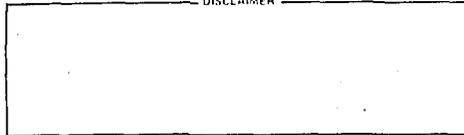


Bent approximations to synchrotron radiation optics

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## Bent approximations to synchrotron radiation optics

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Ideal optical elements can be approximated by bending flats or cylinders. This paper considers the applications of these approximate optics to synchrotron radiation. Analytic and raytracing studies are used to compare their optical performance with the corresponding ideal elements. It is found that for many applications the performance is adequate, with the additional advantages of lower cost and greater flexibility. Particular emphasis is placed on obtaining the practical limitations on the use of the approximate elements in typical beam-line configurations. Also considered are the possibilities for approximating very long length mirrors using segmented mirrors.

Introduction

A typical x-ray beamline has up to three major optical elements. The first element is a collimating mirror to match the synchrotron divergence to the acceptance of the monochromator. Since most monochromators are vertically dispersing, only the vertical divergence must be collimated, and either singly bent parabolic cylinders or paraboloids of revolution can be used. Figure 1 compares the acceptance angles of commonly used monochromator crystals with

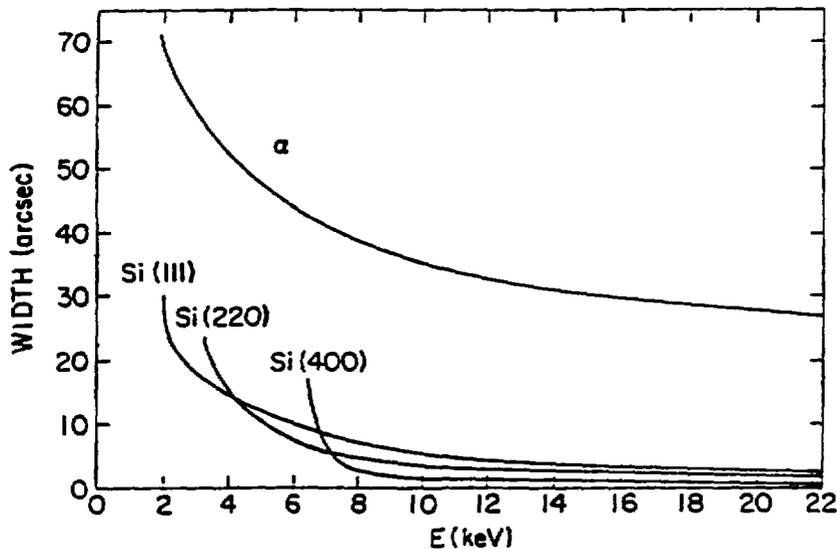


Figure 1. Comparison of the synchrotron radiation opening angle  $\alpha$  with the Darwin width of some Si reflections.

the vertical output angle of the x-ray ring at the NSLS. From this graph it is seen that a significant gain in efficiency is possible if the radiation can be made more parallel. Following the monochromator a focusing mirror is required. If no collimator is used an ellipsoidal figure is required. An ellipsoid also does a good job of focusing the beam if a parabolic cylinder is used to collimate the vertical divergence, although the vertical height at the focus is somewhat larger than the ideal. For a paraboloid collimator a second paraboloid produces a very good focus.

For high reflectivity at x-ray energies, these mirrors must operate at extreme grazing

angles. For example, a Pt surface must have a grazing angle 48 mr in order to reflect 10 keV x-rays. This results in very long mirrors. Table 1 gives the mirror lengths needed to collect a horizontal divergence  $\omega_h$  for three common figures. These formulas assume no vertical

Table 1. Expressions for the mirror length  $l_h$ , necessary to collect a horizontal divergence,  $\omega_h$ . The vertical divergence is assumed to be zero.

Mirror Figure	$l_h$
Paraboloid	$\frac{\omega_h^2 F_1}{16\theta^2 + \omega_h^2}$
Ellipsoid	$\frac{M+1}{2M} \frac{\omega_h^2 F_1}{8\theta^2 + \omega_h^2}$
Cylinder	$\left( -\omega_h^2 F_1 + 4R_s \theta \left[ 1 - \sqrt{\left( 1 - \frac{F_1 \omega_h^2}{2R_s \theta} - \frac{F_1^2 \omega_h^2}{4R_s^2} \right)} \right] \right) / (4\theta^2 + \omega_h^2)$

extent to the beam. Thus, for a real source a factor  $W_v/\theta$  must be added to these formulas where  $W_v$  is the vertical width of the beam at the mirror and  $\theta$  is the glancing angle. For typical applications, mirror lengths can exceed 1 m. Since off-axis paraboloids and ellipsoids can be difficult and expensive to produce in such lengths, this paper examines the possibilities of approximating these ideal elements with bent flat and cylindrical mirrors.

