MULTIQUARK BARYONS WITH BROKEN FLAVOUR SYMMETRY I.

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Abstract: The calculation of the spectrum of $4q\bar{q}$ multiquark baryons is carried out, taking into account that SU(3) flavour is broken. To handle this problem, which includes manipulation of giant expressions for the wavefunctions, methods suitable for programming in SCHOONSCHIP is developed and employed.

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1. Introduction.

We are all familiar with the success of the old quark model introduced in the beginning of the sixties by Gell-Mann[1)], Zweig[2)] and others[3)]. As a consequence of this model, it was for many years believed that every resonance is built up of 3 quarks (baryons) or of $q\bar{q}$ pairs (mesons). This belief , of course, was also born out of the experimental situation in the late sixties and early seventies: no convincing experimental evidence for the existence of exotic resonances had been found.

It was however early discovered that the quark model in its simplest version contained various defects. The $\Delta^{++}(1232)$-resonance in the $J_z = +3/2$ state, was for instance described by a quark wave function in which all three quark spins were aligned: $|\Delta^{++}, J_z = 3/2\rangle = |u\uparrow u\uparrow u\uparrow\rangle$. The structure of this wave-function poses a serious problem: the wave function is symmetric in the quark and spin indices. On the other hand one expects the space-part of the Δ^{++} wave-function to be symmetric, since the Δ^{++} is the ground state of the three u-quarks. In this case we find that the wave-function is totally symmetric, which contradicts the Pauli-principle. An answer to this spin-statistics puzzle was given by introducing coloured quarks[4,5)]: each quark flavour comes in three different "colours", say red, white and blue[*)]. In the Δ^{++}-state the wavefunction is thus antisymmetric in the colour-indicies. This gives the right overall antisymmetry of the wavefunction, as demanded by the Pauli-principle. The world of hadrons discovered thus far can be described by the five quark flavours u,d,s,c,b each coming in three colours. Although not established experimentally, a sixth flavour, t, is demanded by different unified models.

The quarks are triplets under the group $SU_C(3)$. According to the decomposition of products of SU(3)-representations as given by

$$
\begin{aligned}
3 \otimes 3 &= \bar{3} \oplus 6 \\
3 \otimes 3 \times 3 &= 1 \oplus 8' \oplus 8 \oplus 10 \\
3 \otimes \bar{3} &= 1 \oplus 8
\end{aligned}
\qquad (1.1)
$$

it is seen that a consistent way to describe the hadron spectrum would be to suppose that all hadrons are singlets under $SU_C(3)$. It should be noted that only systems of the kind $q\bar{q}$, qqq, $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, ...

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*) Of course there are many more reasons to introduce colour. The different reasons for colour are well documented in a recent review-article by E. Reya[12)].

can be singlets, while qq, qqqq, ... never can be such. The postulate of confinement is thus equivalent to stating that all physical observables are colour-singlets.

The colour singlets in the qqq sector is given by

$$|qqq,\ \text{singlet}\rangle = \sqrt{\tfrac{1}{6}}\,\varepsilon_{ijk}\,|q^i q^j q^k\rangle \tag{1.2}$$

where i,j,k = red, white, blue. The coloursinglet in the $q\bar{q}$ sector is given by

$$|q\bar{q},\ \text{singlet}\rangle = \sqrt{\tfrac{1}{3}}\,(r\bar{r} + w\bar{w} + b\bar{b}) \tag{1.3}$$

The colour-degree of freedom is not seen in the spectrum of ordinary $q\bar{q}$ and qqq hadrons, because in these minimal systems the colour is frozen. The chromomagnetic interaction as given by*)

$$H_{CM} = -\sum_{i<j} c_{ij}\lambda_i^a\,\vec{\sigma}_i\,\lambda_j^a\,\vec{\sigma}_j \tag{1.4}$$

can though be used to understand many features of the usual hadron spectrum. By using eq.(1.4) and calculating the hyperfine splitting for the $J^P = 1^-$ vector mesons (ρ, ω, ϕ, ...) one finds [8)]

$$\langle H_{CM}\rangle = \frac{16}{3}\,c \tag{1.5}$$

(where we have put all $c_{ij} = c$). For the $J^P = 0^-$ pseudoscalar mesons (π,η,η',...) one finds [8)]

$$\langle H_{CM}\rangle = -\,16\,c \tag{1.6}$$

thus explaining the fact that $M(\rho) > m(\pi)$ etc.

In the baryon sector one similarly finds for the $J^P = \tfrac{1}{2}^+$ baryons (p, n, Σ, Λ,...) that [8)]

$$\langle H_{CM}\rangle = -\,8\,c \tag{1.7}$$

For the $J^P = \frac{3}{2}$ baryons (Δ, Σ^*, Λ^*, ...) one finds [8)]

$$\langle H_{CM}\rangle = 8\,c \tag{1.8}$$

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*) This form of the colour-magnetic interaction is considered standard and is extensively used in multiquark calculations. See for instance 6, 7, 8 or 9).

thus explaining that $m(\Delta) > m(N)$ etc. Introducing flavour symmetry-breaking it is further possible to explain more details like for instance the Σ - Λ mass difference, which vanish in the limit where all c_{ij}'s are equal. Because we will be interested in the colour-magnetic interaction with broken flavour symmetry, let us estimate the values of the different c_{ij}'s from the 3q-sector.

Applying the formulas for the colour-magnetic interaction in the 3q system as given by Tsou[11] to the mass differences of N - Δ, Σ - Λ, Ω - N and Λ - N one finds [13]

$$c_{nn} = 18.3 \text{ Mev},\ c_{ns} = 11 \text{ MeV},\ c_{ss} = 7 \text{ MeV},\ \Delta m = 175 \text{ MeV}, \quad (1.9)$$

where Δm is the mass difference between a strange and a non-strange quark. These values of the coefficients will be used in all the numerical calculations in this paper; i.e. we assume that wavefunction effects are of such a nature that the operator over colour-spin space can be approximated by eqs.(1.4) and (1.9) also in states with many quarks.

Let us now turn to multiquark (more than three quarks) systems. In these systems which are of the form $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, $qqqqq\bar{q}\bar{q}$, ... the colourdegree of freedom is not frozen any more, and the colour should then prove its existence by giving nature colourisomers. In the $qq\bar{q}\bar{q}$ systems we find that the colour multiply according to

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

We thus see that we can get two different colour singlets giving rise to isomers. In the $qqqq\bar{q}$ sector we similarly find

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3} = 1 \oplus 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus \ldots$$

We therefore see that in this sector we have three different ways of making colour singlets. This should lead to a much richer spectrum than without colour, i.e. we get colour isomers.

In the last few years there has been considerable interest in predicting the masses of multiquark states. In most cases this has been done in a flavour-symmetric limit[8,9,10,18], but some authors also include flavour-symmetry breaking in systems consisting of two or three quarks [11,13,14,15,16]. It is very difficult to include flavour-symmetry breaking when the number of quarks is large, because the wave-functions are very cumbersome. In this paper we solve this problem with a computer, using the program SCHOONSCHIP developed by Veltman[17]; thus completely solving the problem of predicting masses for the $qqqq\bar{q}$-states in the S-wave with broken flavour-symmetry.

To calculate the masses of these S-wave $4q\bar{q}$ states we use the constituent quark masses:

$$m_u = 360 \text{ MeV}, \ m_d = 360 \text{ MeV}, \ m_s = 535 \text{ MeV} \tag{1.10}$$

(which gives $m_s - m_{u,d} = 175$ MeV as demanded from (1.9)).
Within the $4q\bar{q}$ cluster we then calculate the colour-magnetic energy given by (1.4) between all the five quarks using different c_{ij}'s according to (1.9). The mass formula then reads

$$M = 1800 \text{ MeV} + n_s \cdot 175 \text{ MeV} + H_{CM} \tag{1.11}$$

where n_s is the number of strange quarks. In this paper we only consider $4q\bar{q}$ states composed of u, d and s quarks, i.e. no c or b quarks enter our calulations.

A discussion of the experimental situation in the light of this model is postponed to a forthcoming paper.

2. The $4q\bar{q}$ S-states.

The 3060-dimensional representation of totally antisymmetric colour-spin-flavour states of four quarks can be decomposed into the irreducible representations given in appendix A. The physical interesting representations are the ones with c = 3, since these are the only ones which can give coloursinglets when coupled to an antiquark which transforms under the representation $\bar{3}$. The colour triplet 4q representations are (A2-A5)

$$(3,\ 3,\ \bar{6}),\ (3,\ 3,\ 3),\ (3,\ 1,\ 3),\ (3,\ 5,\ 15)$$
$$(3,\ 3,\ 15),\ (3,\ 1,\ 15) \text{ and } (3,\ 3,\ 15') \tag{2.1}$$

where the numbers indicate colour, spin and flavour representations respectively. When these multiplets are coupled to the $\bar{q}$ we get

$$
\begin{array}{llll}
(3, 3, \bar{6}) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2 \oplus 4, & \overline{18}) \\
(3, 3, 3) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2 \oplus 4, & 9) \\
(3, 1, 3) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2, & 9) \\
(3, 5, 15) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 4 \oplus 6, & 45) \\
(3, 3, 15) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2 \oplus 4, & 45) \\
(3, 1, 15) \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2, & 45) \\
(3, 3, 15') \oplus (\bar{3}, 2, \bar{3}) & = & (1 \oplus 8, & 2 \oplus 4, & 45')
\end{array} \qquad (2.2)
$$

We only give the total flavour multiplicity because we assume magic mixing to occur between the SU(3) flavour multiplets. Instead of having the usual *SU(3) direct decomposition*

$$
\begin{aligned}
9 &= 1 \oplus 8 \\
\overline{18} &= 8 \oplus \overline{10} \\
45 &= 8 \oplus 10 \oplus 27 \\
45' &= 10 \oplus 35
\end{aligned} \qquad (2.3)
$$

we will now get the *magically mixed* decomposition into multiplets with no hidden strangeness ($0s\bar{s}$) and multiplets *with one* $s\bar{s}$ *pair* ($1s\bar{s}$)

$$
\begin{aligned}
9 &= 6_{0s\bar{s}} + 3_{1s\bar{s}} \\
\overline{18} &= 13_{0s\bar{s}} + 5_{1s\bar{s}} \\
45 &= 33_{0s\bar{s}} + 12_{1s\bar{s}} \\
45' &= 35_{0s\bar{s}} + 10_{1s\bar{s}}
\end{aligned} \qquad (2.4)
$$

This decomposition of magically mixed flavour multiplets is shown in fig. 2.1.

Before we proceed and look more specifically at the different states, let us mention some regularities in the spectrum.

We will have no mixing between states with different J^P. Because we assume magic mixing, we will furthermore have no mixing between states with different numbers of $s\bar{s}$-pairs. The $4q\bar{q}$ states with one $s\bar{s}$-pair, isospin I and hypercharge Y are all made up of the same isospin I, hypercharge $Y - \frac{2}{3}$, 4q clusters as the $4q\bar{q}$ states with no $s\bar{s}$-pair, isospin $I + \frac{1}{2}$ and hypercharge $Y - 1$. The only difference between these states is therefore the difference between $\bar{s}$ and $\bar{d}(\bar{u})$. Colour and spin is not changed going from the states with one $s\bar{s}$-pair to the states with no $s\bar{s}$-pair. We thus understand that the H_{CM}-matrix for the states with no hidden strangeness, isospin $I + \frac{1}{2}$ (containing 4q-clusters with isospin I) and hypercharge $Y - 1$ can be obtained from the H_{CM}-matrix of the

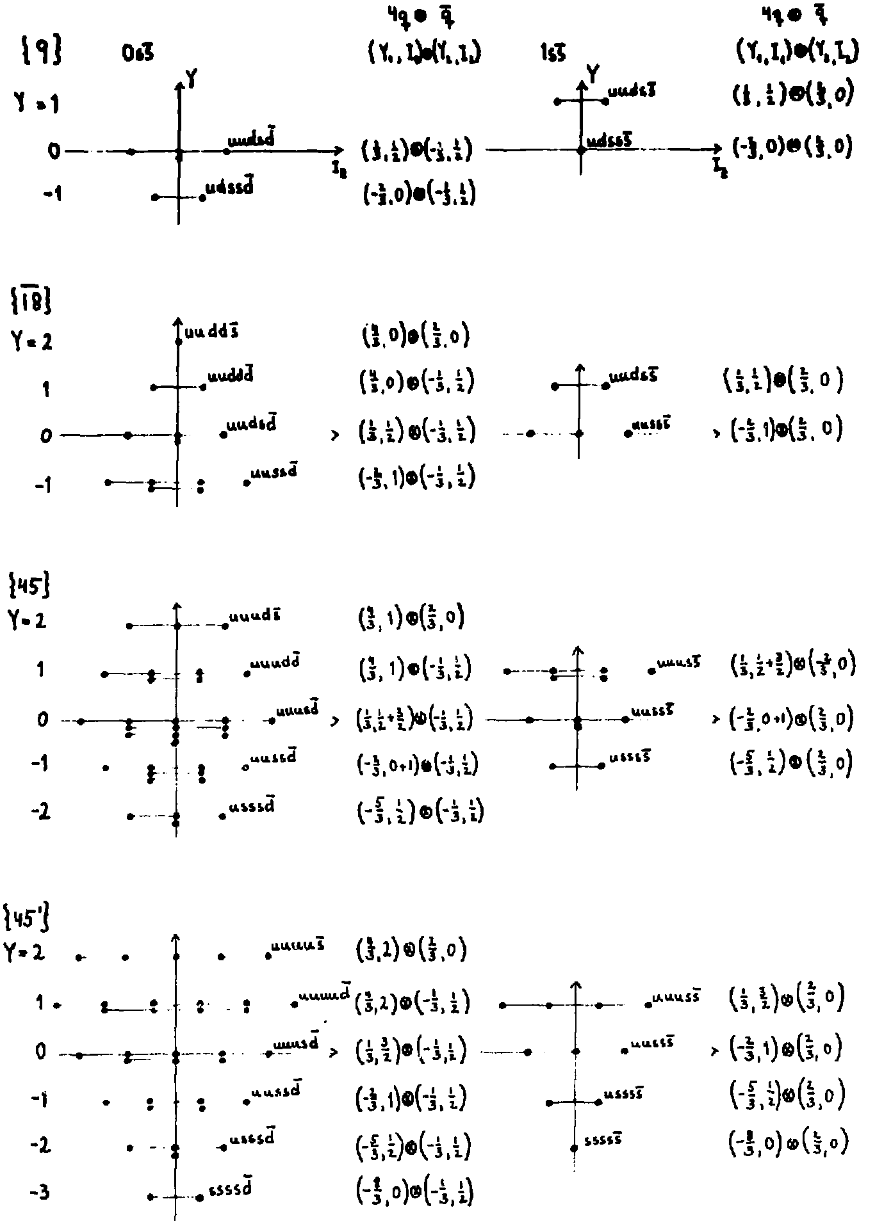


Fig. 2.1. The various magically mixed flavour multiplets in $4q\bar{q}$-states.

states with one hidden $s\bar{s}$-pair, isospin I and hypercharge Y by changing $C_{Q\bar{s}}$ into $C_{Q\bar{q}}$ in the matrix (Q = u,d or s and q = u or d). This regularity will be mentioned every time it occurs.

