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THE INTELLECTUAL QUADRANGLE: MACH-BOLTZMANN-PLANCK-EINSTEIN*).

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ABSTRACT

These four men were influential in the transition from classical to modern physics. They interacted as scientists, often antagonistically. Thus Boltzmann was the greatest champion of the atom, while Mach remained unconvinced all his life. As a physicist, Einstein was greatly influenced by both Mach and Boltzmann, although Mach in the end rejected relativity as well. Because of his work on statistical mechanics, fluctuations, and quantum theory, Einstein has been called the natural successor to Boltzmann. Planck also was influenced by Mach at first. Hence he and Boltzmann were adversaries until Planck converted to atomistics in 1900 and used the statistical interpretation of entropy to establish his radiation law. Planck accepted relativity early, but in quantum theory he was for a long time partly opposed to Einstein, and vice versa -- Einstein considered Planck's derivation of his radiation law as unsound, while Planck could not accept the light quantum. In the case of all four physicists, science was interwoven with philosophy. Boltzmann consistently fought Mach's positivism, while Planck and Einstein moved from positivism to realism. All were also, though in very different ways, actively interested in public affairs.

CONTENTS

1. INTRODUCTION

Let me first justify the title of this lecture, which you have so kindly invited me to give here at CERN. Again and again in my rather long life devoted to science I have noticed that colleagues, especially of the younger generations, are only little aware how the great concepts in physics have been worked out. This is rather a pity. It is delightful to follow the ways and means of the great minds in our science and to analyse their interactions. But also on the level of concrete scientific achievement it may be argued that awareness of the past, with its successes and failures, is a great help and source of inspiration in choosing the future.

The behaviour of the leading scientists at intellectual crossways has, as it will turn out, been strongly influenced by philosophical (more exactly: by epistemological) considerations. Conversely, our picture of the world, our "Weltanschauung", has been determined by the decisions taken by the great minds at the crossroads. Clearly all this has great importance.

I do not belong to those who think that the study of the history of science can be divorced from that of the social environment. Scientific ideas did not arise and evolve merely through the interaction of scientists in their professional work; the scientists were also greatly influenced by historical events outside. Practical life posed questions to the scientists, and also provided materials and tools for them. On the other hand, the achievements of the scientists have been of crucial importance to human society. This is a point that will hardly be disputed, however.

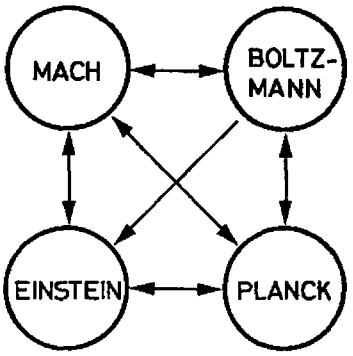
Right at the beginning I must make an apology. I am not a physicist, even less a theoretical physicist, but only a poor physical chemist and, more particularly, a biophysical chemist. I am sadly aware of my lack of knowledge in advanced theoretical physics and, to put it more drastically, of my ability to follow it in all its ramifications. For this reason alone I may be unable to answer some of your questions. Yet I do not feel that specialization in theoretical physics is indispensable for a grasp of the meaning of the great events in the field. At least not in the period with which we are going to deal today.

I shall submit to you some of my thoughts during quite a long period. Under the impact of the socio-political events, I began to study Boltzmann's work and its implications in 1942, during the Second World War, and I continued to do so later[1,2]. Further, the Albert Einstein anniversary year, 1979, was a good pretext to take up the problem of the relationship of the two giants, Boltzmann and Einstein[3-6]. In this connection, Ernst Mach could not and should not be bypassed. Inevitably, Max Planck also came in, a towering and deeply tragic figure in his own right.

All our four heroes came from a similar cultural environment, namely, 19th century Central Europe. Two were Austrians and two were Germans. How deeply rooted even Einstein was in that civilization can be seen from the fact that he preferred the German language to the end to express his thoughts, although he had lived in America during the last 22 years of his life, and although he had come not only to hate the Nazi barbarians, but also to reject all invitations to visit post-war Germany.

For this talk I chose the picture of a quadrangle with the four physicists in the corners. In our topology the quadrangle has no particular shape. All the four men can be

connected directly by sides or by diagonals. They all acted on each other, and with one regrettable exception (Einstein → Boltzmann) the interaction was mutual.



The Quadrangle

2. MACH AND BOLTZMANN

Let us start with Ernst Mach (1838-1916), the oldest of the Gang of Four[7,8]. He was an experimental physicist of great merits, professor at the Universities of Graz and of Prague -- at that time the most important provincial town in the Austro-Hungarian Empire. Mach was also successful in sense physiology. He did not consider himself a philosopher. Nevertheless, in Vienna where he had his last appointment, from 1895, his chair was in philosophy, with the official designation "History and Theory of the Inductive Sciences". Indeed his critical writings in the history of science had great epistemological influence. Because of a stroke, Mach had to retire prematurely in 1901, but his mind remained inquisitive and active, and he continued to write. Ironically, the chair of Mach was subsequently taken up by Boltzmann, who all along had been his great adversary in philosophy. Boltzmann at the same time remained the one professor of theoretical physics at Vienna University.

While Mach and Boltzmann personally esteemed each other, and had, for instance, friendly correspondence, they nevertheless had a running fight both in physics and in philosophy for decades. This struggle became a possibility, nay a necessity, after Mach in 1872, in Prague, had finally turned away from atomistics in his famous paper on "The History and the Root of the Theorem of the Conservation of Work". Let it be recalled that atoms were by no means generally accepted by the physicists in those times. Not only Mach, but also Pierre Duhem[9] in France (1861-1916), the powerful physical chemist Wilhelm Ostwald[10,11] in Leipzig (1853-1932) and even the young Max Planck[12-14] were opponents of atomistics. It may be added here that this continued enmity to atomistics in physics is all the more curious as in chemistry the atoms had become indispensable since the beginning of the 19th century, especially in organic chemistry. Apparently success in chemical industry required acceptance of atoms. That so many physicists rejected atomistics, while the chemists worked with atoms as a matter of course, is really an interesting case of intellectual schizophrenia.