Most existing x-ray beamlines have used bent optical elements.<sup>1-4</sup> Ellipsoids are approximated by bent cylindrical mirrors and focusing in one dimension is accomplished by bending flat mirrors. The flats are usually bent in segments to minimize the difficulty of bending and supporting a long plate. It has also been shown that a plate can be bent to parabolic or elliptical cylinders with high accuracy by an appropriate contouring prior to bending.<sup>5-7</sup> The next sections compare the performance of these approximations with the ideal figures for both single and segmented bent mirrors.

### Flats

The bending of flat plates to approximate parabolic or elliptical cylinders has one important advantage over prefigured mirrors; the glancing angle can be easily changed by re-adjusting the applied bending moments. This allows the cutoff energy of the mirror to be adjusted in order to reject harmonics or to minimize the heat load reaching the next optical element, typically a monochromator crystal. Another advantage is the possibility of adjusting the mirror to compensate for thermal distortions. However, this is offset by the fact that a bent mirror must be relatively unconstrained and may be harder to cool.

Singly bent mirrors are generally not suited for collecting the horizontal divergence of a synchrotron source since due to the grazing angles required their effective aperture is very small. However, they are well suited for collimating or focusing the already collimated vertical divergence of a synchrotron source. In fact, in many cases a cylindrical approximation to an ellipse or parabola is nearly ideal. Thus, aberrations are rarely the limiting factors in the use of such mirrors. More important are considerations of adjustability and cooling as discussed above, along with the difficulty in constructing a suitable bending device as compared to obtaining prefigured mirrors. Therefore, the choice of bent flat mirrors is governed mainly by engineering rather than optical considerations. This is not the case for cylindrical approximations to paraboloids or ellipsoids as discussed below.

### Single cylinders

The geometry of a cylindrical mirror is shown in Fig. 2a. To approximate an ellipsoid or paraboloid the cylinder is bent to a large radius  $R_m$  along its length. For an ellipsoid

$$R_s = \frac{2F_1 F_2}{F_1 + F_2} \sin \theta \quad R_m = \frac{R_s}{\sin^2 \theta}$$

A collimating paraboloid is the limit as  $F_2$  goes to infinity for which  $R_s$  becomes  $2F_1 \sin \theta$ .

In order to evaluate the performance of the approximate optical components, it is

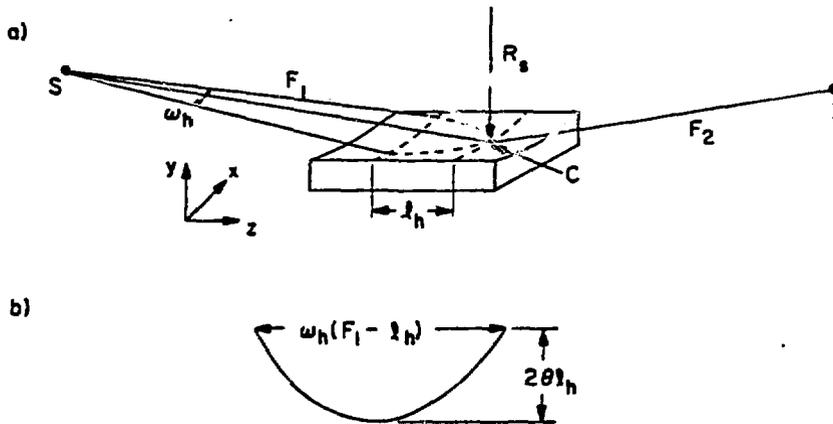


Figure 2a. Geometry of a toroidal mirror at grazing incidence. For paraboloids and ellipsoids  $R_s$  varies along the mirror. The radius  $R_m$  is very large and is not shown. b) Output of the mirror produced by a horizontal plane of radiation.