In the sector with no hidden strangeness we will, to some extent, have degeneracy between states with different isospin. It should be noticed however, that we get degeneracy between states with the same 4q-clusters only. If we have a 4q-cluster with isospin I, the $4q\bar{q}$ states can have isospin either I + ½ or I - ½. These states are degenerate. This can best be understood by a specific example. Let the 4q-cluster have isospin I = ½ and flavourcontent uuds. The spin and colour is not changed during the following discussion, and is therefore not specified here. Let the 4q-state be denoted $|4q\,{}^{I}_{I_Z}\,uuds\rangle$. The $4q\bar{q}$-states are therefore

$$|1\rangle \equiv |4q\bar{q}\;{}^{I=1}_{I_Z=1}\rangle = |4q^{\frac{1}{2}}_{\frac{1}{2}}\;uuds\rangle|\bar{d}\rangle$$

$$|2\rangle \equiv |4q\bar{q}\;{}^{I=0}_{I_Z=0}\rangle = \sqrt{\tfrac{1}{2}}\{-|4q^{\frac{1}{2}}_{\frac{1}{2}}\;uuds\rangle|\bar{u}\rangle - |4q^{\frac{1}{2}}_{-\frac{1}{2}}\;uuds\rangle|\bar{d}\rangle\}$$

The spin and colour-coupling is the same in the two cases. Applying H_{CM} eq.(1.4) to $|1\rangle$ or $|2\rangle$ will therefore lead to

$$H_{CM}|1\rangle = \sum_{\alpha,i,j} k^{ij}_{\alpha} c_{ij} |4q^{\frac{1}{2}}_{\frac{1}{2}}\;uuds\rangle^{\alpha}|\bar{d}\rangle$$

$$H_{CM}|2\rangle = \sum_{\alpha,i,j} k^{ij}_{\alpha} c_{ij} \sqrt{\tfrac{1}{2}}\{-|4q^{\frac{1}{2}}_{\frac{1}{2}}\;uuds\rangle^{\alpha}|\bar{u}\rangle - |4q^{\frac{1}{2}}_{-\frac{1}{2}}\;udds\rangle^{\alpha}|\bar{d}\rangle\}$$

where α runs over different 4q states with I = ½ and the same J^P. The expression $k^{ij}_{\alpha} c_{ij}$ means $k^{ij}_{\alpha} c_{ij} = k^{nn}_{\alpha} c_{nn} + k^{ns}_{\alpha} c_{ns} + k^{ss}_{\alpha} c_{ss} + k^{n\bar{n}}_{\alpha} c_{n\bar{n}} + ..$

The crucial point is whether the same k^{ij}_{α}'s can be used in both cases. This is OK because the states $|1\rangle$ and $|2\rangle$ are equal as far as H_{CM} is concerned. H_{CM} only looks for u,d or s flavour, neglecting the difference between u and d, putting both equal to n in expressions like $c_{us} = c_{ns}$, $c_{ds} = c_{ns}$ etc.

The degeneracy between states with different isospin in the $4q\bar{q}$ sector can thus be understood in much the same way as the degeneracy between different isospin states in the $q\bar{q}$ sector.

2.1 The colourmagnetic interaction for the $J^P = \frac{1}{2}^-$ $4q\bar{q}$ states.

From eq. (2.2) we see that there are six coloursinglet $4q\bar{q}$ representations with $s = \frac{1}{2}$. These are $(1,2,\overline{18})$, (1,2,9) (two different), (1,2,45) (two different) and (1,2,45'). Using the decomposition of the flavour multiplets as given in fig. 2.1 we finally arrive at the $J^P = \frac{1}{2}^-$ $4q\bar{q}$ multiplets as given in fig. 2.1.1. The multiplets are numbered from 1-6 as given in fig. 2.1.1. In the flavoursymmetric case, we know that multiplets 2 and 3 mix under the action of H_{CM}. Multiplets 4 and 5 also mix. The multiplets 1 and 6 are diagonal with respect to H_{CM}.

In the flavoursymmetric case all states belonging to multiplet 1 have [8,18]

$$\langle H_{CM}\rangle = -\frac{56}{3}\,c \tag{2.1.1}$$

Multiplets 2 and 3 mix and we get

$$H_{CM}\begin{bmatrix}|M_2\rangle \\ \\ |M_3\rangle\end{bmatrix} = \begin{bmatrix}-\frac{80}{3}\,c & -8\sqrt{3}\,c \\ \\ -8\sqrt{3}\,c & -16c\end{bmatrix}\begin{bmatrix}|M_2\rangle \\ \\ |M_3\rangle\end{bmatrix} \tag{2.1.2}$$

Diagonalizing this will lead to the eigenvalues -36.181 c and -6.486 c as given by Strottmann[18].

Multiplets 4 and 5 would mix according to

$$H_{CM}\begin{bmatrix}|M_4\rangle \\ \\ |M_5\rangle\end{bmatrix} = \begin{bmatrix}\frac{16}{3}\,c & -8\,c \\ \\ -8\,c & 0\end{bmatrix}\begin{bmatrix}|M_4\rangle \\ \\ |M_5\rangle\end{bmatrix} \tag{2.1.3}$$

which by diagonalizing gives the eigenvalues -5.766 c and 11.099 c[18].

States in multiplet 6 all have [8]

$$\langle H_{CM}\rangle = \frac{88}{3}\,c \tag{2.1.4}$$

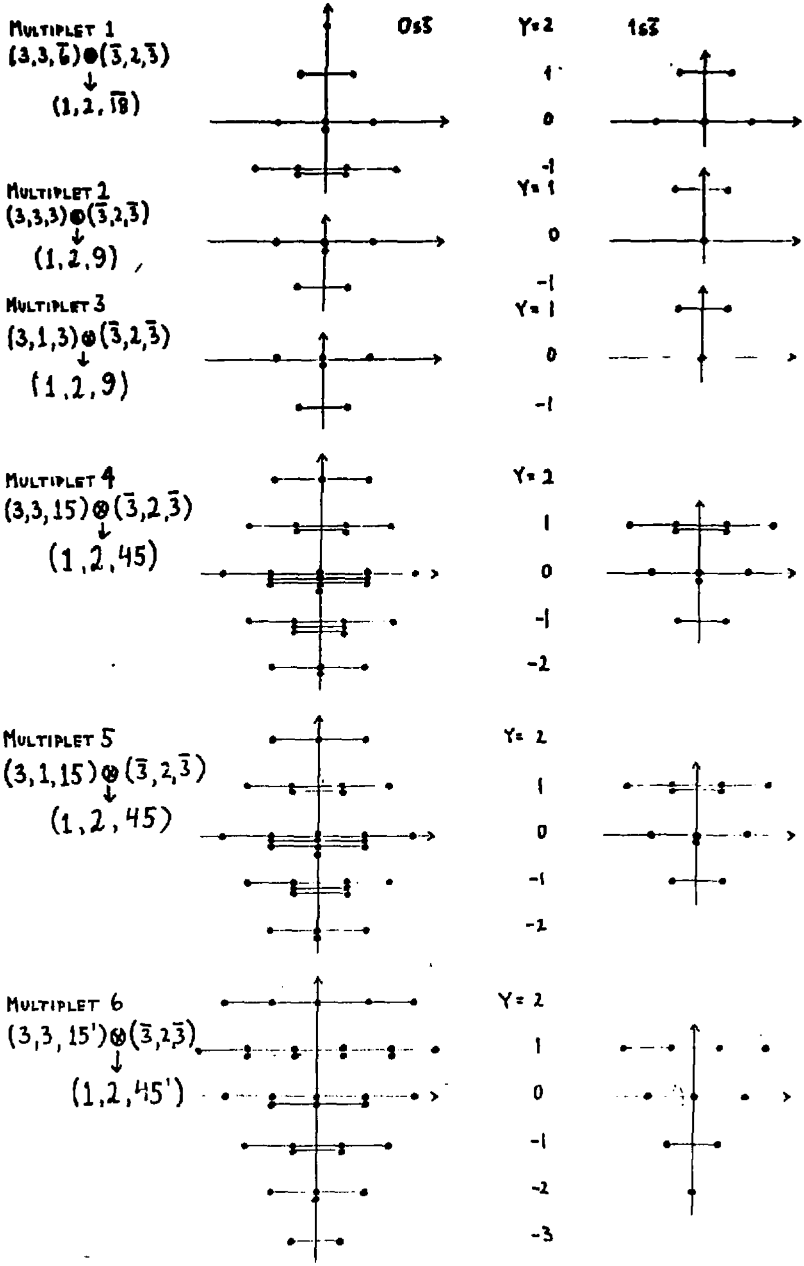


Fig. 2.1.1. The $J^P = \frac{1}{2}^-$ $4q\bar{q}$ flavour multiplets (numbered from 1 to 6).

If we use the average value c = 15 MeV[8] in the flavoursymmetric case, we get the eigenvalues and eigenvectors given in table (2.1.1) after diagonalizing H_{CM} in each case.

Table 2.1.1. Eigenvectors and eigenvalues of H_{CM} for the $J^P = \frac{1}{2}^-$ 4 $q\bar{q}$-states assuming flavoursymmetry.		
Multiplet	Eigenvalue (MeV)	Eigenvector
|M1>	- 280	|M1>
|M2>, |M3>	- 543	0.824 |M2> + 0.566 |M3>
	- 97	0.566 |M2> - 0.824 |M3>
|M4>, |M5>	- 86	0.585 |M4> + 0.811 |M5>
	166	0.811 |M4> - 0.585 |M5>
|M6>	440	|M6>

When we break the flavoursymmetry, we have to put different coefficients into H_{CM} eq. (1.4) according to the flavour of the two quarks in a pair. We also introduce different coefficients for qq and $q\bar{q}$ pairs respectively. See table (2.1.2)

Table 2.1.2. A list of the possible coefficients

Quark pair	Corresponding coefficients	Numbers used in numerical calculations
uu, ud or dd	c_{nn}	18.3 MeV
us, ds	c_{ns}	11.0 MeV
ss	c_{ss}	7.0 MeV
$u\bar{u}$, $u\bar{d}$, $d\bar{u}$, $d\bar{d}$	$c_{n\bar{n}}$	18.3 MeV
$u\bar{s}$, $d\bar{s}$	$c_{n\bar{s}}$	11.0 MeV
$s\bar{u}$, $s\bar{d}$	$c_{s\bar{n}}$	11.0 MeV
$s\bar{s}$	$c_{s\bar{s}}$	7.0 MeV

It should be evident that $c_{s\bar{n}} = c_{n\bar{s}}$. We keep the two different notations however, because this will indicate the flavour of the $\bar{q}$ in each case.

Let us now put the states of multiplet 1-6 into different groups according to their hypercharge, isospin and number of $s\bar{s}$ pairs, and treat these groups one by one.

Within each of these groups, we denote the state belonging to multiplet i with |Mi>.

It is simple to check that the matrix H_{CM} which we calculate with broken flavour symmetry in the following sections, in all cases reduces to the correct value as given by (2.1.1) - (2.1.4), (2.2.1) - (2.2.4) and (2.3.1) in the flavoursymmetric limit.

A1. Y = 2, I = 2, S = ½

a) No hidden strangeness.

From fig. 2.1.1 we see that there is only one state (*) in this group: |M6>. By using the methods developed in the appendices we get

$$\langle H_{CM}\rangle = \frac{56}{3} c_{nn} + \frac{32}{3} c_{n\bar{s}} \tag{2.1.5}$$

which reduces to the appropriate value given by (2.1.4) in the flavour-symmetric case. Using the values of the c_{ij}'s as given in table (2.1.2) we get the values for the eigenvector, eigenvalue and mass as given by table (2.1.3a).

Table 2.1.3a.	$qqqq\bar{s}$-sector	Y = 2, I = 2, S = ½
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
459	|M6>	2434

A2. Y = 2, I = 1, S = ½

a) No hidden strangeness

From fig. (2.1.1) we see that there are two states in this group: |M4> and |M5>. Using these two states as a basis in the sequence |M4>, |M5> gives

$$H_{CM} = \begin{bmatrix} \frac{8}{3}(c_{nn} + c_{n\bar{s}}) & -8\, c_{n\bar{s}} \\ -8\, c_{n\bar{s}} & 0 \end{bmatrix} \tag{2.1.6}$$

(*) There is of course more than one state, because we have $I_z=\pm2, \pm1, 0$ and $S_z=\pm\frac{1}{2}$. These are however, all degenerate under the action of H_{CM}, and we will therefore only talk about the highest state in the isospin and spin multiplets.

which has the correct flavour-symmetric limit as given by (2.1.3).

Using the numerical values of the c_{ij}'s as given in table (2.1.2) we get after diagonalization the eigenvectors, eigenvalues and masses as given in table (2.1.4a).

Table 2.1.4a	$qqq\dot{q}\bar{s}$-sector	Y = 2, I = 1, S = ½
Eigenvalue (MeV)	Eigenvector	Mass (Mev)
- 57	0.545|M4> + 0.838|M5>	1918
135	0.838|M4> - 0.545|M5>	2110

A3. Y = 2, I = 0, S = ½

a) No hidden strangeness

There is only one state in this group: |M1>. We get

$$\langle H_{CM}\rangle = -\frac{16}{3}c_{nn} - \frac{40}{3}c_{n\bar{s}} \qquad (2.1.7)$$

which reduces to (2.1.1) in the flavour-symmetric limit. Numerical values are given in table (2.1.5a).

Table 2.1.5a.	$qqq\bar{s}$ - sector	Y = 2, I = 0, S = ½
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 244	|M1>	1731

B1. Y = 1, I = $\frac{5}{2}$, S = ½

a) No hidden strangeness.

There is only one state in this group: |M6>. We get

$$\langle H_{CM}\rangle = \frac{56}{3}c_{nn} + \frac{32}{3}c_{n\bar{n}} \qquad (2.1.8)$$

which is the same as (2.1.5) with the exception $c_{n\bar{s}} \to c_{n\bar{n}}$. This has to be so, because the only difference in the wavefunctions is given by the substitution $\bar{s} \to \bar{q}$ (q denotes a non-strange quark).

Numerical values are given in table (2.1.6a).

Table 2.1.6a.	$qqqq\bar{q}$-sector.	$Y = 1,\ I = 5/2,\ S = \frac{1}{2}$
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
537	$\|M6\rangle$	2337

B2. $Y = 1,\ I = \frac{3}{2},\ S = \frac{1}{2}$

a) No hidden strangeness.

There are three states in this group. These are $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. The states $|M4\rangle$ and $|M5\rangle$ are made from a 4q state with I = 1 (see fig.2.1), while $|M6\rangle$ has its 4q-cluster in I = 2. This leads to the following block-diagonal form of the matrix H_{CM} in the basis $|M4\rangle$, $|M5\rangle$, $|M6\rangle$:

$$H_{CM} = \begin{bmatrix} \frac{8}{3}(c_{nn} + c_{n\bar{n}}) & -8\,c_{n\bar{n}} & 0 \\ -8\,c_{n\bar{n}} & 0 & 0 \\ 0 & 0 & \frac{56}{3}c_{nn} + \frac{32}{3}c_{n\bar{n}} \end{bmatrix} \qquad (2.1.9)$$

From this we see that the $I = \frac{3}{2}$ state from multiplet 6 has the same $\langle H_{CM}\rangle$ as the $I = \frac{5}{2}$ state from the same multiplet. (See 2.1.8).
Diagonalizing this matrix leads to the eigenvalues, eigenvectors and masses as given in table (2.1.7a).

Table 2.1.7a	$qqqq\bar{q}$-sector.	$Y = 1,\ I = 3/2,\ S = \frac{1}{2}$.
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 106	0.585 $\|M4\rangle$ + 0.811 $\|M5\rangle$	1694
203	0.811 $\|M4\rangle$ - 0.585 $\|M5\rangle$	2003
537	$\|M6\rangle$	2337

b) One hidden $s\bar{s}$ pair.

There are also three states in this group. These are $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. We get

$$H_{CM} = \begin{bmatrix} \frac{26}{3}c_{nn} - 6c_{ns} - \frac{4}{3}c_{n\bar{s}} + 4c_{s\bar{s}} & -\frac{26}{3}c_{n\bar{s}} + \frac{2}{3}c_{s\bar{s}} & \frac{2}{3}\sqrt{2}(c_{nn} - c_{ns} + 2c_{n\bar{s}} - 2c_{s\bar{s}}) \\ & 10\, c_{nn} - 10\, c_{ns} & \frac{2}{3}\sqrt{2}\,(-c_{n\bar{s}} + c_{s\bar{s}}) \\ \text{Symmetric} & & \frac{28}{3}c_{nn} + \frac{28}{3}c_{ns} + 8\, c_{n\bar{s}} + \frac{8}{3}c_{s\bar{s}} \end{bmatrix}$$

(2.1.10)

Diagonalizing this matrix leads to the numerical values given in table (2.1.7b).