Thus Mach in this respect confused contemporary science -- physics, chemistry, and biology. Moreover, he was unable to supply technical practice with a theory on which to build. This also applied to chemical technology and to biotechnology. In retrospect, all this is obvious. How Ostwald, originally a chemist, managed to do without atoms until 1908, when he converted to them, is almost a mystery.

At Vienna University, however, all positions in physics as well as in chemistry were held by atomists. Their leader was Josef Stefan[15] (1835-1893), known through the Stefan-Boltzmann law. By the way, Stefan was also the pioneer of Maxwell's ideas on the electromagnetic field in Central Europe. Stefan later became influential as Vice President of the Imperial Academy of Sciences at Vienna. His colleague and friend was the remarkable Josef Loschmidt[16,17], who in 1865 was the first man to venture a numerical estimate of the dimensions of atoms -- after two and a half millenia of purely speculative atomistics (1821-1895). Stefan's pre-eminent student was Ludwig Boltzmann (1844-1906), the greatest champion of physical atomistics in the whole of the 19th century.

Mach denied the existence of atoms from a philosophical point of view, that of positivism. Mach can really be considered as the founder of neopositivism. Influenced first by Kant, but later by Berkeley[18], the famous Anglican bishop, Mach came to see the task of science merely in the orderly connection of the results of observations, and refused to entertain the notion of an independent, objective, external world, which would be the source of man's sensations. We ought to limit ourselves to the purposeful combination of the "complexes of sensations" or "elements" and not try to go beyond direct experience. These "elements" should be put together in the simplest and most economical way, i.e. by application of the principle of economy of thought.

In respect to Mach, Boltzmann[19] said: "We find ourselves ... in a field where a whole family of modern methodological speculations grew up, expressed in the most precise and spirited way by Mach. He maintains directly that the aim of science is nothing but economy of labour. Almost with equal justification one could, noticing that in business greatest economy is desirable, consider this economy as the purpose of shops and money -- which would, in fact, in a certain sense be correct. Yet one will not easily term as mere economy the investigation of distances, movements, sizes, physical and chemical properties of fixed stars, the invention of microscopes, and the discovery therewith of the causes of our diseases".

In particular, the atoms fell under Mach's axe. He looked at atomistics as a piece of metaphysics. Mach during all his life never converted to atomistics, strange as it may sound. In 1910 Mach[20] wrote: "If the belief in the reality of atoms is so essential for you, I forsake the physical way of thinking, I do not want to be a real physicist ... I thank you very much for the community of the believers. For I prefer the freedom of thought". In his last book, The Principles of Physical Optics[21], written in 1913 and edited posthumously by his faithful and devoted son Ludwig in 1921, Mach was still arguing against what he called the "school" or "church" of atomistics. The wording is given, for example, in Heller's[7] and in Thiele's[22] books. Mach's student, the well-known Czech chemist B. Brauner, also confirmed that Mach remained an anti-atomist to the very end[23].

In contrast, Boltzmann made extensive use of models. They are not pieces of experience, but rather inventions of the human mind, though they must prove their worth in the practice of science. Thus Boltzmann created models to help in the understanding of Maxwell's electromagnetic field. This was all the more important as young physicists in Central Europe mostly learned Maxwell's work from Boltzmann[24]. Above all, Boltzmann never tired of emphasizing the tremendous value of atomistic models in kinetic theory and in statistical mechanics. The powerful exposition by Boltzmann, mainly in his popular writings[25], still deserves to

be read by any and every natural scientist. Let us quote, as an example[26]: "Contemporary atomistics gives a fully adequate picture of all mechanical phenomena. Considering the compactness of the field we shall hardly expect to find there phenomena that will not fit into the frame of the picture. The picture further includes the phenomena of heat. The fact that this cannot be demonstrated quite so clearly is due only to the difficulties in computing molecular motion. In any case, all essential facts are found in the features of our picture. It has proved enormously useful also in the interpretation of the facts of crystallography, of the constant proportions of the masses in chemical compounds, of chemical isomerism and the relationship between optical rotation and chemical constitution, and so on. Atomism is, moreover, capable of further large-scale development". In his obituary of Boltzmann in 1907 Hendrik Antoon Lorentz[27] referred to this passage, and added more items to the already impressive list.

In his models, Boltzmann was careful not to step beyond what he could really justify. "Hypotheses non finxit", to quote Isaac Newton. Boltzmann said[28]: "Simple consideration as well as experience show that it is hopelessly difficult to find the right pictures of the world by mere guessing in the dark. Rather, the pictures always form slowly from individual lucky ideas by fitting. Rightly, epistemology turns against the activities of the many lighthearted producers of hypotheses (Hypothesenschmiede) who hope to find a hypothesis explaining the whole of nature with little effort, as well as against the dogmatic and metaphysical derivation of atomistics".

Thus Boltzmann refrained from speculations about the internal structure of atoms, although at the time the problems of the optical spectra were much discussed among the physicists, and although Boltzmann even lived to witness the remarkable developments in radioactivity. Hypotheses on the internal structure of atoms were not required for Boltzmann's work, and could not be derived from it.

Now a few words about the position of Mach in history, presumably partly determined by his origins and experiences. A wellwishing observer may see Mach's criticism as a reaction against the easy dogmatism of many physicists of his time. There were many indeed who all too readily accepted as axioms far-reaching presumptions which made life easy but which they could not really justify. Thus scepticism and criticism in Mach's style were meritorious, indeed necessary. Mach wanted to draw attention to the lack of a secure basis for many of the current notions. Some of them were in principle inaccessible to experimental tests. Therefore he rejected them as metaphysical and as obstacles to an understanding of physics in depth. When we come to Einstein, the merits of Mach's criticism will be obvious.