important to understand the limitations of their ideal counterparts. This is discussed in detail in reference 8 and only the relevant results are given here. For ellipsoids, the important feature is the quality of focus. The principal aberration is coma which causes an off-axis point to be imaged as a circular arc of length  $4M \phi d$  where

$$\sin \phi = \frac{M+1}{2M} \frac{w_h}{2\theta}, \quad M = F_2/F_1$$

and  $d$  is the distance off axis. For paraboloids, coma can be made negligible, and the important quality is the degree of collimation which is perfect for a point source. For real sources perfect collimation is not possible as shown in Fig. 3. The vertical source height gives a contribution  $\Delta\theta_v = S_v/F_1$ , where  $S_v$  is the vertical source size. For rays with horizontal divergence  $\pm w_h/2$  the horizontal source size,  $S_h$ , also gives a contribution. This is seen in Fig. 3b where parallel rays originating from different parts of the source are shown to intersect the mirror an amount  $\Delta l$  apart. Since the mirror is curved in this direction the outgoing rays can no longer be parallel, but have a  $\Delta\theta_p = w_h S_h / 2F_1 \theta$ . Thus, to find the actual distribution for the output angle, these two terms must be averaged over  $S_v$ ,  $S_h$ , and  $w_h$  for the source being considered.

The simplest approximation to these figures is an unbent cylinder. However, since it is unbent a cylinder provides no collimation in the vertical and is useless as a paraboloid. A short section of a cylinder is a better approximation to an  $M=1$  ellipsoid. Again there is no vertical focusing, but a point source with horizontal divergence  $\pm w_h/2$  will be focused to a "Vee" with dimensions

$$\Delta x = \pm \frac{w_h^3 F_1}{8\theta^2}$$

(1)

$$\Delta y = \frac{w_h^2 F_1}{4\theta}$$

In most cases  $\Delta x$  is small and the limitation on the cylinder approximation to an ellipsoid is the amount of vertical spread which can be tolerated.

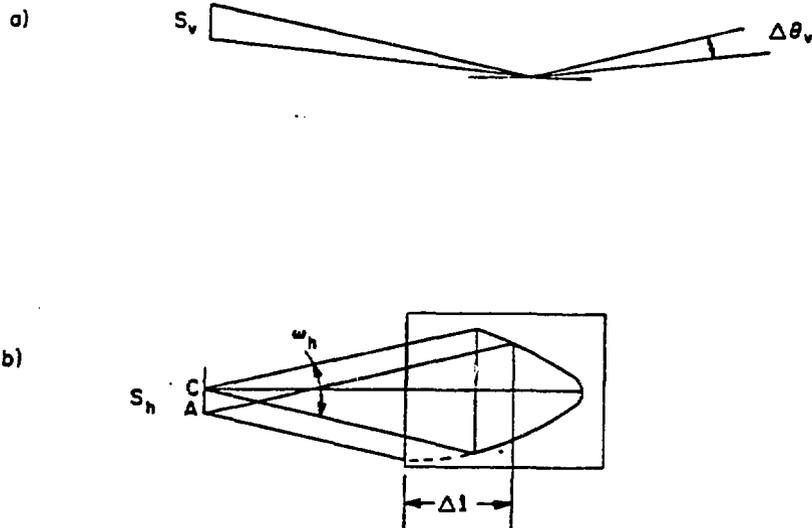


Figure 3. Sources of  $\Delta\theta$  for a paraboloid: a) vertical source size, b) horizontal source size.

If the cylinder is bent, much better results are obtained. For a cylinder bent to a toroidal figure, the results in equation (1) become,<sup>8</sup>

$$\Delta x = \pm \frac{5}{8} \frac{l_h^2}{F_1}$$

(2)

$$\Delta y = \frac{29 l_h^2}{F_1}$$

where  $l_h$  is given in Table 1. In this case the vertical divergence is well focused and the spread at the focus comes from the horizontal divergence  $\omega_h$  accepted by the mirror. Typically, the spread in the focus is unacceptable when  $\omega_h \sim \theta$ , although this depends strongly on the application. Equations (1) and (2) are for the  $M=1$  case. The more complicated expressions for  $M \neq 1$  are given in reference 8.

As approximations to paraboloids, toroids are less successful, but they still may be usable in some cases. The main problem is imperfect collimation. Rays originating from the source with horizontal divergence  $\pm \omega_h/2$  will have additional contribution to the angular spread given by,

$$\Delta\theta = \frac{\omega_h^2}{16\theta}$$

A practical restriction is  $\Delta\theta < \Delta\theta_v = S_v/F_1$  which gives a restriction on the horizontal divergence of  $\omega_h < 16\theta S_v/F_1$ . This is a few milliradians in most cases.