Table 2.1.7b.	$qqqs\bar{s}$-sector. Y = 1, I = 3/2, S = ½.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 3	$0.642\|M4\rangle + 0.767\|M5\rangle - 0.017\|M6\rangle$	2147
181	$0.764\|M4\rangle - 0.641\|M5\rangle - 0.067\|M6\rangle$	2331
381	$0.062\|M4\rangle - 0.031\|M5\rangle + 0.998\|M6\rangle$	2531

B3. Y = 1, I = ½, S = ½.

a) No hidden strangeness.

There are three states in this group: $|M1\rangle$, $|M4\rangle$ and $|M5\rangle$. The 4q-state in $|M1\rangle$ has I = 0 and the 4q-states in $|M4\rangle$ and $|M5\rangle$ have I = 1

This gives the block diagonal form

$$H_{CM} = \begin{bmatrix} -\frac{16}{3}c_{nn} - \frac{40}{3}c_{n\bar{n}} & 0 & 0 \\ 0 & \frac{8}{3}(c_{nn} + c_{n\bar{n}}) & -8\, c_{n\bar{n}} \\ 0 & -8\, c_{n\bar{n}} & 0 \end{bmatrix}$$

(2.1.11)

It should be noted that the block corresponding to |M4> and |M5> is the same for the $I = \frac{1}{2}$ case given above, as it is for the $I = \frac{3}{2}$ case in (2.1.9).

The numerical values are given in table (2.1.8a).

Table 2.1.8a.	$qqqq\bar{q}$-sector. $Y = 1,\ I = \frac{1}{2},\ S = \frac{1}{2}$	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-342	|M1>	1458
-106	0.585 |M4> + 0.811 |M5>	1694
203	0.811 |M4> - 0.585 |M5>	2003

b) One hidden $s\bar{s}$ pair

We have five states in this group: |M1>, |M2>, |M3>, |M4> and |M5>. We get

$$H_{CM} = \begin{bmatrix}
-\frac{8}{3}c_{nn} - \frac{8}{3}c_{ns} - 10c_{n\bar{s}} - \frac{10}{3}c_{s\bar{s}} & \frac{5}{3}\sqrt{6}(c_{nn} - c_{ns} - c_{n\bar{s}} + c_{s\bar{s}}) & 3\sqrt{2}(-c_{n\bar{s}} + c_{s\bar{s}}) & \frac{\sqrt{2}}{3}(-c_{nn} + c_{ns} + 7c_{n\bar{s}} - 7c_{s\bar{s}}) & \frac{7}{3}\sqrt{2}(c_{n\bar{s}} - c_{s\bar{s}}) \\
 & -3c_{nn} - \frac{31}{3}c_{ns} - \frac{23}{3}c_{n\bar{s}} - \frac{17}{3}c_{s\bar{s}} & \sqrt{3}(-\frac{16}{3}c_{n\bar{s}} - \frac{8}{3}c_{s\bar{s}}) & \frac{5}{3}\sqrt{3}(c_{nn} - c_{ns} - c_{n\bar{s}} + c_{s\bar{s}}) & \frac{8}{3}\sqrt{3}(-c_{n\bar{s}} + c_{s\bar{s}}) \\
 & & -5c_{nn} - 11c_{ns} & -2c_{n\bar{s}} + 2c_{s\bar{s}} & 3c_{nn} - 3c_{ns} \\
 & \text{Symmetric} & & -\frac{7}{3}c_{nn} + 5c_{ns} + \frac{11}{3}c_{n\bar{s}} - c_{s\bar{s}} & -\frac{14}{3}c_{n\bar{s}} \quad \frac{10}{3}c_{s\bar{s}} \\
 & & & & -5c_{nn} + 5c_{ns}
\end{bmatrix}$$

(2.1.12)

By diagonalizing H_{CM} given by (2.1.12) we get the numerical values in table (2.1.8b).

Table 2.1.8b.	$qqqs\bar{s}$-sector	Y = 1, I = ½, S = ½
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 392	-0.004|M1> + 0.803|M2> + 0.597|M3> - 0.006|M4> + 0.004|M5>	1758
- 219	0.964|M1> - 0.131|M2> + 0.182|M3> - 0.062|M4> - 0.130|M5>	1931
- 117	0.261|M1> + 0.535|M2> - 0.719|M3> + 0.079|M4> + 0.351|M5>	2033
- 72	- 0.053|M1> + 0.216|M2> - 0.291|M3> - 0.520|M4> - 0.771|M5>	2078
93	- 0.014|M1> - 0.079|M2> + 0.095|M3> - 0.848|M4> + 0.515|M5>	2243

It should be noticed that we have a fairly large deviation from the flavoursymmetric case. In the flavoursymmetric case we have the eigenvalues -542 MeV and -97 MeV from the mixing of |M2> and |M3>. This is changed to -392 MeV and -117 MeV. Other changes are also large (see table (2.1.1)).

C1. Y = 0, I = 2, S = ½

a) No hidden strangeness

In this group we have three states: |M4>, |M5> and |M6>. The only difference between these states and the 3 states with Y = 1, I = 3/2, S = ½ and one hidden $s\bar{s}$ pair is that $\bar{s}$ has been substituted by $\bar{d}$ in the wavefunctions. Therefore the colourmagnetic interaction for these states can be obtained from (2.1.10) by replacing $\bar{s}$ in (2.1.10) by $\bar{n}$.

This gives

$$
H_{CM} = \begin{bmatrix} \frac{26}{3} c_{nn} - 6c_{ns} - \frac{4}{3} c_{n\bar{n}} + 4c_{s\bar{n}} & -\frac{26}{3} c_{n\bar{n}} + \frac{2}{3} c_{s\bar{n}} & \frac{2}{3}\sqrt{2}(c_{nn} - c_{ns} + 2c_{n\bar{n}} - 2c_{s\bar{n}}) \\ & 10\, c_{nn} - 10\, c_{ns} & \frac{2}{3}\sqrt{2}(-c_{n\bar{n}} + c_{s\bar{n}}) \\ \text{Symmetric} & & \frac{28}{3} c_{nn} + \frac{28}{3} c_{ns} + 8c_{n\bar{n}} + \frac{8}{3} c_{s\bar{n}} \end{bmatrix} \tag{2.1.13}
$$

By diagonalizing (2.1.13) we get the numerical values given in table (2.1.9a).

Table 2.1.9a	$qqqs\bar{q}$-sector $\quad Y = 0,\ I = 2,\ S = \frac{1}{2}$	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 60	0.661|M4> + 0.750|M5> - 0.017|M6>	1915
243	0.746|M4> - 0.659|M5> - 0.097|M6>	2218
451	0.083|M4> - 0.052|M5> + 0.995|M6>	2426

We clearly see that |M4> and |M5> are much mixed, while |M6> is almost eigenstate of H_{CM}. The deviation from the flavoursymmetric case is though prominent, at least for the second eigenvalue given above (conf. table (2.1.1)).

C2. $Y = 0,\ I = 1,\ S = \frac{1}{2}$

a) No hidden strangeness

There are altogether 8 states with these quantum numbers. They are |M1>, |M2>, |M3>, |M4>, |M4'>, |M5>, |M5'> and |M6>.

Let |M4'> and |M5'> be the states where the 4q-clusters have $I = \frac{3}{2}$. The state |M6> does also have its 4q-cluster in $I = \frac{3}{2}$. The 4q-clusters in |M1>, |M2>, |M3>, |M4> and |M5> all have $I = \frac{1}{2}$. If we choose to put these basis-vectors in the following sequence |M1>, |M2>, |M3>, |M4>, |M5>, |M4'>, |M5'>, |M6> we get *)

----

*) We used 7430 seconds of computer time running our different SCHOONSCHIP-programs to get this matrix.

$$
H_{CM} = \begin{bmatrix}
\begin{matrix}
-\frac{8}{3}c_{nn}-\frac{8}{3}c_{ns}-10c_{n\bar{n}}-\frac{10}{3}c_{s\bar{n}} & \frac{5}{3}\sqrt{6}(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}) & 3\sqrt{2}(-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{\sqrt{2}}{3}(-c_{nn}+c_{ns}+7c_{n\bar{n}}-7c_{s\bar{n}}) & \frac{7}{3}\sqrt{2}(c_{n\bar{n}}-c_{s\bar{n}}) \\
 & -3c_{nn}-\frac{31}{3}c_{ns}-\frac{23}{3}c_{n\bar{n}}-\frac{17}{3}c_{s\bar{n}} & \sqrt{3}(-\frac{16}{3}c_{n\bar{n}}-\frac{8}{3}c_{s\bar{n}}) & \frac{5}{3}\sqrt{3}(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{8}{3}\sqrt{3}(-c_{n\bar{n}}+c_{s\bar{n}}) \\
 & & -5c_{nn}-11c_{ns} & -2c_{n\bar{n}}+2c_{s\bar{n}} & 3c_{nn}-3c_{ns} \\
 \text{Symmetric} & & & -\frac{7}{3}c_{nn}+5c_{ns}+\frac{11}{3}c_{n\bar{n}}-c_{s\bar{n}} & -\frac{14}{3}c_{n\bar{n}}-\frac{10}{3}c_{s\bar{n}} \\
 & & & & -5c_{nn}+5c_{ns}
\end{matrix} & 0 \\
0 & \begin{matrix}
\frac{26}{3}c_{nn}-6c_{ns}-\frac{4}{3}c_{n\bar{n}}+4c_{s\bar{n}} & -\frac{26}{3}c_{n\bar{n}}+\frac{2}{3}c_{s\bar{n}} & \frac{2}{3}\sqrt{2}(c_{nn}-c_{ns}+2c_{n\bar{n}}-2c_{s\bar{n}}) \\
 & 10c_{nn}-10c_{ns} & \frac{2}{3}\sqrt{2}(-c_{n\bar{n}}+c_{s\bar{n}}) \\
\text{Symmetric} & & \frac{28}{3}c_{nn}+\frac{28}{3}c_{ns}+8c_{n\bar{n}}+\frac{8}{3}c_{s\bar{n}}
\end{matrix}
\end{bmatrix} \qquad (2.1.14)
$$

We notice again the block structure

$$H_{CM} = \left[\begin{array}{c|c} \begin{matrix}5x5\\ I(4q)=\frac{1}{2}\end{matrix} & 0 \\ \hline 0 & \begin{matrix}3x3\\ I(4q)=\frac{3}{2}\end{matrix} \end{array}\right] \qquad (2.1.15)$$

We also notice that the 3x3 block of (2.1.14) is identical to (2.1.13).

Diagonalizing the matrix (2.1.14) leads to the numerical values given in table (2.1.10a).

Table 2.1.10a. $qqqs\bar{q}$-sector. Y = 0, I = 1, S = ½

Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 528	0.073 |M1>+0.815 |M2>+0.574 |M3>+0.018 |M4>+0.031 |M5>	1447
- 306	0.974 |M1>-0.121 |M2>+0.059 |M3>-0.101 |M4>-0.153 |M5>	1669
- 117	0.209 |M1>+0.141 |M2>-0.285 |M3>+0.476 |M4>+0.793 |M5>	1858
- 60	0.661 |M4'> + 0.750 |M5'> - 0.017 |M6>	1915
- 53	0.050 |M1>+0.543 |M2>-0.756 |M3>-0.302 |M4>-0.200 |M5>	1922
153	0.016 |M1>+0.085 |M2>-0.118 |M3>+0.820 |M4>-0.554 |M5>	2128
243	0.746 |M4'> - 0.659 |M5'> - 0.097 |M6>	2218
451	0.083 |M4'> - 0.052 |M5'> + 0.995 |M6>	2426

Let us take a closer look at the deviations we get compared to the flavoursymmetric case. The interesting mixing of |M2> and |M3> in the flavoursymmetric case gave the big massefect of -543 MeV and -97 MeV. With broken flavoursymmetry we see that the other states still contribute little to the eigenstates that are mainly a mixture of |M2> and |M3>. This leads to eigenvalues of -528 MeV and -53 MeV.

It is also interesting to take a look at the mixing of multiplets 4 and 5. In the flavoursymmetric case this gave eigenvalues -86 MeV and 166 MeV. Each of these are doubly degenerated. From table (2.1.10a) we see that the mainly |M4> and |M5> states mix and give eigenvalues -117 MeV and 153 MeV, while the mainly |M4'> and |M5'> states mix and give eigenvalues -60 MeV and 243 MeV. For this case we thus see that introducing broken flavoursymmetry leads to splitting of degenerate levels. In this case -86 MeV splits into -117 MeV and -60 MeV, while 166 MeV splits into 153 Mev and 243 Mev.

b) One hidden $s\bar{s}$ pair

We have four states in this group: $|M1\rangle$, $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. We find

$$H_{CM}=\begin{bmatrix} \frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}-\frac{20}{3}c_{n\bar{s}}-\frac{20}{3}c_{s\bar{s}} & \frac{2}{9}\sqrt{2}(-c_{nn}+c_{ss}+14c_{n\bar{s}}-14c_{s\bar{s}}) & \frac{28}{9}\sqrt{3}(c_{n\bar{s}}-c_{s\bar{s}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) \\ & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}+\frac{4}{3}c_{n\bar{s}}+\frac{4}{3}c_{s\bar{s}} & -4(c_{n\bar{s}}+c_{s\bar{s}}) & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}+4c_{n\bar{s}}-4c_{s\bar{s}}) \\ & & \frac{8}{3}(c_{nn}-2c_{ns}+c_{ss}) & \frac{4}{9}\sqrt{6}(-c_{n\bar{s}}+c_{s\bar{s}}) \\ & \text{Symmetric} & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})+\frac{16}{3}(c_{n\bar{s}}+c_{s\bar{s}}) \end{bmatrix} \qquad (2.1.16)$$

Diagonalizing (2.1.16) gives the numerical values of table (2.1.10b).