Here some remarks about Mach's social situation may be in order. He came from a humble family and knew what poverty meant. All his life he remained conscious of the need for struggle to improve the lot of the working class and of other underprivileged people. He saw the enormous difference between the standard of life of most subjects of the Habsburg emperors, on the one hand, and of the rich on the other hand -- the owners of the big estates, the mines, and the factories, of the rich that were often idle as well. The roots of Mach's passionate antimetaphysics may be sought in his opposition to the system of values that in the Habsburg empire (and elsewhere, of course) had been imposed on the largely ignorant masses of the population, and which could not be justified rationally, but only metaphysically. In this context, in Austria a strong role was played by the all-powerful Roman

Catholic Church, utterly reactionary in those times and a pillar of the Habsburg empire. Mach was an outspoken atheist.

In Mach's time it was practically unheard of for an academic scientist to support the Socialists. Mach did. After 1914 Mach was also one of the very few, along with Einstein, to withhold his support from the terrible First World War and to take a stand against nationalism. This was duly pointed out by Einstein in his obituary of Mach in 1916 [29]. Boltzmann, who had died long before that war, likewise was a democrat and republican, but in his writings there is no word about socialism or against wars. He had come from a comfortable middle-class family, and he never left that orbit. It is quite possible that precisely this fact added to his despair when he clashed with the bearers of traditionalist views, notably in respect to atomism, materialism, Darwinism, and also republicanism. The resulting unhappiness may well have been one of the factors that in a dark hour drove Boltzmann to lay hand on himself.

It was Mach's tragic fate that he exaggerated his refusal to go beyond experience. He threw out the baby with the bath water. Thus he did not see the enormous heuristic value which freely-invented hypotheses and models can have, notably the hypothesis of the existence of atoms. He did not take note of the fact that through millenia atomism had been connected with a progressive outlook. As late as 1624 the King of France had still threatened the teachers of atomistics with the death penalty[30].

3. MACH AND PLANCK

Now let us turn to Max Planck (1858-1947). His social origins were utterly different from those of Mach[12,13,31]. He came from a family of respected members of the upper middle class in Northern Germany. None of them had a quarrel to pick with existing conditions in the Hohenzollern empire. All his life Planck remained a conservative not only in politics but also in his scientific work. Only his intellectual integrity induced him to accept fundamental innovations in science. Planck tended to minimize the impact of his own quantum hypothesis on physics in general and would have loved to explain the quantum in classical terms[32]. As has been said, in physics he was a revolutionary against his own will.

Similarly, Planck's admirable objectivity must have motivated him, as the editor of the leading professional journal, Annalen der Physik, to accept for publication Einstein's first paper on relativity in 1905 without difficulty. Indeed he became one of the earliest and most consistent supporters of relativity. Planck may have seen here a final contribution towards the wonderful structure of classical, Newtonian and Maxwellian, physics rather than an intellectual revolution. Nevertheless, initially Einstein's paper must have been quite a shock for Planck. In the support of relativity, Planck was followed quite early by his student Max von Laue (1889-1960).

I feel that it was consistent with Planck's upbringing and general situation in life that basically he should have had no sympathy with Mach's positivism. All the same, Planck admitted -- indeed Planck stressed -- that in his young days he was influenced by Mach[14,33]. Einstein also probably referred to Planck when in his obituary of Mach[29] in 1916 he said: "I even think that those who consider themselves as opponents of Mach hardly know how much of Mach's way of looking at things they so-to-say have sucked up with their mother's milk." It is plausible that the well-known attraction of the "absolute" for Planck not only made

him admire thermodynamics and accept the phenomenological approach to it, but also, because of its uncertainty, reject the use of models involved in realism.

But in the first years of this century Planck turned away from Mach. Planck's philosophical opposition to Mach reached a first high point in his famous address to students at Leyden[34] in 1908, where he ended with the words of the Sermon on the Mount, warning mankind of the false prophets: "You shall recognize them by their fruit!" Planck here referred to the notorious barrenness of positivism. Mach's reply[20] led to an even more strongly worded attack by Planck[33]. Planck continued the polemic against positivism for the rest of his life[13].

This last paragraph had already been written when I came across an important lecture by Res Jost[14] on Mach and Planck. Jost argues convincingly that the influence of Mach on Planck had at first been stronger than is now generally suspected. The intense, indeed violent, aversion of Planck to Mach in his later years was fuelled by deep resentment that Planck had, for many years, under the influence of Mach missed the importance of atomism and more particularly of the achievements of Boltzmann. Planck now looked at Mach as a seducer -- a false prophet, as he had said in 1908. At the age of 82, Planck wrote in 1940 [35]: "Fifty years ago I counted myself still among the convinced positivists .... At that time I was, under the influence of Ernst Mach, inclined towards a rejection of atomism. Thus I attracted, to my regret, the opposition of Ludwig Boltzmann, who at that time, almost alone in Germany devoted his main effort to the construction of the kinetic gas theory, and with whom I should have liked to feel associated in the struggle for the Second Law of Thermodynamics against the flat energism of Ostwald." Boltzmann, though an Austrian, was professor in Munich for a time, and in any case appeared in Germany often. We shall return to the relationship of Planck and Boltzmann below.

The political development of Planck was likewise determined by his conservatism, on the one hand, and by his integrity, on the other. Thus in 1914 he felt obliged to sign the ill-famed Manifesto of 93 intellectual leaders, scientists and artists, in explicit favour of German militarism. The term was used directly. After the Nazis had come to power in 1933, Planck's conservative views on State authority, inherited from his family and his milieu, induced him to attempt some accommodation even with the new government, however much he disliked it. So until 1937 he remained President of the prestigious and rich Kaiser-Wilhelm-Gesellschaft zur Förderung der Wissenschaften, founded in 1911 on the initiative of Walther Nernst, and renamed Max-Planck-Gesellschaft later, after the defeat in the Second World War. He did not resign as Secretary of the Prussian Academy of Sciences in Berlin until 1938. Planck was not capable of head-on resistance against established power. Even more. To quite an extent he succumbed to official slogans. Thus, on 13 April 1933, he wrote from his holiday place in Sicily to a close and highly regarded colleague in the Prussian Academy, Heinrich von Ficker, about "the infamous hate and atrocity propaganda (from abroad: E.B.) that has been poured over Germany since the nomination of Hitler as Reich Chancellor"[36].

