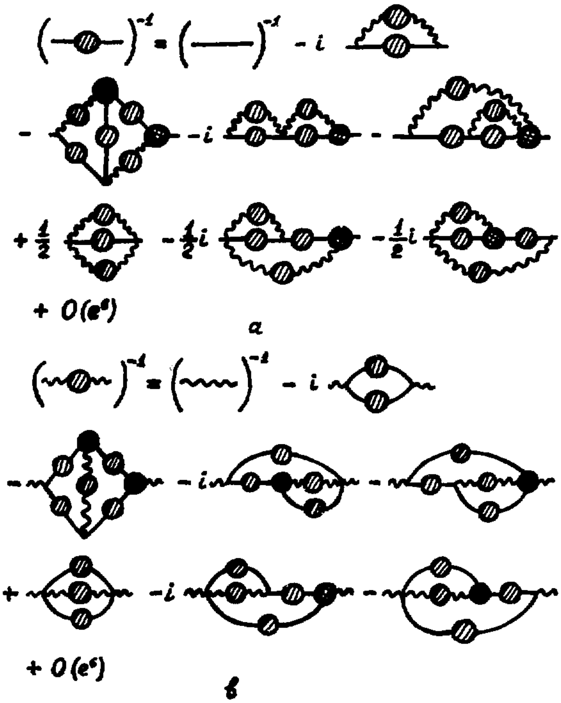

Fig. 3. The two-loop approximation of the Schwinger-Dyson equations. $O(e^6)$ denotes three-loop terms.

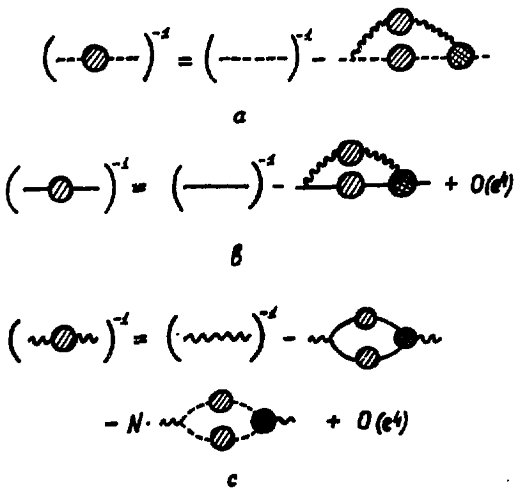

Fig. 4. The set of equations for electrodynamics of vector and spinor particles in the one-loop approximation. The dotted line corresponds to the charged fermion propagator. $O(e^4)$: two-loop contributions.

Цена 12 коп.

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