Table 2.1.10b.	$qqss\bar{s}$-sector. Y = 0, I = 1, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass(MeV)
-175	$0.980\|M1\rangle -0.119\|M4\rangle -0.162\|M5\rangle +0.004\|M6\rangle$	2150
- 35	$0.201\|M1\rangle +0.553\|M4\rangle +0.808\|M5\rangle -0.013\|M6\rangle$	2290
113	$0.007\|M1\rangle +0.822\|M4\rangle -0.565\|M5\rangle -0.074\|M6\rangle$	2438
313	$-0.001\|M1\rangle +0.068\|M4\rangle -0.031\|M5\rangle +0.997\|M6\rangle$	2638

C3. Y = 0, I = 0, S = ½

a) No hidden strangeness

In this group we have 5 states: |M1>, |M2>, |M3>, |M4> *and* |M5>. They all have 4q clusters with I = ½, *and we get*

$$H_{CM} = \begin{bmatrix} -\frac{8}{3}c_{nn}-\frac{8}{3}c_{ns}-10c_{n\bar{n}}-\frac{10}{3}c_{s\bar{n}} & \frac{5}{3}\sqrt{6}(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}) & 3\sqrt{2}(-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{\sqrt{2}}{3}(-c_{nn}+c_{ns}+7c_{n\bar{n}}-7c_{s\bar{n}}) & \frac{7}{3}\sqrt{2}(c_{n\bar{n}}-c_{s\bar{n}}) \\ & -\frac{1}{3}(9c_{nn}+31c_{ns}+23c_{n\bar{n}}+17c_{s\bar{n}}) & -\frac{8}{3}\sqrt{3}(2c_{n\bar{n}}+c_{s\bar{n}}) & \frac{5}{3}\sqrt{3}(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{8}{3}\sqrt{3}(-c_{n\bar{n}}+c_{s\bar{n}}) \\ & & -5c_{nn}-11c_{ns} & 2(-c_{n\bar{n}}+c_{s\bar{n}}) & 3(c_{nn}-c_{ns}) \\ & \text{Symmetric} & & -\frac{7}{3}c_{nn}+5c_{ns}+\frac{11}{3}c_{n\bar{n}}-c_{s\bar{n}} & -\frac{14}{3}c_{n\bar{n}}-\frac{10}{3}c_{s\bar{n}} \\ & & & & 5(-c_{nn}+c_{ns}) \end{bmatrix}$$

(2.1.17)

which is identical to the 5x5 block of (2.1.14). Diagonalization of H_{CM} gives the numerical values as given in table (2.1.11a)

Table 2.1.11a	$qqqs\bar{q}$-sector Y = 0, I = 0, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-528	0.073|M1>+0.815|M2>+0.574|M3>+0.018|M4>+0.031|M5>	1447
-306	0.974|M1>-0.121|M2>+0.059|M3>-0.101|M4>-0.153|M5>	1669
-117	0.209|M1>+0.141|M2>-0.285|M3>+0.476|M4>+0.793|M5>	1858
- 53	0.050|M1>+0.543|M2>-0.756|M3>-0.302|M4>-0.200|M5>	1922
153	0.016|M1>+0.085|M2>-0.118|M3>+0.820|M4>-0.554|M5>	2128

b) One hidden $s\bar{s}$-pair

We have four states in this group: M1>, M2>, M4> and M5>. We get

$$H_{CM} = \begin{bmatrix} -\frac{14}{3}c_{nn}-\frac{34}{3}c_{ns}+\frac{8}{3}c_{ss} -\frac{34}{3}c_{n\bar{s}}-2c_{s\bar{s}} & \frac{8}{3}\sqrt{3}(-2c_{n\bar{s}} -c_{s\bar{s}}) & \frac{10}{3}(c_{nn}-c_{ns} -c_{n\bar{s}}+c_{s\bar{s}}) & \frac{16}{3}(-c_{n\bar{s}} +c_{s\bar{s}}) \\ & -3c_{nn}-16c_{ns} + 3c_{ss} & \frac{4}{3}\sqrt{3}(-c_{n\bar{s}} + c_{s\bar{s}}) & \frac{\sqrt{3}}{3}(5c_{nn}-4c_{ns} - c_{ss}) \\ & & -\frac{14}{3}c_{nn}+\frac{14}{3}c_{ns} + \frac{8}{3}c_{ss} + \frac{14}{3}c_{n\bar{s}}-2c_{s\bar{s}} & -\frac{4}{3}c_{n\bar{s}}-\frac{20}{3}c_{s\bar{s}} \\ \text{Symmetric} & & & \frac{1}{3}(-19c_{nn} + 8c_{ns}+11c_{ss}) \end{bmatrix} \quad (2.1.18)$$

Diagonalization of H_{CM} given by (2.1.18) leads to the numerical values given in table (2.1.11b)

Table 2.1.11b	$qqss\bar{s}$-sector Y = 0, I = 0, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 417	0.840|M1>+0.542|M2>-0.008|M4>+0.013|M5>	1908
- 137	0.480|M1>-0.753|M2>+0.093|M4>+0.440|M5>	2188
- 84	0.235|M1>-0.353|M2>-0.496|M4>-0.757|M5>	2241
59	-0.091|M1>+0.117|M2>-0.863|M4>+0.482|M5>	2384

D1. Y = -1, I = 3/2, S = ½

a) No hidden strangeness

We have four states in this group: $|M1\rangle$, $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. They all have 4q-clusters with I = 1. The only difference between these states and the four states with one $s\bar{s}$ pair and Y = 0, I = 1, S = ½ is that $\bar{s}$ is substituted by $\bar{d}$ (if we take the states with $I_z = 3/2$). Therefore H_{CM} can be obtained from (2.1.16) by replacing $c_{x\bar{s}}$ with $c_{x\bar{n}}$. We get

$$H_{CM}=\begin{bmatrix} \frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}-\frac{20}{3}c_{n\bar{n}}-\frac{20}{3}c_{s\bar{n}} & \frac{2}{9}\sqrt{3}(-c_{nn}+c_{ss}+14c_{n\bar{n}}-14c_{s\bar{n}}) & \frac{28}{9}\sqrt{3}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) \\ & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}+\frac{4}{3}c_{n\bar{n}}+\frac{4}{3}c_{s\bar{n}} & -4(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}+4c_{n\bar{n}}-4c_{s\bar{n}}) \\ & & \frac{8}{3}(c_{nn}-2c_{ns}+c_{ss}) & \frac{4}{9}\sqrt{6}(-c_{n\bar{n}}+c_{s\bar{n}}) \\ \text{Symmetric} & & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})+\frac{16}{3}(c_{n\bar{n}}+c_{s\bar{n}}) \end{bmatrix} \qquad (2.1.19)$$

Diagonalizing this matrix leads to the numerical values given in table (2.1.12a)

Table 2.1.12a	$qq\bar{s}s\bar{q}$-sector Y = -1, I = 3/2, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass (Mev)
-260	$0.961\|M1\rangle-0.174\|M4\rangle-0.217\|M5\rangle+0.005\|M6\rangle$	1890
- 63	$0.278\|M1\rangle+0.570\|M4\rangle+0.773\|M5\rangle-0.014\|M6\rangle$	2087
164	$0.011\|M1\rangle+0.798\|M4\rangle-0.593\|M5\rangle-0.107\|M6\rangle$	2314
374	$0.0004\|M1\rangle+0.095\|M4\rangle-0.052\|M5\rangle+0.994\|M6\rangle$	2524

D2. $Y = -1$, $I = \frac{1}{2}$, $S = \frac{1}{2}$

a) No hidden strangeness

We have 8 states with these quantum numbers. They are |M1>, |M2>, |M3>, |M4>, |M4'>, |M5>, |M5'> and |M6>. The 4q-clusters have $I = 1$ in |M1>, |M4'>, |M5'> and |M6>. The 4q-clusters of |M2>, |M3>, |M4> and |M5> have $I = 0$. We therefore choose the basis in the following sequence |M1>, |M4'>, |M5'>, |M6>, |M2>, |M3>, |M4>, |M5> which gives H_{CM} as in eq. (2.1.20) (next page).

$$
H_{CM} = \left[\begin{array}{cccc|cccc}
\frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}-\frac{20}{3}c_{n\bar{n}}-\frac{20}{3}c_{s\bar{n}} & \frac{2}{9}\sqrt{3}(-c_{nn}+c_{ss}+14c_{n\bar{n}}-14c_{s\bar{n}}) & \frac{28}{9}\sqrt{3}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) & & & & \\
 & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}+\frac{4}{3}c_{n\bar{n}}+\frac{4}{3}c_{s\bar{n}} & -4(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}+4c_{n\bar{n}}-4c_{s\bar{n}}) & & 0 & & \\
 & & \frac{8}{3}(c_{nn}-2c_{ns}+c_{ss}) & \frac{4}{9}\sqrt{6}(-c_{n\bar{n}}+c_{s\bar{n}}) & & & & \\
 & \text{Symmetric} & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})+\frac{16}{3}(c_{n\bar{n}}+c_{s\bar{n}}) & & & & \\
\hline
 & & & & -\frac{14}{3}c_{nn}-\frac{34}{3}c_{ns}+\frac{8}{3}c_{ss}-\frac{34}{3}c_{n\bar{n}}-2c_{s\bar{n}} & \frac{8}{3}\sqrt{3}(-2c_{n\bar{n}}-c_{s\bar{n}}) & \frac{10}{3}(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{16}{3}(-c_{n\bar{n}}+c_{s\bar{n}}) \\
 & 0 & & & & -3c_{nn}-16c_{ns}+3c_{ss} & \frac{4}{3}\sqrt{3}(-c_{n\bar{n}}+c_{s\bar{n}}) & \frac{\sqrt{3}}{3}(5c_{nn}-4c_{ns}-c_{ss}) \\
 & & & & & & -\frac{14}{3}c_{nn}+\frac{14}{3}c_{ns}+\frac{8}{3}c_{ss}+\frac{14}{3}c_{n\bar{n}}-2c_{s\bar{n}} & -\frac{4}{3}c_{n\bar{n}}-\frac{20}{3}c_{s\bar{n}} \\
 & & & & & \text{Symmetric} & & \frac{1}{3}(-19c_{nn}+8c_{ns}+11c_{ss})
\end{array}\right] \quad (2.1.20)
$$

The upper block is identical to (2.1.19). The lower block can be obtained from (2.1.18) by changing $\bar{s}$ in (2.1.18) to $\bar{n}$. (This reflects the difference between the corresponding wavefunctions).

Diagonalizing the matrix (2.1.20) gives the masses etc. of all Ξ-particles with no hidden strangeness as given in table (2.1.13a).

Table 2.1.13a	$qqss\bar{q}$-sector Y = -1, I = ½, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-560	0.847|M2>+0.529|M3>+0.022|M4>+0.046|M5>	1590
-260	0.961|M1>-0.174|M4'>-0.217|M5'>+0.005|M6>	1890
-131	0.199|M2>-0.404|M3>+0.398|M4>+0.799|M5>	2019
- 64	0.483|M2>-0.731|M3>-0.376|M4>-0.303|M5>	2086
- 63	0.278|M1>+0.570|M4'>+0.773|M5'>-0.014|M6>	2087
111	-0.100|M2>+0.150|M3>-0.837|M4>+0.517|M5>	2261
164	0.011|M1>+0.798|M4'>-0.593|M5'>-0.107|M6>	2314
374	0.0004|M1>+0.095|M4'>-0.052|M5'>-0.994|M6>	2524

b) One hidden $s\bar{s}$ pair

We have three states in this group: \|M4>, \|M5> and \|M6>. For these states we get

$$H_{CM} = \begin{bmatrix} \frac{26}{3}c_{ss} - 6c_{ns} - \frac{4}{3}c_{s\bar{s}} + 4c_{n\bar{s}} & -\frac{26}{3}c_{s\bar{s}} + \frac{2}{3}c_{n\bar{s}} & \frac{2}{3}\sqrt{2}(-c_{ns} + c_{ss} - 2c_{n\bar{s}} + 2c_{s\bar{s}}) \\ & 10c_{ss} - 10c_{ns} & \frac{2}{3}\sqrt{2}(+c_{n\bar{s}} - c_{s\bar{s}}) \\ \text{Symmetric} & & \frac{28}{3}c_{ns} + \frac{28}{3}c_{ss} + \frac{8}{3}c_{n\bar{s}} + 8c_{s\bar{s}} \end{bmatrix} \quad (2.1.21)$$

Diagonalizing the matrix (2.1.21) we get the numerical values given in table (2.1.13b).

Table 2.1.13b	*$qsss\bar{s}$-sector* Y = -1, I = ½, S = ½	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-69	0.477|M4>+0.879|M5>-0.006|M6>	2431
58	0.877|M4>-0.477|M5>-0.060|M6>	2558
254	0.056|M4>-0.023|M5>+0.998|M6>	2754

E1. $Y = -2,\ I = 1,\ S = \frac{1}{2}$

a) *No hidden strangeness*

We have three states in this group: $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. The colour-magnetic interaction for these states can be obtained from the matrix (2.1.21) by doing the substitution $\bar{s} \to \bar{n}$ in (2.1.21). We find

$$H_{CM} = \begin{bmatrix} -6c_{ns} + \frac{26}{3}c_{ss} + 4c_{n\bar{n}} - \frac{4}{3}c_{s\bar{n}} & \frac{2}{3}c_{n\bar{n}} - \frac{26}{3}c_{s\bar{n}} & \frac{2}{3}\sqrt{2}(-c_{ns} + c_{ss} - 2c_{n\bar{n}} + 2c_{s\bar{n}}) \\ & -10c_{ns} + 10c_{ss} & \frac{2}{3}\sqrt{2}(c_{n\bar{n}} - c_{s\bar{n}}) \\ \text{Symmetric} & & \frac{28}{3}c_{ns} + \frac{28}{3}c_{ss} + \frac{8}{3}c_{n\bar{n}} + 8c_{s\bar{n}} \end{bmatrix} \quad (2.1.22)$$

It should be noticed that we can also get (2.1.22) from (2.1.13) by letting $n \to s$ and $s \to n$ in (2.1.13). This can be understood from the fact that we have totally antisymmetric 4q-clusters with the same spin and colour in each case. The only difference in the wavefunctions is that we have flavour qqqs and sssq respectively (q = u or d).

Diagonalizing (2.1.22) gives the numerical values of table (2.1.14a).

Table 2.1.14a	$qsss\bar{q}$-sector $Y = -2,\ I = 1,\ S = \frac{1}{2}$	
Eigenvalue (MeV)	Eigenvector	Mass (Mev)
- 89	$0.506\,|M4\rangle + 0.862\,|M5\rangle - 0.007\,|M6\rangle$	2236
100	$0.859\,|M4\rangle - 0.505\,|M5\rangle - 0.091\,|M6\rangle$	2425
307	$0.082\,|M4\rangle - 0.039\,|M5\rangle + 0.996\,|M6\rangle$	2632

E2. $Y = -2,\ I = 0,\ S = \frac{1}{2}$

a) No hidden strangeness

There are three states in this group: $|M4\rangle$, $|M5\rangle$ and $|M6\rangle$. They all have 4q-clusters with $I = \frac{1}{2}$ (the same 4q-clusters which make up the $Y = -2$, $I = 1$, $S = \frac{1}{2}$ states). The H_{CM}-matrix in this case is identical to (2.1.22), and the numerical values is therefore the same as the values given in table (2.1.14a).

b) One hidden $s\bar{s}$-pair

We have only one state in this group: $|M6\rangle$. We find

$$\langle H_{CM}\rangle = \frac{56}{3}c_{ss} + \frac{32}{3}c_{s\bar{s}} \qquad (2.1.23)$$

which leads to the numerical values given in table (2.1.14b)

Table 2.1.14b	$ssss\bar{s}$-sector	$Y = -2,\ I = 0,\ S = \frac{1}{2}$
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
205	$\|M6\rangle$	2880

F1. $Y = -3,\ I = \frac{1}{2},\ S = \frac{1}{2}$

a) No hidden strangeness

We have only one stat[illegible] group: $|M6\rangle$. $\langle H_{CM}\rangle$ can be obtained from (2.1.23) by the substit[illegible] $\rightarrow \bar{n}$, or from (2.1.8) by the substitution $n \rightarrow s$. We thus find

$$\langle H_{CM}\rangle = \frac{56}{3}c_{ss} + \frac{32}{3}c_{s\bar{n}} \qquad (2.1.24)$$

The numerical values are given in table (2.1.15a)

Table 2.1.15a	$ssss\bar{q}$-sector	$Y = -3,\ I = \frac{1}{2},\ S = \frac{1}{2}$
Eigenvalue (MeV)	Eigenvector	Mass (Mev)
248	$\|M6\rangle$	2748

2.2. The colour-magnetic interaction for the $J^P = 3/2^- 4q\bar{q}$ states.

From eq. (2.2) we see that there are five coloursinglet $4q\bar{q}$ representations with $s = \frac{3}{2}$. These are $(1,4,\overline{18})$, $(1,4,9)$ $(1,4,45)$ (two different) and $(1,4,45')$. Using the decomposition of the flavour multiplets as given in fig. 2.1 we finally arrive at the $J^P = 3/2^-$ $4q\bar{q}$ multiplets as given in fig. 2.2.1. The multiplets are numbered from 1 to 5 as given in fig. 2.2.1. In the flavoursymmetric case we know that multiplets 3 and 4 are mixed under the action of H_{CM}. The other multiplets are all diagonal with respect to H_{CM}. In the flavour-symmetric case we thus have [8,18]

$$\langle H_{CM} \rangle = \frac{4}{3}\, c \qquad (2.2.1)$$

for all states in multiplet 1.

$$\langle H_{CM} \rangle = -\frac{20}{3}\, c \qquad (2.2.2)$$

for all states in multiplet 2.
Multiplets 3 and 4 mix and we get

$$H_{CM} \begin{bmatrix} |M3\rangle \\ |M4\rangle \end{bmatrix} = \begin{bmatrix} 0 & -4\sqrt{10}c \\ -4\sqrt{10}c & \frac{4}{3}\, c \end{bmatrix} \begin{bmatrix} |M3\rangle \\ |M4\rangle \end{bmatrix} \qquad (2.2.3)$$

Diagonalizing this leads to the eigenvalues $-12c$ and $\frac{40}{3}\, c$ [18].