Yet Planck tried to mitigate the harsh measures of the Nazi régime. As we shall see, he spoke out for Einstein in public. Planck's intention to prevent the worst also moved him to speak in favour of some of Nazism's victims during an interview with Hitler early in 1933 [37]. However, as soon as Hitler noticed Planck's intentions he embarked on one of his

vicious monologues. The interview was fruitless. On the contrary, Planck's upright stand may well have contributed to Hitler's ready confirmation of the death sentence against Planck's only surviving son (and child) from his first marriage -- and his best friend, as he said -- in 1944 for conspiracy against the Nazi régime. Planck's eldest son had been killed on the battlefield in the First War.

Perhaps here we should add that Planck, as he insisted, was deeply religious all his life[13], but that he did not believe in a personal, still less in the Christian, God. He stated this himself a short time before death[38]. This fact may come as a surprise to some, especially after hearing Planck quote the Sermon on the Mount, i.e. the Gospel of St. Matthew. That Planck took part in the activities of the Confessing Church in Berlin-Dahlem under the Nazi régime was surely due to his wish, in those dark times, to help where he could.

I have long felt that a critical biography of Planck, not only as a scientist, but also as a man within the historical, political, and social context, would be desirable. Such a book would give a moving picture of the tragic history of the intellectual leadership during Germany's way from the Hohenzollern régime to Hitlerism, with all its contradictions. Much material has already been provided by the late Kurt Mendelssohn in his excellent work on Walther Nernst[39].

4. BOLTZMANN AND PLANCK

At first, Planck's relationship to Boltzmann was cool, as has been seen. Planck was a thermodynamicist through and through, and disliked atomistic, statistical models. Here the influence of Mach was important. Candid as always, Planck later acknowledged that Boltzmann in the earlier days had no reason to be pleased with him.

Outwardly at least the nadir of the relationship was maybe reached in 1896 when Planck's student, the mathematician Ernst Zermelo, had published his recurrence objection to the statistical interpretation of the Second Law[40], fully developed in Boltzmann's fundamental papers in 1872, on the H theorem, and in 1877, on statistical mechanics. By the way, the term "recurrence objection" (Wiederkehreinwand) is due to Paul and Tatyana Ehrenfest[41], to whom we shall later return. If one considers the Second Law as absolutely valid, the recurrence objection is lethal for the atomistic hypothesis.

Briefly, Zermelo had argued that according to a theorem by Henri Poincaré, in itself undisputed, an ensemble of particles must after sufficient time take up any configuration. More exactly, in the words of Brush[42], the theorem states that every mechanical system must almost certainly return infinitely many times as close as one wishes to its starting position. If probability is identified with entropy according to $S = k \cdot \log W$ -- Boltzmann's Principle, in Einstein's[43] words -- this means that an isolated finite system consisting of particles must sooner or later return to a condition of decreased entropy, also macroscopically. Today we are all aware of the central role of the fluctuation concept in the life works both of Boltzmann and of Einstein, and here we need not go into the matter further. But the fact was that Planck had authorized Zermelo's article. Boltzmann[44] answered hotly, and bluntly advised Zermelo to give up gas theory if he was unable to grasp its theorems clearly.

[We may interpolate. Qualitatively, Boltzmann had been anticipated in the explicit statement of recurrence. The idea had been brought up, apparently unknown to Zermelo, by

Poincaré as early as 1893. At the same time, long before Boltzmann, Poincaré had also envisaged cosmic fluctuations as an escape from heat death. These facts have recently been recalled, or unearthed, by Brush[42,45].]

Yet, in spite of the clash about Zermelo, the approach of Planck to Boltzmann was coming. The anti-atomist Ostwald, with the help of Georg Helm, had put forward the idea of "energism" (Energetik), with energy as the ultimate entity. Energism was a counterproposal to atomism. Boltzmann of course was much opposed to energism[25]. The energists were routed by Boltzmann in the famous meeting of the "Society of German naturalists and physicians" at Lübeck in 1895, where Ostwald according to the title of his lecture meant to overcome (überwinden) scientific materialism. Sommerfeld's[46] description may be recalled: "Externally and internally the struggle Boltzmann versus Ostwald was like the struggle of the bull with the flexible fighting man. But this time the bull won against the torero in spite of all his fighting skill. We all, the then younger mathematicians, stood at Boltzmann's side."

Planck, though still lacking sympathy with atomistics, definitely refrained from associating with Ostwald. Planck recalled: "It was impossible to prevail against the authority of men like W. Ostwald, G. Helm, E. Mach.... I could ... (in the struggle between Boltzmann and Ostwald: E.B.) only play the role of a supporter of Boltzmann. True, my services were not at all recognized by Boltzmann, not even looked at favourably ...." Already we have referred to the view taken of these events by Planck in retrospect in 1940, in connection with the influence of Mach on him.

The question of the attitude of Mach to Ostwald arises here. As seen by Mach's foremost biographer, J.D. Blackmore[8], Mach needed Ostwald as an ally against atomism, but basically Mach opposed both "atomism" and "energeticism" (I prefer energism: E.B.). In Sommerfeld's[46] words, Mach's natural philosophy stood behind Ostwald and Helm at Lübeck. But Mach kept in the background and withheld clear support from Ostwald.

[Ostwald invited Mach to a chair in his University at Leipzig in 1897 [10,47]. Mach did not accept, but Boltzmann later did. He spent the years 1901-1903 at Leipzig. In spite of his scientific and philosophical opposition to Ostwald[25], Boltzmann was on excellent personal terms with Ostwald. There was a lot of friendly correspondence between the two men[47]. Many polite letters, more restrained, were also exchanged between Ostwald and Planck[47]. It may be pointed out, by the way, that Ostwald was, in many respects, a man of considerable merits.]