#### Segmented mirrors

As optical design is pushed to collect large  $\omega_h$ , segmented mirrors become more attractive. They can provide large effective mirror lengths from less expensive and more available short segments. Figure 4 shows two arrangements for approximating a long mirror from shorter bent cylindrical segments. In both cases the outer segments are lower in order to intercept the beam at the same distance from the source as the inner segment. The arrows indicate the direction of the applied bending moments. In Fig. 4b the segments are bent as a unit. This

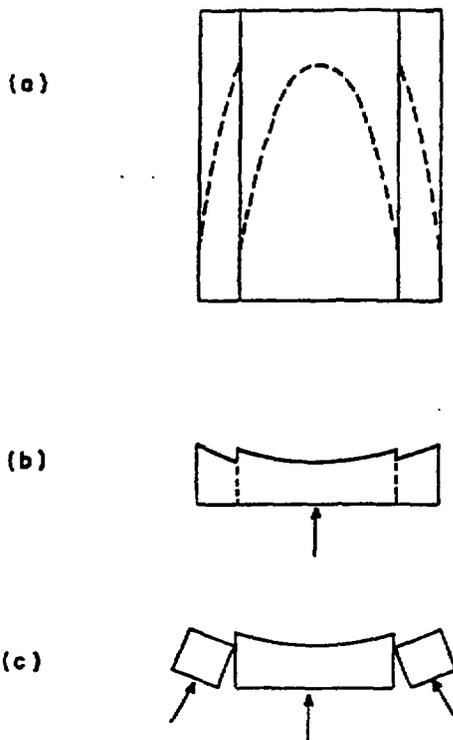


Figure 4. Two types of segmented mirrors: a) The intersection of a horizontal plane of radiation with both types of mirrors. In (b) all three segments are bent about the same axis while in (c) they are bent separately.

has the advantage of simplicity, but the aberrations are the same as a single bent cylinder with the same collecting power.

A better design is shown in Fig. 4c. In this case, each is bent separately. The direction of the bending moments all point to a common center a distance  $R_s$  above the center mirror. Also, the outer segments must be made slightly nonparallel to the center segment in order to bring all the foci into coincidence. Obviously such a mirror is more difficult to align. However, with exception of the bend, all of the alignments can be done optically prior to installation in the beamline. The aberrations are much reduced, and are determined for each segment by the horizontal divergence collected by it alone. For equal length segments, the center segment collects the most milliradians and, thus, dominates the aberrations. For bent cylinders, these are strong functions of  $\omega_h$ , and are, therefore, greatly reduced by segmentation.

Table 2 compares the performance of an ideal ellipsoidal mirror with two approximations: a single toroidal mirror and a three segment toroidal mirror with the segments bent separately. The design parameters appropriate to the NSLS x-ray ring source are used,  $2\sigma_x = .75$  mm and  $2\sigma_y = .25$  mm. This result shows the significant length reduction and improved performance which can be achieved using a segmented cylinder.

#### Conclusions

This paper has shown the limitations in applying bent mirrors to x-ray beamlines. These limitations are fairly severe for approximations to paraboloids, and bent cylinders are likely to have only limited use in this application. For focusing applications, bent cylinders are good approximations when  $M \ll 1$ . Typically a horizontal divergence equal to the

Table 2. Comparison of bent cylindrical and segmented bent cylindrical approximations to an ellipsoidal focusing mirror. The mirror parameters are  $\theta = 5$  mr,  $F_1 = 10$  m and  $M = 1$ . The source parameters are  $\omega_h = 5$  mr,  $2\sigma_x = 0.75$  mm,  $2\sigma_y = 0.25$  mm and  $E = 10$  keV.

	L	Focus:	
Ellipsoid	1447 mm	.67 mm	.45 mm
Bent Cylinder	1450	1.03	.67
Segmented Bent Cylinder (3 segments)	828	.84	.35

grazing angle can be collected without serious aberrations. A larger collection angle can be achieved by using segmented mirrors to approximate very long mirrors. In this case, the limitation is not the optical aberrations, but rather the increased mechanical complexity involved in mounting and bending a number of segments. However, much of this can be done optically prior to mounting in the beamline. The performance of singly bent flats can be made nearly ideal. Their use depends more on engineering rather than optical factors.

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