All states in multiplet 5 have

$$\langle H_{CM} \rangle = \frac{40}{3}\, c \qquad (2.2.4)$$

If we use the average value $c = 15$ MeV in the flavoursymmetric case, we get the eigenvalues and eigenvectors given in table (2.2.1).

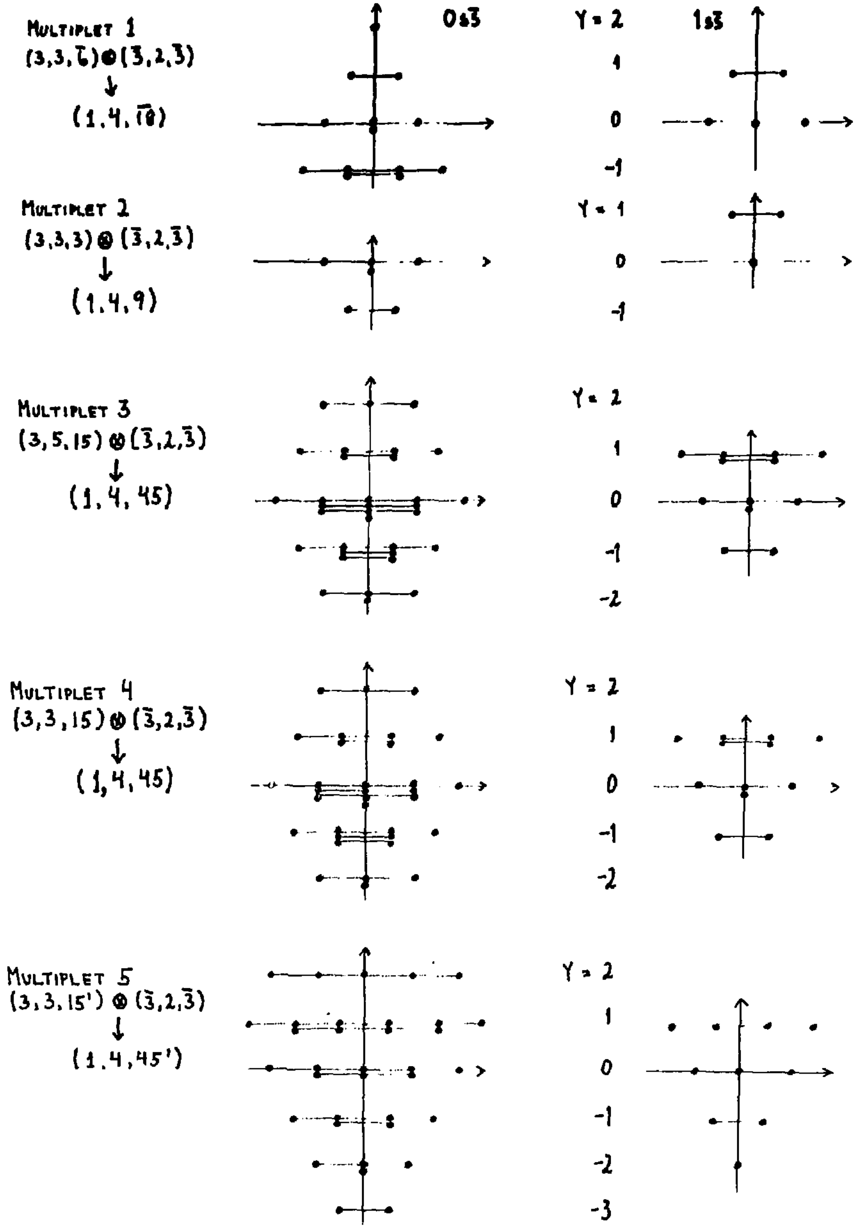


Fig. 2.2.1. The $J^P = 3/2^-$ $4q\bar{q}$ flavour multiplets (numbered from 1 to 5).

Table 2.2.1. Eigenvectors and eigenvalues of H_{CM} for the $J^P=3/2^-$ $4q\bar{q}$ states assuming flavoursymmetry		
Multiplet	Eigenvalue (MeV)	Eigenvector
|M1>	20	|M1>
|M2>	-100	|M2>
|M3>, |M4>	-180	0.725|M3>+0.688|M4>
	200	0.688|M3>-0.725|M4>
|M5>	200	|M5>

We now proceed as for the $J^P = \frac{1}{2}^-$ states, and introduce flavour-symmetry breaking. Let us divide the states with $J^P = 3/2^-$ into different groups, according to the value of Y, I and the number of $s\bar{s}$ pairs. Within each of these groups, we denote a state in multiplet i by |Mi> (see fig. 2.2.1).

A1. Y = 2, I = 2, S = 3/2

a) No hidden strangeness

From fig. 2.2.1 we see that there is only one state in this group: |M5>. We get

$$\langle H_{CM}\rangle = \frac{56}{3} c_{nn} - \frac{16}{3} c_{n\bar{s}} \tag{2.2.5}$$

which reduces to (2.2.4) in the flavoursymmetric case.
Numerical values are given in table (2.2.2a).

Table 2.2.2a	$qqqq\bar{s}$-sector	Y = 2, I = 2, S = 3/2
Eigenvalue (MeV)	Eigenvector	Mass
283	|M5>	2258

A2. Y = 2, I = 1, S = 3/2

a) No hidden strangeness

From fig. (2.2.1) we see that there are two states in this group: $|M3\rangle$ and $|M4\rangle$. Using these states as a basis we get

$$H_{CM} = \begin{bmatrix} 8c_{nn} - 8c_{n\bar{s}} & -4\sqrt{10}\, c_{n\bar{s}} \\ -4\sqrt{10}\, c_{n\bar{s}} & \frac{8}{3} c_{nn} - \frac{4}{3} c_{n\bar{s}} \end{bmatrix} \qquad (2.2.6)$$

Diagonalizing the matrix (2.2.6) leads to the numerical values given in table (2.2.3a).

Table 2.2.3a.	$qqqq\bar{s}$-sector.	Y = 2, I = 1, S = 3/2.
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 93	$0.676|M3\rangle+0.737|M4\rangle$	1882
186	$0.737|M3\rangle-0.676|M4\rangle$	2161

The fairly large deviations from the flavoursymmetric case given in table (2.2.1) should be noticed.

A3. Y = 2, I = 0, S = 3/2

a) No hidden strangeness

There is only one state in this group: $|M1\rangle$. We get

$$\langle H_{CM}\rangle = -\frac{16}{3} c_{nn} + \frac{20}{3} c_{n\bar{s}} \qquad (2.2.7)$$

The numerical values are given in table (2.2.4a)

Table 2.2.4a.	$qqqq\bar{s}$-sector	Y = 2, I = 0, S = 3/2
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 24	$|M1\rangle$	1951

B1. $Y = 1, I = 5/2, S = 3/2$

a) No hidden strangeness

There is only one state in this group: $|M5\rangle$. We get

$$\langle H_{CM}\rangle = \frac{56}{3} c_{nn} - \frac{16}{3} c_{n\bar{n}} \qquad (2.2.8)$$

which is the same as (2.2.5) with the exception $c_{n\bar{s}} \rightarrow c_{n\bar{n}}$. This has to be so because the only difference in the wavefunctions are given by the substitution $\bar{s} \rightarrow \bar{q}$ (q denotes a *nonstrange* quark).
The numerical values are given in table (2.2.5a).

Table 2.2.5a.	$qqqq\bar{q}$-sector	$Y = 1, I = 5/2, S = 3/2$
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
244	$\|M5\rangle$	2044

We also notice that the interaction-energy given by (2.2.8) leads to a state with lower mass than the interaction-energy given by (2.1.8). We therefore see that $M(\Uparrow\uparrow) < M(\Uparrow\downarrow)$, where $\Uparrow$ indicate a spin 1 4q-cluster, and $\uparrow$ the spin ½ $\bar{q}$.

This is opposite to what we find for the $q\bar{q}$ states π and ρ and the qqq states N and Δ where we have

$$M_\pi(\uparrow\downarrow) < M_\rho(\uparrow\uparrow).$$

$$M_N(\uparrow\downarrow\uparrow) < M_\Delta(\uparrow\uparrow\uparrow).$$

This is an illustration of the more complex structure of the CS-symmetry one has in multiquark states compared to the minimum states $q\bar{q}$ and qqq. It is not any longer possible to look at the spinsymmetry only, as we do for the $q\bar{q}$ and qqq systems, because the colour is not "frozen" any more.

B2. Y = 1, I = 3/2, S = 3/2

a) No hidden strangeness

There are three states in this group: |M3>, |M4> and |M5>. The 4q-clusters of |M3> and |M4> have I = 1 (see fig. 2.1) and the 4q-cluster of |M5> has I = 2. This gives a block-matrix if we choose the basisvectors in the sequence |M3>, |M4>, |M5>. We get

$$H_{CM} = \left[\begin{array}{cc|c} 8(c_{nn} - c_{n\bar{n}}) & -4\sqrt{10}c_{n\bar{n}} & 0 \\ -4\sqrt{10}c_{n\bar{n}} & \frac{8}{3}c_{nn} - \frac{4}{3}c_{n\bar{n}} & 0 \\ \hline 0 & 0 & \frac{56}{3}c_{nn} - \frac{16}{3}c_{n\bar{n}} \end{array}\right] \tag{2.2.9}$$

If we look at the matrix (2.2.9) and the value of H_{CM} given in eq. (2.2.8) we clearly see the degeneracy between the I = 3/2 and I = 5/2 states with Y = 1 of multiplet 5.

Diagonalizing the matrix (2.2.9) leads to the numerical values given in table (2.2.6a).

Table 2.2.6a. $qqqq\bar{q}$-sector. Y = 1, I = 3/2, S = 3/2.

Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-220	0.725|M3>+0.688|M4>	1580
244	0.688|M3>-0.725|M4>	2044
244	|M5>	2044

It should also be noticed that the two degenerate levels in the flavour-symmetric case with eigenvalue 200 MeV survives and moves to 244 MeV when we break the flavoursymmetry. This is of course so, because we have only non-strange quarks in this sector (we see that this leads to the new eigenvalue by multiplying the old one with c_{nn}/c = 18.3/15).

b) One hidden $s\bar{s}$ pair

We have three states in this group: |M3>, |M4> and |M5>. They all have 4q-clusters with I = 3/2. We find

$$H_{CM} = \begin{bmatrix} 8(c_{nn} - c_{s\bar{s}}) & \frac{4}{3}\sqrt{10}(-c_{n\bar{s}} - 2c_{s\bar{s}}) & \frac{8}{3}\sqrt{5}(-c_{n\bar{s}} + c_{s\bar{s}}) \\ & \frac{26}{3}c_{nn} - 6c_{ns} + \frac{2}{3}c_{n\bar{s}} - 2c_{s\bar{s}} & \frac{2}{3}\sqrt{2}(c_{nn} - c_{ns} - c_{n\bar{s}} + c_{s\bar{s}}) \\ \text{Symmetric} & & \frac{28}{3}(c_{nn} + c_{ns}) - 4c_{n\bar{s}} - \frac{4}{3}c_{s\bar{s}} \end{bmatrix} \qquad (2.2.10)$$

Diagonalizing H_{CM} in (2.2.10) leads to the table (2.2.6b).

Table 2.2.6b.	$qqqs\bar{s}$-sector. Y = 1, I = 3/2, S= 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 18	0.702|M3>+0.709|M4>+0.061|M5>	2132
184	0.612|M3>-0.645|M4>+0.456|M5>	2334
231	-0.363|M3>+0.283|M4>+0.888|M5>	2381

In this case we again see that we have fairly large deviations from the flavoursymmetric results. In the flavoursymmetric case we have one state with a massdefect of -180 MeV. This is changed to -18 MeV, which changes the mass and the stability of the corresponding state considerably.

B3., Y = 1, I = ½, S = 3/2

a) No hidden strangeness

We have three states in this group: |M1>, |M3> and |M4>. The 4q-cluster of |M1> has I = 0, and the 4q-clusters of |M3> and |M4> have I = 1. This gives a block-diagonal matrix when the basis is chosen in the sequence |M1>, |M3>, |M4>.

We get

$$H_{CM} = \begin{bmatrix} -\frac{16}{3}c_{nn}+\frac{20}{3}c_{n\bar{n}} & 0 & 0 \\ 0 & 8(c_{nn}-c_{n\bar{n}}) & -4\sqrt{10}c_{n\bar{n}} \\ 0 & -4\sqrt{10}c_{n\bar{n}} & \frac{8}{3}c_{nn}-\frac{4}{3}c_{n\bar{n}} \end{bmatrix} \qquad (2.2.11)$$

We also notice the degeneracy between the $I = \frac{1}{2}$ and $I = 3/2$ states of multiplets 3 and 4 (see (2.2.11) and (2.2.9)). Diagonalizing the matrix (2.2.11) leads to the numerical values given in table (2.2.7a)

Table 2.2.7a.	$qqqq\bar{q}$-sector. $Y = 1$, $I = \frac{1}{2}$, $S = 3/2$	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 220	0.725 |M3>+0.688 |M4>	1580
24	|M1>	1824
244	0.688 |M3>-0.725 |M4>	2044

b) One hidden $s\bar{s}$ pair

We find four states in this group: |M1>, |M2>, |M3> and |M4>. We get

$$H_{CM} = \begin{bmatrix} -\frac{8}{3}c_{nn}-\frac{8}{3}c_{ns}+5c_{n\bar{s}}+\frac{5}{3}c_{s\bar{s}} & \sqrt{\frac{2}{3}}(5c_{nn}-5c_{ns}+\frac{5}{2}c_{n\bar{s}}-\frac{5}{2}c_{s\bar{s}}) & \frac{\sqrt{5}}{3}(c_{n\bar{s}}-c_{s\bar{s}}) & \frac{\sqrt{2}}{6}(-2c_{nn}+2c_{ns}-7c_{n\bar{s}}+7c_{s\bar{s}}) \\ & -3c_{nn}-\frac{31}{3}c_{ns}+\frac{23}{6}c_{n\bar{s}}+\frac{17}{6}c_{s\bar{s}} & \frac{\sqrt{30}}{6}(c_{n\bar{s}}-c_{s\bar{s}}) & \frac{5}{6}\sqrt{3}(2c_{nn}-2c_{ns}+c_{n\bar{s}}-c_{s\bar{s}}) \\ & & 2c_{nn}+6c_{ns}-9c_{n\bar{s}}+c_{s\bar{s}} & \frac{\sqrt{10}}{6}(-23c_{n\bar{s}}-c_{s\bar{s}}) \\ \text{Symmetric} & & & -\frac{7}{3}c_{nn}+5c_{ns}-\frac{11}{6}c_{n\bar{s}}+\frac{1}{2}c_{s\bar{s}} \end{bmatrix} \qquad (2.2.12)$$

Diagonalizing H_{CM} in (2.2.12) leads to the numerical values given in table (2.2.7b)

Table 2.2.7b.	qqqs$\bar{s}$-sector. Y = 1, I = ½, S = 3/2	
Eigenvalue (MeV)	**Eigenvector**	**Mass (MeV)**
- 151	0.187|M1>-0.569|M2>+0.523|M3>+0.606|M4>	1999
- 104	0.278|M1>-0.748|M2>-0.455|M3>-0.395|M4>	2046
2	0.941|M1>+0.337|M2>-0.004|M3>+0.029|M4>	2152
142	0.045|M1>-0.057|M2>+0.720|M3>-0.690|M4>	2292

C1. Y = 0, I = 2, S = 3/2

a) No hidden strangeness

We have three states in this group: |M3>, |M4> and |M5>. They all have 4q-clusters with I = 3/2. H_{CM} is obtained from (2.2.10) by doing the substitution $\bar{s} \rightarrow \bar{n}$. We find

$$H_{CM} = \begin{bmatrix} 8(c_{nn}-c_{s\bar{n}}) & \frac{4}{3}\sqrt{10}(-c_{n\bar{n}} - 2c_{s\bar{n}}) & \frac{8}{3}\sqrt{5}(-c_{n\bar{n}} + c_{s\bar{n}}) \\ & \frac{26}{3}c_{nn} - 6c_{ns} + \frac{2}{3}c_{n\bar{n}} - 2c_{s\bar{n}} & \frac{2}{3}\sqrt{2}(c_{nn}-c_{ns} - c_{n\bar{n}} + c_{s\bar{n}}) \\ \text{Symmetric} & & \frac{28}{3}(c_{nn}+c_{ns}) - 4c_{n\bar{n}} - \frac{4}{3}c_{s\bar{n}} \end{bmatrix} \quad (2.2.13)$$

Diagonalizing (2.2.13) leads to the numerical values given in table (2.2.8a).