In the late 1890's Planck took up the burning question of the composition of the radiation in equilibrium with a black body. The theory of light radiation had not only academic importance, but great practical and financial interests were involved as well. First, there was the enormous economic importance of energy transfer and radiation losses from hot bodies. Secondly, the big electrical firms centred on Berlin (AEG, Siemens, Osram) intended to improve the technology of light production. Not by coincidence the world's foremost experimentalists in the measurement of light radiation had their home in the Physikalisch-Technische Reichsanstalt at Berlin, founded in 1887 on the basis of a gift by Werner von Siemens. The first Director was Hermann von Helmholtz, the "Reich Chancellor of German Physics", as he was jokingly called. Planck had the benefit of daily contact with these eminent men.

At first, Planck meant to explain the dependence of the intensity of the radiation on wavelength and temperature merely on the basis of classical thermodynamics and electromagnetism, without using statistical mechanics. He made considerable progress. Especially he succeeded in deriving a fundamental relationship between the energy content of presumed oscillators in the black body and that of the radiation field. The oscillators were considered quite formally and had nothing to do with atoms. Nevertheless, a final solution of the problem eluded Planck. Planck's initial failure brought him, in 1900, to the decision to apply Boltzmann statistics, i.e. atomistics, to his oscillators. After some weeks of the most strenuous work, as he later said[13], Planck succeeded in setting up the law that accounted well for the experimental results of his colleagues, and has remained valid ever since. Planck's law contains the quantum of action, h, the numerical value of which is, and has remained, empirical. h is "Planck's constant". Planck also derived the numerical value of k, and called it "Boltzmann's constant".

Boltzmann had every reason to be happy with this development. The atomistic approach and the Boltzmann principle had at last made possible the solution of the old, important, and difficult problem of the spectral composition of the black-body radiation. The contribution of Boltzmann to this achievement was not limited to his writings, with which Planck was familiar. Hans Kangro[48] has shown that close personal contact about the radiation question existed between Boltzmann and Planck already during the naturalists' annual meeting at Munich in 1899. Jokingly, Alfred Kastler (personal communication) likes to say that Planck may have been the father of quantum theory, but that Boltzmann was the mother. Here Kastler partly refers to the derivation by Boltzmann in 1877 of the distribution of energy among the atoms of a gas by introduction of finite energy intervals, a sort of quantization of the energy. Though in the further steps of computation Boltzmann let the energy intervals (energy quanta, one might say) approach the limit zero, i.e. though Boltzmann's procedure may be viewed as a mere mathematical trick, this procedure no doubt was known to Planck. Presumably he was guided by it.

During the Second World War, the correspondence between Boltzmann and Planck, of utmost importance to the history of science, was lost. We only know from Planck that he wrote about his atomistic achievements in radiation theory to Boltzmann[12], and that now Boltzmann's irritation (gereizter Ton) gave way to friendly agreement (freundliche Zustimmung). In particular, after Planck had sent Boltzmann his great quantum paper, the latter showed his interest and his acceptance in principle of Planck's approach[13]. In his later writings, Planck[13] abundantly expressed his admiration for the genius of Boltzmann.

One might have thought Boltzmann would, after 1900, in lectures and writings refer to Planck's work on radiation, made possible by adoption of his own, Boltzmann's, statistical methods. This was not the case, however. While Boltzmann continued to lecture and to publish till his tragic death in 1906, we find no word about Planck or about his work. Nor did Boltzmann refer to Planck in his classes, as told by his student Lise Meitner[49]. By the way, Boltzmann did not refer to Einstein either[49], although Einstein's explanation, in 1905, of Brownian movement as a fluctuation phenomenon, to which we shall return later, had been another triumph of Boltzmann's ideas. It looks as if Boltzmann had quite systematically shut himself up in his own old world, the world of the 19th century, about which he had spoken in glowing words[50].

5. MACH AND EINSTEIN

We are arriving at the last and greatest of our heroes, Albert Einstein (1879-1955). It is common knowledge that Mach's criticism of classical mechanics was a great help to Einstein in the establishment of the special theory of relativity, and also that Mach's principle had an important role in the thinking that led to the general theory of relativity. Mach's principle is the idea that the result of Newton's famous bucket experiment might be accounted for through the action of the mass of the Universe on the water in the rotating bucket[51]. Einstein did not tire of expressing his gratitude to Mach. In 1909, he signed a note to Mach "Ihr verehrender Schüler", your admiring student[52]. In his only autobiography, published in 1949, Einstein[53] wrote: "It was Ernst Mach who, in his History of Mechanics, shook this dogmatic faith (in mechanics as the secure basis of physics: E.B.); this book exercised a profound influence upon me ... while I was a student."

In the circumstances it is not surprising that Einstein was influenced by Mach's positivist philosophy, too[53]: "In my younger years ... Mach's epistemological position also influenced me very greatly." However, in time the views of Einstein on Mach changed. One factor may have been that Einstein was a convinced atomist from the beginning[54]. In fact he specifically stated in his autobiography[56] that he "developed the statistical mechanics ... to find facts which would guarantee as much as possible the existence of atoms of definite finite size." He predicted the existence of the fluctuations. At first tentatively, he saw an example in Brownian movement.

[It may be pointed out that Boltzmann had considered the possibility of the observation of fluctuations in gases[44], and that he also knew the fact of Brownian movement and thought it important[57], but he apparently did not connect the two things. Poincaré already in 1904, one year before Einstein, spoke of Brownian movement as an example of fluctuations. Here he followed much earlier ideas of Léon Gouy, likewise in France. Of course, Poincaré did not supply any equations. He did not work with atoms, and was doubtful about them[42,45].]

Thus the rigidly negative and inflexible attitude of Mach towards atomism, clearly due to his positivist prejudices, must have disappointed Einstein. Also in the only personal meeting between Mach and Einstein, which took place in Vienna probably in 1911, no agreement was reached between the two men in respect to the atoms. It is curious that in the only source about that interesting meeting, the book by Einstein's friend and successor in Prague, the Viennese Philipp Frank[58], we do not find any indication what was said there about relativity. It is hard to believe that this problem was not touched on. Frank, by the way, was a strong supporter of Mach, though he learned physics from Boltzmann during the latter's last years[4].

However, Mach himself informed us in the end in no uncertain terms that he had come to reject relativity. After he had expressed himself favourably privately and -- in a more subdued way -- even publicly on relativity for many years, and after many physicists had come to consider Mach as a supporter of relativity[8,59], the great shock came in 1921. The time bomb had been ticking, as Holton[59] said. In Mach's posthumous book on optics, mentioned before, he rejected relativity just as strongly as atomistics[7,22]. The circumstances have been discussed in great detail by Blackmore[8]. The rejection must have hurt Einstein, who shortly afterwards, in Paris, called Mach "un philosophe déplorable"[60].

In respect to positivist philosophy Einstein had written already in 1917 to his lifelong friend, Michelangelo Besso in Switzerland[61]: "I do not scold Mach's little (hobby) horse. You know how I think about it. But it cannot give birth to anything living, only exterminate harmful vermin." In 1930 Einstein*) wrote about (Mach's) positivism to Planck's former student, the eminent physicist-philosopher Moritz Schlick at Vienna, six years later murdered by a right-wing fanatic in the University: "Looked at generally, your representation does not correspond to my views insofar as I find all your view too positivistic, so-to-say ... . I tell you directly: Physics is an attempt at a conceptual construction of a model of the real world and of the laws of its structure". Finally in Einstein's autobiography his judgment was[54]: "Mach's epistemological position ... today appears to me to be untenable. For he did not place in the correct light the essentially constructive and speculative nature of thought and more especially of scientific thought; as a consequence of which he condemned theory on precisely those points where its constructive-speculative character unconcealably comes to light, as for example in the kinetic atomic theory." Further remarks of Einstein about positivism, in communications with M. Solovine and with K.R. Popper, will be found elsewhere[3,4,5].

6. BOLTZMANN AND EINSTEIN

The influence of Ludwig Boltzmann on young Einstein was also extremely strong. Along with Helmholtz, Hertz, Kirchhoff, Lorentz, Mach, and Maxwell, Boltzmann was the author most carefully studied by Einstein and his friends of the so-called "Akademie Olympia" in Berne[3]. Yet Einstein knew only part of the enormous work of Boltzmann, as he confessed in his autobiography[56]. It is quite likely that he was entirely ignorant of Boltzmann's philosophical writings, which had not appeared in a readily accessible form[25] before 1905. Nor did Einstein ever have personal contact with the great master of statistics. The expert, third class, in the Federal Patent Office in Berne surely was too shy for an initiative. So Einstein apparently later was not aware to what extent he had gradually embraced the philosophical views characteristic of Boltzmann. That was the philosophy of realism, as we have seen in connection with Boltzmann's polemic against Mach. The philosophy was also called materialism by Boltzmann in his magnificent lecture against Schopenhauer[62] in 1905. The original title of that lecture was: "Proof that Schopenhauer is a mindless ignorant philosophaster who scribbles nonsense and debases the heads through empty word trash fundamentally and for ever." Originally these very words had been used by Schopenhauer against Hegel.

The word "materialism" certainly did not appeal to Einstein, who considered himself as religious. I have discussed his views on religion in some detail elsewhere[5,6]. Here I shall limit myself to the statement that Einstein did not believe either in a personal God or in one of the established religious creeds, including Judaism, of course. Einstein was a pantheist and identified himself with the beliefs of Spinoza. Apart from terminology and emphasis, however, it does not appear that the views of Boltzmann and of Einstein diverged. The pantheist Spinoza has also been attacked or praised, as the case may be, as a materialist.

While Einstein probably was ignorant of Boltzmann's philosophy, he wholly accepted Boltzmann's atomistics, as he made clear again and again in his autobiography. In the words

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*) I thank Dr. O. Nathan, The Estate of Albert Einstein, for this text. An English translation had been given previously by G. Holton[59], where I also learned about the existence of the letter to Schlick.

of Einstein's favourite collaborator in the 1920's, the Hungarian Cornelius Lanczos, Einstein was the "natural successor" to Boltzmann[63]. The first papers by Einstein were all in the Boltzmann tradition[64], and during the whole of his life Einstein remained fascinated by Boltzmann's principle.

In Einstein's so-called annus mirabilis, 1905, Einstein demonstrated Boltzmannian fluctuations, as we have seen, and made possible the numerical estimate of Avogadro's number by Jean Perrin on the basis of the extent of these fluctuations in 1908. Further, in the same year, 1905, Einstein through his light quantum hypothesis introduced atomistics into the modern theory of the radiation field. The concept of the light quantum, of tremendous importance not only to physics but also to chemistry and even biology, was further followed up in Einstein's papers for many years. We may especially refer to one of the "most beautiful feats of scientific thinking in the entire history of human thought"[63], the paper "Quantum theory of radiation" that appeared in Physikalische Zeitschrift in 1917.

It might be explained here why the light quantum having energy $E = h\nu$ is crucial in biology. If the energy of light were diluted uniformly with increasing distance from the source, its energy at a point in space would tend to fall below the value needed for any particular photochemical reaction. The insufficiency of energy available would be reached the more easily the smaller the strength of the light source. Hence photosynthesis, which is a photochemical reaction, could not take place at all in most conditions. In fact, it is not clear how it could take place at all. Thus the conservation of the quality of the light in energy packages, the light quanta, is indispensable for photosynthesis. Photosynthesis has existed on Earth for some 3 gigayears now, and has become the basis of contemporary life[65].