Table 2.2.8a.	qqsq̄-sector. Y = 0, I = 2, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 103	0.734|M3>+0.670|M4>+0.111|M5>	1872
176	-0.204|M3>+0.373|M4>-0.905|M5>	2151
254	-0.648|M3>+0.642|M4>+0.410|M5>	2229

C2. Y = 0, I = 1, S = 3/2

a) No hidden strangeness

There are altogether 7 states with these quantum numbers. They are |M1>, |M2>, |M3>, |M3'>, |M4>, |M4'> and |M5>. Let |M3'> and |M4'> be the states where the 4q-clusters have I = 3/2. The 4q-cluster of |M5> also has I = 3/2. The states |M1>, |M2>, |M3> and |M4> all have 4q-clusters with I = ½ (see fig. 2.1 and 2.2.1). If we choose to put the 7 basis-vectors in the following sequence:
|M1>, |M2>, |M3>, |M4>, |M3'>, |M4'>, |M5> we get a block-diagonal form on the colour-magnetic interaction (see eq. 2.2.14 next page).

The lower 3x3 block matrix of H_{CM} in (2.2.14) is identical to the I = 2 matrix given in (2.2.13), making the three I = 2 states degenerate with these three corresponding I = 1 states. The upper 4x4 block matrix can be obtained from H_{CM} given in (2.2.12) by doing the substitution $\bar{s} \to \bar{n}$.

Diagonalizing the matrix (2.2.14) leads to the numerical values given in table (2.2.9a).

Table 2.2.9a.	$qqs\bar{q}$-sector. Y = 0, I = 1, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-266	-0.043|M1>+0.142|M2>-0.721|M3>-0.677|M4>	1709
-103	0.734|M3'>+0.670|M4'>+0.111|M5>	1872
- 82	0.362|M1>-0.917|M2>-0.164|M3>-0.041|M4>	1893
49	0.928|M1>+0.369|M2>-0.024|M3>+0.044|M4>	2024
176	-0.204|M3'>+0.373|M4'>-0.905|M5>	2151
197	0.075|M1>-0.058|M2>+0.673|M3>-0.734|M4>	2172
254	-0.648|M3'>+0.642|M4'>+0.410|M5>	2229

We see that the lowest state has a mass defect of -266 MeV which is 86 MeV lower than the flavour-symmetric result of -180 MeV.

$$
H_{CM}=\left[\begin{array}{cccc|ccc}
-\frac{8}{3}c_{nn}-\frac{8}{3}c_{ns}+5c_{n\bar{n}}+\frac{5}{3}c_{s\bar{n}} & \sqrt{\frac{2}{3}}\left(5c_{nn}-5c_{ns}+\frac{5}{2}c_{n\bar{n}}-\frac{5}{2}c_{s\bar{n}}\right) & \frac{\sqrt{5}}{3}\left(c_{n\bar{n}}-c_{s\bar{n}}\right) & \frac{\sqrt{2}}{6}\left(-2c_{nn}+2c_{ns}-7c_{n\bar{n}}+7c_{s\bar{n}}\right) & & & \\
 & -3c_{nn}-\frac{31}{3}c_{ns}+\frac{23}{6}c_{n\bar{n}}+\frac{17}{6}c_{s\bar{n}} & \frac{\sqrt{30}}{6}\left(c_{n\bar{n}}-c_{s\bar{n}}\right) & \frac{5}{6}\sqrt{3}\left(2c_{nn}-2c_{ns}+c_{n\bar{n}}-c_{s\bar{n}}\right) & & 0 & \\
 & \text{Symmetric} & 2c_{nn}+6c_{ns}+c_{s\bar{n}}-9c_{n\bar{n}} & \frac{\sqrt{10}}{6}\left(-23c_{n\bar{n}}-c_{s\bar{n}}\right) & & & \\
 & & & -\frac{7}{3}c_{nn}+5c_{ns}-\frac{11}{6}c_{n\bar{n}}+\frac{1}{2}c_{s\bar{n}} & & & \\
\hline
 & & & & 8c_{nn}-8c_{s\bar{n}} & \frac{4}{3}\sqrt{10}\left(-c_{n\bar{n}}-2c_{s\bar{n}}\right) & \frac{8}{3}\sqrt{5}\left(-c_{n\bar{n}}+c_{s\bar{n}}\right) \\
 & 0 & & & & \frac{26}{3}c_{nn}-6c_{ns}+\frac{2}{3}c_{n\bar{n}}-2c_{s\bar{n}} & \frac{2}{3}\sqrt{2}\left(c_{nn}-c_{ns}-c_{n\bar{n}}+c_{s\bar{n}}\right) \\
 & & & & \text{Symmetric} & & \frac{28}{3}\left(c_{nn}+c_{ns}\right)-4c_{n\bar{n}}-\frac{4}{3}c_{s\bar{n}}
\end{array}\right] \qquad (2.2.14)
$$

b) One hidden $s\bar{s}$ pair

There are four states in this group: $|M1\rangle$, $|M3\rangle$, $|M4\rangle$ and $|M5\rangle$. We find

$$H_{CM}=\begin{bmatrix} \frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}+\frac{10}{3}(c_{n\bar{s}}+c_{s\bar{s}}) & \frac{2}{9}\sqrt{30}(c_{n\bar{s}}-c_{s\bar{s}}) & \frac{2}{9}\sqrt{3}(-c_{nn}+c_{ss}-7c_{n\bar{s}}+7c_{s\bar{s}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) \\ & \frac{8}{3}(c_{nn}+c_{ns}+c_{ss})-4(c_{n\bar{s}}+c_{s\bar{s}}) & -2\sqrt{10}(c_{n\bar{s}}+c_{s\bar{s}}) & \frac{16}{9}\sqrt{15}(-c_{n\bar{s}}+c_{s\bar{s}}) \\ & & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}-\frac{2}{3}c_{n\bar{s}}-\frac{2}{3}c_{s\bar{s}} & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}-2c_{n\bar{s}}+2c_{s\bar{s}}) \\ \text{Symmetric} & & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})-\frac{8}{3}(c_{n\bar{s}}+c_{s\bar{s}}) \end{bmatrix} \quad (2.2.15)$$

Diagonalizing the matrix (2.2.15) leads to the numerical values given in table (2.2.9b).

Table 2.2.9b.	$qqss\bar{s}$-sector. Y = 0, I = 1, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
- 89	$0.061\|M1\rangle+0.711\|M3\rangle+0.696\|M4\rangle+0.072\|M5$	2236
10	$0.993\|M1\rangle-0.114\|M3\rangle+0.031\|M4\rangle-0.014\|M5\rangle$	2335
131	$-0.094\|M1\rangle-0.584\|M3\rangle+0.654\|M4\rangle-0.471\|M5\rangle$	2456
180	$-0.040\|M1\rangle-0.373\|M3\rangle+0.294\|M4\rangle+0.879\|M5\rangle$	2505

C3. Y = 0, I = 0, S = 3/2

a) No hidden strangeness

We have four states in this group: |M1>, |M2>, |M3> and |M4>. They all have 4q-clusters with I = ½, and we find

$$H_{CM} = \begin{bmatrix} -\frac{8}{3}c_{nn}-\frac{8}{3}c_{ns}+5c_{n\bar{n}}+\frac{5}{3}c_{s\bar{n}} & \sqrt{\frac{1}{6}}(10c_{nn}-10c_{ns}+5c_{n\bar{n}}-5c_{s\bar{n}}) & \frac{\sqrt{5}}{3}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{\sqrt{2}}{6}(-2c_{nn}+2c_{ns}-7c_{n\bar{n}}+7c_{s\bar{n}}) \\ & -3c_{nn}-\frac{31}{3}c_{ns}+\frac{23}{6}c_{n\bar{n}}+\frac{17}{6}c_{s\bar{n}} & \frac{\sqrt{30}}{6}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{5\sqrt{3}}{6}(2c_{nn}-2c_{ns}+c_{n\bar{n}}-c_{s\bar{n}}) \\ & & 2c_{nn}+6c_{ns}-9c_{n\bar{n}}+c_{s\bar{n}} & \frac{\sqrt{10}}{6}(-23c_{n\bar{n}}-c_{s\bar{n}}) \\ & \text{Symmetric} & & -\frac{7}{3}c_{nn}+5c_{ns}-\frac{11}{6}c_{n\bar{n}}+\frac{1}{2}c_{s\bar{n}} \end{bmatrix} \qquad (2.2.16)$$

which is identical to the upper 4x4 block of (2.2.14).
Diagonalizing this matrix leads to the numbers given in table (2.2.10a).

Table 2.2.10a.	$qqqs\bar{q}$-sector. Y = 0, I = 0, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-266	-0.043|M1>+0.142|M2>-0.721|M3>-0.677|M4>	1709
- 82	0.362|M1>-0.917|M2>-0.164|M3>-0.041|M4>	1893
49	0.928|M1>+0.369|M2>-0.024|M3>+0.044|M4>	2024
197	0.075|M1>-0.058|M2>+0.673|M3>-0.734|M4>	2172

b) One hidden $s\bar{s}$ pair

We have three states in this group: $|M2\rangle$, $|M3\rangle$ and $|M4\rangle$. We find

$$H_{CM} = \begin{bmatrix} -\frac{14}{3}c_{nn}-\frac{34}{3}c_{nn}+\frac{8}{3}c_{ss}+\frac{17}{3}c_{n\bar{s}}+c_{s\bar{s}} & \frac{\sqrt{10}}{3}(c_{n\bar{s}}-c_{s\bar{s}}) & \frac{5}{3}(2c_{nn}-2c_{ns}+c_{n\bar{s}}-c_{s\bar{s}}) \\ & -\frac{4}{3}c_{nn}+\frac{20}{3}c_{ns}+\frac{8}{3}c_{ss}-10c_{n\bar{s}}+2c_{s\bar{s}} & -\frac{\sqrt{10}}{3}(11c_{n\bar{s}}+c_{s\bar{s}}) \\ \text{Symmetric} & & -\frac{14}{3}c_{nn}+\frac{14}{3}c_{ns}+\frac{8}{3}c_{ss}-\frac{7}{3}c_{n\bar{s}}+c_{s\bar{s}} \end{bmatrix} \quad (2.2.17)$$

Diagonalizing (2.2.17) leads to the numerical values given in table (2.2.10b).

Table 2.2.10b.	$qqss\bar{s}$-sector. Y = 0, I = 0, S = 3/2.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-178	$0.418\|M2\rangle -0.615\|M3\rangle -0.668\|M4\rangle$	2147
-112	$0.904\|M2\rangle +0.349\|M3\rangle +0.245\|M4\rangle$	2213
105	$0.083\|M2\rangle -0.707\|M3\rangle +0.703\|M4\rangle$	2430

D1. Y = -1, I = 3/2, S = 3/2

a) No hidden strangeness

There are four states in this group: $|M1\rangle$, $|M3\rangle$, $|M4\rangle$ and $|M5\rangle$. These states contain the same 4q-clusters as the Y = 0, I = 1, S = 3/2 states with one $s\bar{s}$-pair. We thus get the matrix for the present case by substituting $\bar{s} \rightarrow \bar{n}$ in (2.2.15).

We get

$$
H_{CM}=\begin{bmatrix}
\frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}+\frac{10}{3}(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{2}{9}\sqrt{30}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{2}{9}\sqrt{3}(-c_{nn}+c_{ss}-7c_{n\bar{n}}+7c_{s\bar{n}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) \\
 & \frac{8}{3}(c_{nn}+c_{ns}+c_{ss})-4(c_{n\bar{n}}+c_{s\bar{n}}) & -2\sqrt{10}(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{16}{9}\sqrt{15}(-c_{n\bar{n}}+c_{s\bar{n}}) \\
 & & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}-\frac{2}{3}c_{n\bar{n}}-\frac{2}{3}c_{s\bar{n}} & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}-2c_{n\bar{n}}+2c_{s\bar{n}}) \\
 \text{Symmetric} & & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})-\frac{8}{3}(c_{n\bar{n}}+c_{s\bar{n}})
\end{bmatrix}
\tag{2.2.18}
$$

By diagonalizing (2.2.18) we get the numerical values given in table (2.2.11a).

Table 2.2.11a.	$qqss\bar{q}$-sector. Y = -1, I = 3/2. S = 3/2.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-191	-0.034|M1>-0.745|M3>-0.656|M4>-0.118|M5>	1959
45	0.983|M1>-0.133|M3>+0.110|M4>-0.059|M5>	2195
127	0.125|M1>+0.202|M3>-0.395|M4>+0.887|M5>	2277
206	-0.128|M1>-0.622|M3>+0.634|M4>+0.442|M5>	2356

D2. Y = -1, I = ½, S = 3/2

a) No hidden strangeness

We have seven states in this group: |M1>, |M2>, |M3>, |M3'>, |M4>, |M4'> and |M5>. The 4q-clusters of |M1>, |M3'>, |M4'> and |M5> have I = 1, while the 4q-clusters of |M2>, |M3> and |M4> all have I = 0. This gives rise to a block-diagonal form on H_{CM} if we choose the basis in the following sequence: |M1>, |M3'>, |M4'>, |M5>, |M2>, |M3>, |M4>. We find

$$
H_{CM}=\begin{bmatrix}
\frac{26}{9}c_{nn}-\frac{100}{9}c_{ns}+\frac{26}{9}c_{ss}+\frac{10}{3}(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{2}{9}\sqrt{30}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{2}{9}\sqrt{3}(-c_{nn}+c_{ss}-7c_{n\bar{n}}+7c_{s\bar{n}}) & \frac{2}{9}\sqrt{2}(-c_{nn}+2c_{ns}-c_{ss}) & & & \\
 & \frac{8}{9}(c_{nn}+c_{ns}+c_{ss})-4(c_{n\bar{n}}+c_{s\bar{n}}) & -2\sqrt{10}(c_{n\bar{n}}+c_{s\bar{n}}) & \frac{16}{9}\sqrt{15}(-c_{n\bar{n}}+c_{s\bar{n}}) & & 0 & \\
 & & \frac{10}{3}c_{nn}-4c_{ns}+\frac{10}{3}c_{ss}-\frac{2}{3}c_{n\bar{n}}-\frac{2}{3}c_{s\bar{n}} & \frac{2}{9}\sqrt{6}(c_{nn}-c_{ss}-2c_{n\bar{n}}+2c_{s\bar{n}}) & & & \\
 & \text{Symmetric} & & \frac{28}{9}(c_{nn}+4c_{ns}+c_{ss})-\frac{8}{3}(c_{n\bar{n}}+c_{s\bar{n}}) & & & \\
 & & & & -\frac{14}{3}c_{nn}-\frac{34}{3}c_{ns}+\frac{8}{3}c_{ss}+\frac{17}{3}c_{n\bar{n}}+c_{s\bar{n}} & \frac{1}{3}\sqrt{10}(c_{n\bar{n}}-c_{s\bar{n}}) & \frac{5}{3}(2c_{nn}-2c_{ns}+c_{n\bar{n}}-c_{s\bar{n}}) \\
 & 0 & & & & -\frac{4}{3}c_{nn}+\frac{20}{3}c_{ns}+\frac{8}{3}c_{ss}-10c_{n\bar{n}}+2c_{s\bar{n}} & -\frac{\sqrt{10}}{3}(11c_{n\bar{n}}+c_{s\bar{n}}) \\
 & & & & \text{Symmetric} & & -\frac{14}{3}c_{nn}+\frac{14}{3}c_{ns}+\frac{8}{3}c_{ss}-\frac{7}{3}c_{n\bar{n}}+c_{s\bar{n}}
\end{bmatrix}
\qquad (2.2.19)
$$

The upper 4x4 block of (2.2.19) is identical to (2.2.18). The lower 3x3 block can be obtained from (2.2.17) by the substitution $\bar{s} \to \bar{n}$. By diagonalizing we find the numerical values given in table (2.2.12a).