Now a short remark on a further problem that has been considered recently[5]. Beginning in middle age, Boltzmann devoted himself increasingly, along with philosophy, to biology[1,25]. He explained the meaning of photosynthesis on the basis of the Second Law. Above all, he was a fervent admirer of Darwin and called the 19th century 'Darwin's century'[66]. In 1886 [67] and again in 1904 [57] he put forward Darwinian ideas about the origin of life that are quite consistent with our present ideas, a century later. He developed most interesting and fruitful ideas about the evolution of the mental functions in animals and man[57,68]. The polemic of Boltzmann against the obscurantists who denied the existence of Darwinian evolution bordered on violence[69].

Is is a striking fact that nothing of the kind applied to Einstein. Of course, he accepted Darwinism, as every serious scientist had done for a long time. But Einstein clearly was not really interested. While especially in his time in Berlin he must have been well acquainted with biologists at the University and at the Academy, we do not find many references to biology in the voluminous writings and in the letters of Einstein, as far as they have been published. Any remarks are occasional, and do not refer to fundamental problems. Why was it so?

Whatever may have been the reason, it was an interesting consequence that Boltzmann could try to develop ideas on the origin of morality on an evolutionist basis, while Einstein here failed to provide consistent ideas[6]. This is all the more remarkable as Einstein in practice certainly devoted more effort and time to the moral needs of mankind and even to politics than most great scientists[70]. In this respect, too, he deserves the lasting gratitude of all mankind.

Given Einstein's selfless personality, the exceptional strength of his involvement in public affairs can be understood on the basis of social factors. The son of an impoverished, uprooted, and non-conformist father, Einstein early learned about some of the dark sides of life. When he was forced to witness the horrors of the First War and later the atrocities committed by reactionary and fascist movements and régimes in many countries, he had his eyes open, and was not easily misled. Understanding and sense of responsibility for suffering mankind combined to motivate Einstein.

7. PLANCK AND EINSTEIN

This may be the place to emphasize the differences between Planck's original quantum concept and Einstein's concept. Planck established the quantum of action[32,48,71,72], but for many years he did not accept the quantization of the radiation field itself. This fact can again be seen as a consequence of Planck's conservatism. As late as 1913 the eminent authors of the proposal to the Prussian Academy of Sciences to appoint young Einstein, now in Zürich, as a professor and successor to the great J.H. Van 't Hoff, still wrote[73]: "He (Einstein) should not be blamed too much for occasionally exaggerating his speculations, for instance in his hypothesis of the light quanta. Not even in the most exact science can real innovation be introduced without venturing risks." The signatures under this extraordinary document were those of Walther Nernst, Heinrich Rubens and Emil Warburg, along with that of Planck. Among these excellent men, Planck presumably was the leader.

The idea of the light quantum [named photon by G.N. Lewis in 1926 [74]], with all its tremendous consequences, indeed was Einstein's[72,74]. The light quantum began to be accepted only at the famous Solvay meeting in Brussels in 1912, and acceptance was complete only after the discovery of the Compton effect in 1923, when the momentum of the photons could be measured directly.

The differences in the early period between Planck's and Einstein's ideas on quanta were so great that T.S. Kuhn[75] concluded that Planck did not really establish the existence of energy quanta at all. Rather, in Kuhn's view Planck only claimed quantal absorption and emission of light. According to Kuhn, the true originators of the concept of the energy quantum, also in respect to the oscillators in the black body, were Einstein and Paul Ehrenfest. The views of Kuhn have been disputed by Martin Klein[76], who feels that Planck did propose that the energy of his oscillators is composed of quanta. We shall not go into this matter further. Certainly the concept of the light quantum, of the quantized field, was due to Einstein. The discussion on the light quanta between Planck, Einstein, Ehrenfest, Lorentz and others, carried through with great fairness, was extremely fruitful. We may point, for example to the plenary lecture by Einstein[77], at that time still almost unknown outside Switzerland as a living person, to the Salzburg meeting of the German naturalists in 1909, where Planck was prominent. There was a lively discussion[32], and Einstein was, as he reported, fêted. For the first time, at this age, Einstein personally met a living theoretical physicist, as he said to Infeld[78]. Planck's ready acceptance of the theory of relativity has been mentioned before.

So it is not surprising that real friendship developed between Planck and Einstein after the latter had moved to Berlin in 1914. Planck and Einstein always spoke about each other with admiration. Their friendship also found expression when Einstein wrote to

Ehrenfest in 1919. In these letters, consideration for Planck was the chief argument used by Einstein why he could not follow Ehrenfest's friendly invitation to move from suffering post-war Germany to Leyden in prosperous and peaceful Holland[79]; in 1912 Ehrenfest had become the successor of Lorentz there. On the other hand, Planck, along with Nernst and others, courageously defended Einstein in public when he was viciously attacked, mostly on thinly disguised racial and political grounds, by reactionary obscurantists in Germany in the early 1920's.

In the fateful year 1933, Germany again resumed a position of political and military power, and moved into a moral abyss. As we have pointed out, Planck did not accept antisemitism or the vicious attacks against Einstein. It took extraordinary courage to state in solemn words in a plenary session of the Berlin Academy on 11 May 1933, after so many German democrats, social democrats, and communists had been murdered and after the concentration camps had been set up[36]: "I believe to express the view of my colleagues in the Academy, as well as that of the overwhelming majority of all German physicists, in saying: Mr. Einstein is not only one among many outstanding physicists, but Mr. Einstein is the physicist, by whose works published in our Academy physical knowledge in our century has reached a depth, the importance of which can be measured only by the achievements of Johannes Kepler and Isaac Newton." In 1948, Einstein wrote a moving memorial article for his dead friend[80].

8. BOLTZMANN'S STUDENTS AND EINSTEIN

To which extent was the interaction between the four great physicists continued in the succeeding generations?

Among Mach's students there were no important physicists. Philipp Frank was no exception. While he became a follower of Mach in philosophy, he had learned his physics from Boltzmann. Planck also had few students, though we may recall the important figures of Zermelo, Laue, and Schlick. However, Boltzmann had quite a number of eminent students who could later interact with Einstein. At Graz University in the 1880's there was not only Svante Arrhenius but also Walther Nernst, who in 1913 together with Planck personally travelled to Zürich to persuade Einstein to move to Berlin. Einstein was much impressed, though not wholly charmed, by Nernst's unique personality. In his laudatory obituary for Nernst[81] in 1942 Einstein, now in America, wrote that he "was one of the most characteristic and most interesting scholars ... I have never met any one who resembled him in any essential way."

Another eminent student of Boltzmann -- in the years 1901-1904 -- was Paul Ehrenfest[79,83], born in 1880, whom we have mentioned already. Until Ehrenfest's tragic death by suicide in 1933, he was probably Einstein's best friend. Directly after they had met at Prague in 1912, Einstein wrote to him[79]: "Within a few hours we were true friends, as if our dreams and efforts were thought for each other." Ehrenfest's work was determined by Boltzmann. He became well known through his contribution to the Enzyklopädie der mathematischen Wissenschaften on the conceptual basis of statistical mechanics[41]. He wrote it, together with his wife, Tatyana Afanaseva, during their stay in Russia in 1907-1912. After the death of Boltzmann, who at first had been supposed to supply the article[84], the Ehrenfests were entrusted with the task by the Editor-in-Chief of the Encyclopedia, Felix Klein.

It is interesting that in their important work the Ehrenfests were rather cool towards Gibbs, who independently of Boltzmann had established statistical mechanics and also had coined the term -- later adopted by Boltzmann. Could the Ehrenfests have been influenced by the fact that Gibbs had been translated by nobody else but Ernst Zermelo? Incidentally, Paul Ehrenfest was also friendly with his fellow student, under Boltzmann, Lise Meitner, whom Einstein later, in jest, called "our Marie Curie"[85], and with Philipp Frank. Paul (and Tatyana) Ehrenfest's Encyclopedia article on statistical mechanics and his early work on the quanta may have been the reasons for Lorentz proposing young Ehrenfest for his former chair at Leyden. Victor Weisskopf did his Ph.D. Thesis essentially under Ehrenfest[86].

Further I would like to name the scientific descendants of Fritz Hasenöhrl. This student of Boltzmann at Vienna University was a theoretical physicist of the first rank, and became the successor to his teacher. His foremost achievement was the discovery, though only for special cases, of the equivalence of energy and mass[87]. This was in 1904, one year before the more fundamental contribution of Einstein. Unfortunately, Hasenöhrl fell a victim to the First World War. His student and successor at Vienna University was Hans Thirring, well known not only as an authority in relativity and a splendid teacher, but also as a courageous enemy of Nazism and a pacifist; he was friendly with Einstein for decades, and they both took part in the foundation of the international Pugwash movement for world peace.

By the way, Hasenöhrl also was a close friend of Marian von Smoluchowski[88]. Independently of Einstein, Smoluchowski developed the quantitative theory of Brownian movement on the basis of fluctuations. A curious fact that does not seem to have been discussed so far: Smoluchowski, who had done his thesis under Stefan, but called himself a student of Boltzmann, had been interested in Brownian movement, to be explained through fluctuations, since 1900. That was 6 years before Boltzmann's death. Is it possible that Boltzmann was not aware of Smoluchowski's promising investigations?

Thirring's fellow student and friend was Erwin Schrödinger[89]. When Schrödinger arrived in Berlin in 1927 as successor to Planck, he immediately formed close links with Einstein, whom he had known before. The electrifying influence of Schrödinger has been described impressively by Philipp Frank[58]. Incidentally, Planck had come to Berlin as successor to Gustav Robert Kirchhoff, after Boltzmann -- then in Graz -- had declined the chair in curious circumstances. He was afraid that he might not be able to adapt to the Prussian way of life.

9. CONCLUSION

As a conclusion we may ask for the final result of our story. Surely story-telling cannot be an end in itself. What has been the over-all result of our analysis? It is my feeling that the struggles of our four great men contain a great deal of the events that led to the breakthrough of modern atomism in physics. The struggles often, but by no means always, were carried through in a friendly spirit. Philosophical idealism and positivism lost, and realism and atomism have emerged as the adequate and fruitful world view of the modern scientist. That world view has demonstrated its truth and worth in human practice, not only at the desk and in the laboratory in physics and bioscience but also in technology. The great decisions were taken in the successful establishment of Boltzmann's Principle and of Planck's radiation law, of Einstein's mass/energy relationship, and of the colour dependence of the photoeffect.

The later developments, those of quantum mechanics, I have left out. The establishment and the interpretation of quantum mechanics belongs to a later period, and here we find new protagonists. While Einstein, Planck, Laue, Schrödinger, and -- during most of the time -- de Broglie were reluctant to abandon determinism, they were opposed by Heisenberg, Bohr, Born, and Pauli. Forgive me for as much as mentioning these well-known facts. Many fine articles and books have been written about the conceptual basis and the implications of quantum mechanics, burning questions of science and epistemology. It is really a scientific and also a human drama. I have not set myself the task here of entering into these questions.

I do not want to end, however, without emphasizing that our knowledge of the corpuscular atom and of the light quantum is now threatening the further existence of mankind in a quite literal sense. This thought also darkened the last decades of the greatest and most human of our heroes -- Albert Einstein -- although the weapons now in the arsenals of the big powers exceed the weapons known to Einstein by orders of magnitude, quantitatively and qualitatively. Paraphrasing Bertolt Brecht*), somebody might say that a talk about the great and peaceful achievements of the human mind is a crime, if it takes the place of a consideration of the threatening nuclear holocaust. However, it is not I, Broda, who say this.

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*) Berthold Brecht, "An die Nachgeborenen, I", ... Was sind das für Zeiten, wo ein Gespräch über Bäume fast ein Verbrechen ist/weil es ein Schweigen über soviele Untaten einschliesst, ..."

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