Table 2.2.12a.	$qqss\bar{q}$-sector. $Y = -1$, $I = \frac{1}{2}$, $S = 3/2$.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-299	-0.135 |M2>+0.731|M3>+0.668 |M4>	1851
-191	-0.034 |M1>-0.745 |M3'>-0.656 |M4'>-0.118 |M5>	1959
- 75	0.986 |M2>+0.163 |M3>+0.020 |M4>	2075
45	0.983 |M1>-0.133 |M3'>+0.110 |M4'>-0.059 |M5>	2195
127	0.125 |M1>+0.202 |M3'>-0.395 |M4'>+0.887 |M5>	2277
157	-0.094 |M2>+0.662 |M3>-0.743 |M4>	2307
206	-0.128 |M1>-0.622 |M3'>+0.634 |M4'>+0.442 |M5>	2356

It should be noticed that the lowest lying state has been lowered with ~120 MeV as compared to the flavoursymmetric case (see table (2.2.1)).

b) <u>One hidden $s\bar{s}$-pair</u>

We have three states in this group: |M3>, |M4> and |M5>. We find

$$H_{CM} = \begin{bmatrix} 8(c_{ss} - c_{n\bar{s}}) & \frac{4}{3}\sqrt{10}(-c_{s\bar{s}} - 2c_{n\bar{s}}) & \frac{8}{3}\sqrt{5}(-c_{s\bar{s}} + c_{n\bar{s}}) \\ & \frac{26}{3}c_{ss} - 6c_{ns} + \frac{2}{3}c_{s\bar{s}} - 2c_{n\bar{s}} & \frac{2}{3}\sqrt{2}(c_{ss} - c_{ns} - c_{s\bar{s}} + c_{n\bar{s}}) \\ \text{Symmetric} & & \frac{28}{3}(c_{ss} + c_{ns}) - 4c_{s\bar{s}} - \frac{4}{3}c_{n\bar{s}} \end{bmatrix} \quad (2.2.20)$$

By diagonalizing (2.2.20) we get the numerical values given in table (2.2.12b).

Table 2.2.12b.	$qsss\bar{s}$-sector. $Y = -1$, $I = \frac{1}{2}$, $S = 3/2$.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-151	0.722|M3>+0.689|M4>-0.062|M5>	2349
88	0.617|M3>-0.682|M4>-0.393|M5>	2588
133	0.314|M3>-0.246|M4>+0.917|M5>	2633

E1. $Y = -2$, $I = 1$, $S = 3/2$

a) *No hidden strangeness*

We find three states in this group: |M3>, |M4> and |M5>. All these states have 4q-clusters with $I = \frac{1}{2}$, and H_{CM} in this case can be obtained from H_{CM} for $Y = -1$, $I = \frac{1}{2}$, $S = 3/2$ and one $s\bar{s}$-pair as given in (2.2.20) by doing the substitution $\bar{s} \rightarrow \bar{n}$. We thus find

$$H_{CM} = \begin{bmatrix} 8(c_{ss}-c_{n\bar{n}}) & \frac{4}{3}\sqrt{10}(-c_{s\bar{n}} - 2c_{n\bar{n}}) & \frac{8}{3}\sqrt{5}(-c_{s\bar{n}} + c_{n\bar{n}}) \\ & \frac{26}{3}c_{ss} - 6c_{ns} + \frac{2}{3}c_{s\bar{n}} - 2c_{n\bar{n}} & \frac{2}{3}\sqrt{2}(c_{ss} - c_{ns} - c_{s\bar{n}} + c_{n\bar{n}}) \\ \text{Symmetric} & & \frac{28}{3}(c_{ss}+c_{ns}) - 4c_{s\bar{n}} - \frac{4}{3}c_{n\bar{n}} \end{bmatrix} \qquad (2.2.21)$$

One should notice the relationship between (2.2.21) and (2.2.13). This is the same relationship as for the $J^P = \frac{1}{2}^-$ case (see E1 under section 2.1 for further comments).

Diagonalizing this matrix leads to the numerical values given in table (2.2.13a).

Table 2.2.13a.	$qsss\bar{q}$-sector. Y = -2, I = 1, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
-268	0.755|M3>+0.649|M4>-0.095|M5>	2057
89	-0.235|M3>+0.403|M4>+0.885|M5>	2414
154	0.612|M3>-0.645|M4>+0.456|M5>	2479

E2. Y = -2, I = 0, S = 3/2

a) No hidden strangeness

There are three states in this group: |M3>, |M4> and |M5>. They all have 4q-clusters with I = ½ (the same 4q-clusters as the ones that made up the Y = -2, I = 1, S = 3/2 states). The H_{CM}-matrix in this case is identical to (2.2.21), and the numerical values are therefore also given in table (2.2.13a).

b) No hidden $s\bar{s}$-pair

We have only one state in this group: |M5>. We find

$$\langle H_{CM}\rangle = \frac{56}{3}\, c_{ss} - \frac{16}{3}\, c_{s\bar{s}} \qquad (2.2.22)$$

Numerical values are given in table (2.2.14b).

Table 2.2.14b	$ssss\bar{s}$-sector Y = -2, I = 1, S = 3/2	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
93	|M5>	2768

F1. Y = -3, I = ½, S = 3/2

a) No hidden strangeness

There is only one state in this group: |M5>. The 4q-cluster of this state is the same as the 4q-cluster of the Y = -2, I = 0, S = 3/2 state with one $s\bar{s}$-pair. From (2.2.22) we get after replacing $\bar{s} \to \bar{n}$:

$$\langle H_{CM}\rangle = \frac{56}{3}\, c_{ss} - \frac{16}{3}\, c_{s\bar{n}} \qquad (2.2.23)$$

One should notice the close relation between (2.2.23) and (2.2.8). Numerical values are given in table (2.2.15a).

Table 2.2.15a. ssss$\bar{q}$-sector.	Y = -3, I = ½, S = 3/2.	
Eigenvalue (MeV)	Eigenvector	Mass (MeV)
72	|M5>	2572

2.3. The colour-magnetic interaction for the $J^P = 5/2^-$ states

From eq. (2.2) we see that there is only one coloursinglet $4q\bar{q}$ representation with $S = \frac{5}{2}$. That is (1,6,45) obtained from the product $(3,5,15) \otimes (\bar{3},2,\bar{3})$. Using the decomposition of the flavour multiplet given in fig. 2.1 we arrive at the $J^P = \frac{5}{2}^-$ multiplets as given in fig. 2.3.1.

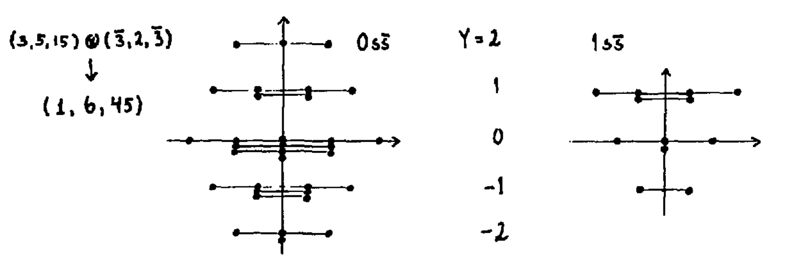


Fig. 2.3.1. The $J^P = \frac{5}{2}^-$ $4q\bar{q}$ flavour multiplets.

In the flavour-symmetric case all states belonging to the multiplet (1,6,45) have [8,18)]

$$\langle H_{CM} \rangle = \frac{40}{3} c \tag{2.3.1}$$

When flavour-symmetry is broken, we treat the states one by one as done for J = ½ and J = 3/2.

It is more simple to find the eigenvalues of H_{CM} for $J^P = 5/2^-$ than for $J^P = \frac{1}{2}^-$, because we will get no mixing between different multiplets now. All the states given in fig. (2.3.1) are eigenstates, as for the flavoursymmetric case, but we will of course not get the same eigenvalues for all the states.

All eigenvalues are given in table (2.3.1a) and (2.3.1b).

Table 2.3.1. Eigenvalues and masses of the $J^P = 5/2^-$ states. I(4q) denotes the isospin of the corresponding 4q-multiplet.

a) No hidden $s\bar{s}$ pair

Y	I(tot)	I(4q)	$\langle H_{CM}\rangle$	$\langle H_{CM}\rangle$(MeV)	Mass (MeV)
2	1	1	$8c_{nn}+\frac{16}{3}c_{n\bar{s}}$	205	2180
1	3/2	1	$8c_{nn}+\frac{16}{3}c_{n\bar{n}}$	244	2044
"	½	1	-"-	244	2044
0	2	3/2	$8c_{nn}+\frac{16}{3}c_{s\bar{n}}$	205	2180
"	1	3/2	- " -	205	2180
"	1	½	$2c_{nn}+6c_{ns}+6c_{n\bar{n}}-\frac{2}{3}c_{s\bar{n}}$	205	2180
"	0	½	- " -	205	2180
-1	3/2	1	$\frac{8}{3}(c_{nn}+c_{ns}+c_{ss}+c_{n\bar{n}}+c_{s\bar{n}})$	175	2325
"	½	1	- " -	175	2325
"	½	0	$\frac{1}{3}(-4c_{nn}+20c_{ns}+8c_{ss}+20c_{n\bar{n}}-4c_{s\bar{n}})$	175	2325
-2	1	½	$8c_{ss}+\frac{16}{3}c_{n\bar{n}}$	154	2479
"	0	½	- " -	154	2479

(continued)

b) One hidden $s\bar{s}$ pair					
Y	I(tot)	I(4q)	$\langle H_{CM}\rangle$	$\langle H_{CM}\rangle$(MeV)	Mass (MeV)
1	3/2	3/2	$8c_{nn}+\frac{16}{3}c_{s\bar{s}}$	184	2334
"	½	½	$2c_{nn}+6c_{ns}+6c_{n\bar{s}}-\frac{2}{3}c_{s\bar{s}}$	164	2314
0	1	1	$\frac{8}{3}(c_{nn}+c_{ns}+c_{ss}+c_{n\bar{s}}+c_{s\bar{s}})$	145	2470
"	0	0	$\frac{1}{3}(-4c_{nn}+20c_{ns}+8c_{ss}+20c_{n\bar{s}}-4c_{s\bar{s}})$	132	2457
-1	½	½	$8c_{ss}+\frac{16}{3}c_{n\bar{s}}$	115	2615

We see that all the eigenvalues in table (2.3.1) properly reduce to the value given in eq. (2.3.1) in the flavoursymmetric limit. We also see that we have degeneracy between some of the isospin-multiplets with the same hyper-charge in the sector of no hidden strangeness.

3. Conclusions

In this paper masses of all possible $4q\bar{q}$ states have been calculated. We calculate the masses in a scheme where flavoursymmetry is broken. The masses we get when flavoursymmetry is broken differs significantly from the flavoursymmetric results previously obtained by other authors[8,18]. Even if flavoursymmetry is broken, some degeneracy is left in the spectrum. This degeneracy is the one we find between different $4q\bar{q}$ states which differs with one unit of isospin and is built with the same 4q clusters. As seen from the tables presented in the text this gives for instance rise to five $4q\bar{q}$ Λ states which is degenerate with five $4q\bar{q}$ Σ states (see tables 2.1.10a and 2.1.11a). This is contrary to what is found for the 3q baryons, where we will find a mass-splitting between Σ and Λ if we break flavoursymmetry.

Whether any of these $4q\bar{q}$-states will show themselves as usual resonances or as poles in Jaffe and Low's P-matrix is still a matter of uncertainty[18,27). Before making contact with experiment it is of course important to have as good theoretical values as possible. We think that our calculations with broken flavoursymmetry is a step in this direction. One should of course also proceed, and include the annihilation diagrams in the calculations here, as done elsewhere for other configurations[14]. This work is in progress. At last one should calculate branching ratios. This has recently been done with broken flavoursymmetry by Bickerstaff and Wybourne[28].

In summary one might say that the colourmagnetic interaction matrices given in this paper is quite general, and can be used as a starting point for hyperfine spectroscopy in any desirable model using the values for c_{ij} as demanded by model/experiment.

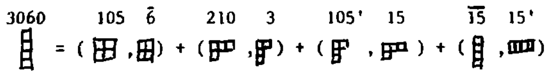
4. Acknowledgement

It is a pleasure to thank Dr. H. Högaasen for many stimulating and clarifying discussions.

Appendix A

Decomposition of the completely antisymmetric 4q-state into its irreducible colour ⊗ spin ⊗ flavour representations.

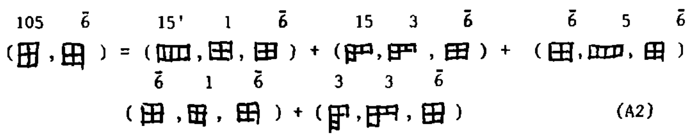
The completely antisymmetric 4q-state for the group $SU(18)_{CSF}$ is represented by the Young-tableau  . Its decomposition into $SU_{CS}(6) \otimes SU_F(3)$ is according to the tables of Itzykson and Nauenberg[19] given by



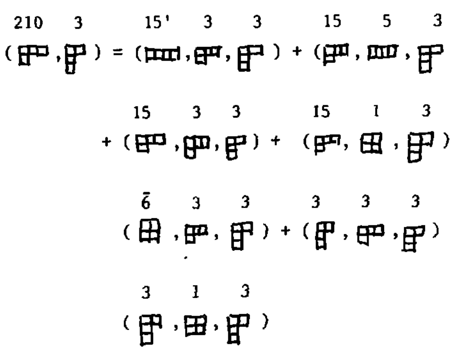
(A1)

where the dimensionality appears on top of the Young tableaux.

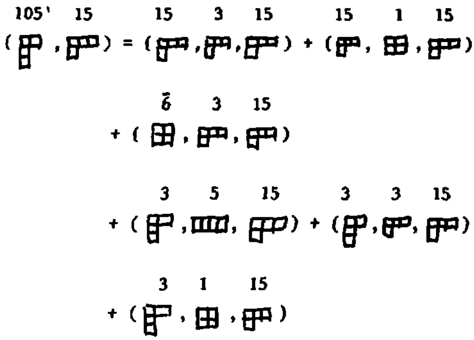
By further reduction of the CS ⊗ F representations into C ⊗ S ⊗ F representations we get



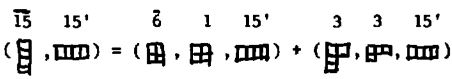
(A2)



(A3)



(A4)



(A5)

We see from these decompositions that the 4q colour {3} representation appears altogether 7 times. These are the physically interesting representations if we want to couple these 4q states to an antiquark to form colour-singlets.

Appendix B

The construction of colour {3} 4q-wavefunctions.

The fundamental triplet in $SU_C(3)$ will be represented by $\begin{pmatrix} r \\ w \\ b \end{pmatrix}$, the fundamental doublet in $SU_S(2)$ by $\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$ and the fundamental triplet in $SU_F(3)$ by $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$. This leads to the fundamental 18-plet in $SU_{CSF}(18)$ given by

$$\begin{pmatrix} r \\ w \\ b \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \text{(B1)}$$

which also can be represented by a 18-dimensional column:

$$\begin{bmatrix} r\uparrow u \\ r\uparrow d \\ r\uparrow s \\ r\downarrow u \\ r\downarrow d \\ r\downarrow s \\ w\uparrow u \\ w\uparrow d \\ w\uparrow s \\ w\downarrow u \\ w\downarrow d \\ w\downarrow s \\ b\uparrow u \\ b\uparrow d \\ b\uparrow s \\ b\downarrow u \\ b\downarrow d \\ b\downarrow s \end{bmatrix} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{14} \\ q_{15} \\ q_{16} \\ q_{17} \\ q_{18} \end{bmatrix}. \qquad \text{(B2)}$$

We will need the step-operators in the colour, spin and flavour spaces separately. In colour-space we define the step-operators I_-^C and U_-^C by

$$I_-^C\ q(C,S,F) = \delta_{rC}\ q(w,S,F) \tag{B3}$$

$$U_-^C\ q(C,S,F) = \delta_{wC} q(b,S,F) \tag{B4}$$

(where C = r,w,b, S = ↑, ↓ and F = u,d,s)
In flavour-space we define I_-^F and U_-^F by

$$I_-^F\ q(C,S,F) = \delta_{uF}\ q(C,S,d) \tag{B5}$$

$$U_-^F\ q(C,S,F) = \delta_{dF}\ q(C,S,s) \tag{B6}$$

In spin-space we define S_- by

$$S_- q(C,S,F) = \delta_{\uparrow S}\ q(C,\downarrow,F) \tag{B7}$$

It should be noticed that the step-operators are defined for the fundamental representations (i.e. on quark-level). We also choose the matrix-elements of the step-operators defined above to be positive. This is the usual phase convention. (See Haacke et al.[20] or Baird et al.[21])

The other step-operators are obtained by using the commutation relations between the step-operators given by Haacke et al.[20] (originally used by Weyl[22]). The relative signs within a multiplet is thus fixed, and after having chosen the sign of the highest state in the multiplet the rest follows by the phase convention for the step-operators.

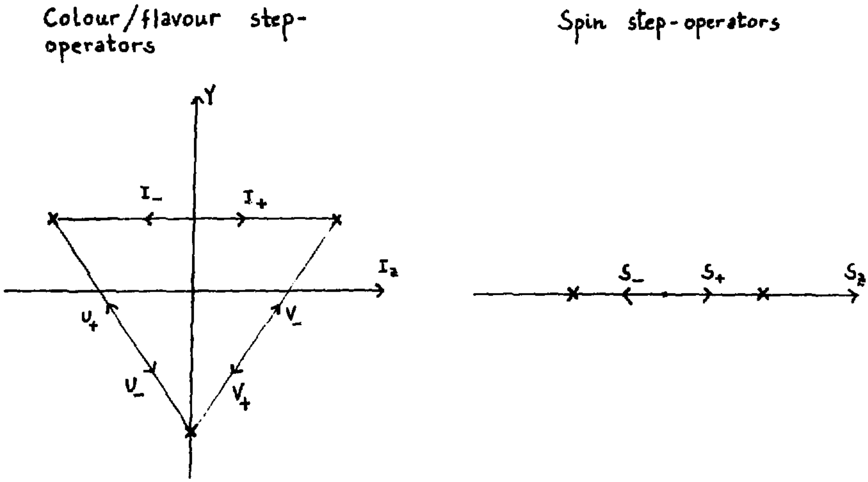


Fig. B1. Figures showing the effect of the different step-operators in a colour/flavour weight diagram and a spin weight diagram.

At this stage we recapitulate the weight-diagrams of the flavour-representations that can occur together with a colour(3)-state. According to A2-A5 these are $\bar{6}$, 3, 15 and 15'. The weight-diagrams of these are shown in fig. B2

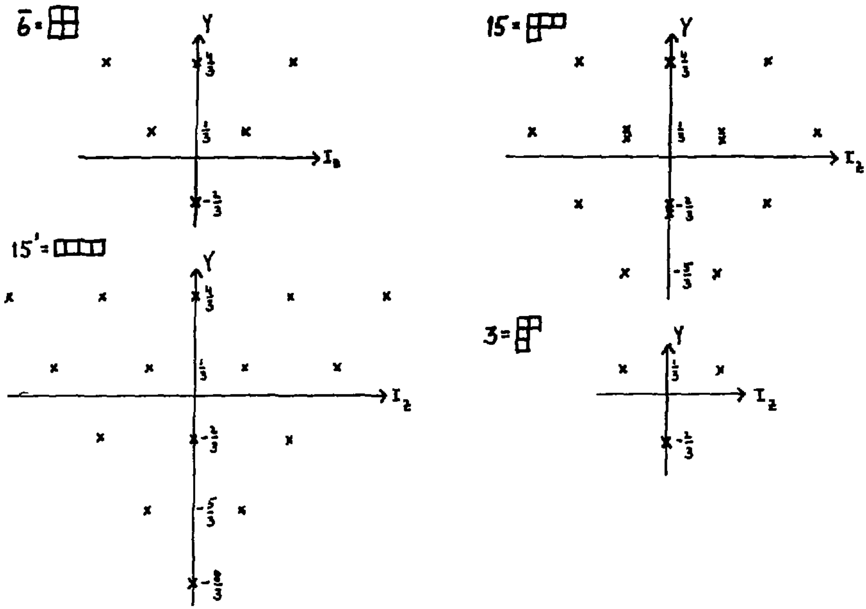


Fig. B2. Weight diagrams for $\bar{6}$, 15, 15' and 3 in SU(3)

From the preceeding we are now able to construct the totally antisymmetric 4q-wavefunction in any of the representations occuring in A2-A5. All the wavefunctions will be linear superpositions of basis-vectors which are themselves totally antisymmetric. Let therefore

$$\begin{array}{|c|}\hline a\\\hline b\\\hline c\\\hline d\\\hline\end{array} \equiv \sqrt{\frac{1}{24}}\,\varepsilon_{abcd}\, q_a\, q_b\, q_c\, q_d \qquad \text{(B8)}$$

where ε_{abcd} is the completely antisymmetric Levi-Civita symbol.

As an example let us find the 4q-wave function in the $(3,3,\bar{6})$ representation having the <u>highest</u> weight both in colour, spin and flavour. The colour-content must thus be (rrwb), spin-content (↑↑↑↓) and flavour content (uudd).

We will also have the same colour-spin-flavour content in other states. It appears in $(\bar{6},5,\bar{6})$, $(15,3,\bar{6})$ twice, $(15,3,15)$ twice, $(\bar{6},3,15)$, (3,5,15), (3,3,15) and (3,3,15'). This means that it must be possible to construct 10 linearly independent wave-functions all with the colour-spin-flavour contents given above.

This is again reflected in the fact that it is only possible to find 10 linearly independent basis-vectors with the colour-spin-flavour contents given above which are totally antisymmetric.

These are

$$
|1\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \uparrow & d \\ \hline w & \uparrow & u \\ b & \downarrow & u \end{array} \quad
|2\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \uparrow & d \\ \hline w & \uparrow & d \\ b & \downarrow & u \end{array} \quad
|3\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \uparrow & d \\ \hline w & \downarrow & u \\ b & \uparrow & d \end{array} \quad
|4\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \uparrow & d \\ \hline w & \downarrow & d \\ b & \uparrow & u \end{array}
$$

$$
|5\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \downarrow & u \\ \hline w & \uparrow & d \\ b & \uparrow & d \end{array} \quad
|6\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \downarrow & d \\ \hline w & \uparrow & d \\ b & \uparrow & u \end{array} \quad
|7\rangle \equiv \begin{array}{|ccc|} r & \uparrow & u \\ r & \downarrow & d \\ \hline w & \uparrow & u \\ b & \uparrow & d \end{array} \quad
|8\rangle \equiv \begin{array}{|ccc|} r & \uparrow & d \\ r & \downarrow & u \\ \hline w & \uparrow & d \\ b & \uparrow & u \end{array}
$$

$$
|9\rangle \equiv \begin{array}{|ccc|} r & \uparrow & d \\ r & \downarrow & d \\ \hline w & \uparrow & u \\ b & \uparrow & u \end{array} \quad
|10\rangle \equiv \begin{array}{|ccc|} r & \uparrow & d \\ r & \downarrow & u \\ \hline w & \uparrow & u \\ b & \uparrow & d \end{array} \qquad \text{(B9)}
$$

The desired wavefunction $|\psi\rangle \equiv \left|\begin{array}{lll} C = 3 & S = 1 & F = \bar{6} \\ Y_c = \frac{1}{3} & S_z = 1 & Y_F = \frac{4}{3} \\ I_c = \frac{1}{2} & & I_F = 0 \end{array}\right\rangle$

can therefore be written as a linear superposition of the basis in (B9) i.e.

$$|\psi\rangle = \sum_{i=1}^{10} a_i |i\rangle \qquad \text{(B10)}$$

The task is now to find the a_i's. This is done by using the step-operators directly on (B10) demanding

$$
\begin{array}{ll} I_+^C \, |\psi\rangle = 0, & U_+^C \, |\psi\rangle = 0 \\ S_+ \, |\psi\rangle = 0, & I_+^F \, |\psi\rangle = 0 \end{array} \qquad \text{(B11)}
$$

This gives (much more than) enough linear equations which is used together with the normalization requirement $\langle\psi|\psi\rangle = 1$. One gets

$$|\psi\rangle = \frac{\sqrt{3}}{4}\{\ |1\rangle - |2\rangle - |3\rangle + |4\rangle - \frac{2}{3}|5\rangle + \frac{1}{3}|6\rangle + \frac{1}{3}|7\rangle + \frac{1}{3}|8\rangle - \frac{2}{3}|9\rangle + \frac{1}{3}|10\rangle\} \qquad (B12)$$

The other wavefunctions in the $(3,3,\bar{6})$ representation are then found by applying the step-operators on $|\psi\rangle$.

This procedure has to be repeated for the other 6 interesting representations requiring some tedious work. The most laboursome are the (3,1,3) which is the innermost state. Here we have 34 basisvectors and thus 34 coefficients have to be determined.

Here follows a complete list of all the highest state wavefunctions in the 7 interesting C = 3 representations. We use the notation introduced in (B2) and (B8) (HS means Highest State).

$$|3,5,15\rangle^{HS} = \begin{bmatrix}1\\2\\ \\7\\13\end{bmatrix} \qquad (B13)$$

$$|3,3,15\rangle^{HS} = \sqrt{\tfrac{1}{8}}\left\{\begin{bmatrix}1\\2\\ \\7\\16\end{bmatrix} + \begin{bmatrix}1\\2\\ \\10\\13\end{bmatrix} + \begin{bmatrix}1\\4\\ \\7\\14\end{bmatrix} + \begin{bmatrix}1\\4\\ \\8\\13\end{bmatrix} - 2\begin{bmatrix}1\\5\\ \\7\\13\end{bmatrix}\right\} \qquad (B14)$$

$$|3,1,15\rangle^{HS} = \sqrt{\tfrac{1}{12}}\left\{\begin{bmatrix}1\\2\\ \\10\\16\end{bmatrix} + \begin{bmatrix}4\\5\\ \\7\\13\end{bmatrix} - \frac{3}{2}\begin{bmatrix}1\\4\\ \\7\\17\end{bmatrix} + \frac{3}{2}\begin{bmatrix}1\\4\\ \\8\\16\end{bmatrix} - \tfrac{1}{2}\begin{bmatrix}1\\5\\ \\7\\16\end{bmatrix}\right.$$

$$\left. + \tfrac{1}{2}\begin{bmatrix}2\\4\\ \\7\\16\end{bmatrix} + \frac{3}{2}\begin{bmatrix}1\\4\\ \\10\\14\end{bmatrix} - \frac{3}{2}\begin{bmatrix}1\\4\\ \\11\\13\end{bmatrix} - \tfrac{1}{2}\begin{bmatrix}1\\5\\ \\10\\13\end{bmatrix} + \tfrac{1}{2}\begin{bmatrix}2\\4\\ \\10\\13\end{bmatrix}\right\} \qquad (B15)$$

$$|3,3,15'\rangle^{HS} = \begin{bmatrix}1\\4\\ \\7\\13\end{bmatrix} \qquad (B16)$$

$$|3,3,\bar{6}\rangle^{HS} = \frac{\sqrt{3}}{4}\left\{\begin{bmatrix}1\\2\\ \\7\\17\end{bmatrix} - \begin{bmatrix}1\\2\\ \\8\\16\end{bmatrix} - \begin{bmatrix}1\\2\\ \\10\\14\end{bmatrix} + \begin{bmatrix}1\\2\\ \\11\\13\end{bmatrix} - \frac{2}{3}\begin{bmatrix}1\\4\\ \\8\\14\end{bmatrix} + \frac{1}{3}\begin{bmatrix}1\\5\\ \\8\\13\end{bmatrix} + \frac{1}{3}\begin{bmatrix}1\\5\\ \\7\\14\end{bmatrix} + \frac{1}{3}\begin{bmatrix}2\\4\\ \\8\\13\end{bmatrix} - \frac{2}{3}\begin{bmatrix}2\\5\\ \\7\\13\end{bmatrix} + \frac{1}{3}\begin{bmatrix}2\\4\\ \\7\\14\end{bmatrix}\right\}$$

(B17)

$$|3,3,3\rangle^{HS} = \frac{1}{8}\left\{ 3\begin{array}{|c|}\hline 1\\2\\ \hline 7\\18\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\2\\ \hline 9\\16\\ \hline\end{array} - 3\begin{array}{|c|}\hline 1\\3\\ \hline 7\\17\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\3\\ \hline 8\\16\\ \hline\end{array} + 2\begin{array}{|c|}\hline 2\\3\\ \hline 7\\16\\ \hline\end{array} \right.$$

$$- \begin{array}{|c|}\hline 1\\2\\ \hline 10\\15\\ \hline\end{array} + 3\begin{array}{|c|}\hline 1\\2\\ \hline 12\\13\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\3\\ \hline 10\\14\\ \hline\end{array} - 3\begin{array}{|c|}\hline 1\\3\\ \hline 11\\13\\ \hline\end{array} + 2\begin{array}{|c|}\hline 2\\3\\ \hline 10\\13\\ \hline\end{array}$$

$$- \begin{array}{|c|}\hline 1\\5\\ \hline 7\\15\\ \hline\end{array} + \begin{array}{|c|}\hline 2\\4\\ \hline 7\\15\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\5\\ \hline 9\\13\\ \hline\end{array} + \begin{array}{|c|}\hline 2\\4\\ \hline 9\\13\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\6\\ \hline 7\\14\\ \hline\end{array}$$

$$\left. + \begin{array}{|c|}\hline 1\\6\\ \hline 8\\13\\ \hline\end{array} - 2\begin{array}{|c|}\hline 2\\6\\ \hline 7\\13\\ \hline\end{array} - \begin{array}{|c|}\hline 3\\4\\ \hline 7\\14\\ \hline\end{array} - \begin{array}{|c|}\hline 3\\4\\ \hline 8\\13\\ \hline\end{array} + 2\begin{array}{|c|}\hline 3\\5\\ \hline 7\\13\\ \hline\end{array} \right\} \qquad \text{(B18)}$$

$$|3,1,3\rangle^{HS} = \frac{1}{4\sqrt{2}}\left\{ \begin{array}{|c|}\hline 1\\2\\ \hline 10\\18\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\2\\ \hline 12\\16\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\3\\ \hline 10\\17\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\3\\ \hline 11\\16\\ \hline\end{array} + 2\begin{array}{|c|}\hline 2\\3\\ \hline 10\\16\\ \hline\end{array} \right.$$

$$+ \begin{array}{|c|}\hline 4\\5\\ \hline 7\\15\\ \hline\end{array} + \begin{array}{|c|}\hline 4\\5\\ \hline 9\\13\\ \hline\end{array} - \begin{array}{|c|}\hline 4\\6\\ \hline 7\\14\\ \hline\end{array} - \begin{array}{|c|}\hline 4\\6\\ \hline 8\\13\\ \hline\end{array} + 2\begin{array}{|c|}\hline 5\\6\\ \hline 7\\13\\ \hline\end{array}$$

$$+ \begin{array}{|c|}\hline 1\\4\\ \hline 8\\18\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\6\\ \hline 7\\17\\ \hline\end{array} + \begin{array}{|c|}\hline 2\\4\\ \hline 9\\16\\ \hline\end{array} - \begin{array}{|c|}\hline 2\\6\\ \hline 7\\16\\ \hline\end{array} - \begin{array}{|c|}\hline 3\\4\\ \hline 8\\16\\ \hline\end{array}$$

$$+ \begin{array}{|c|}\hline 3\\5\\ \hline 7\\16\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\4\\ \hline 9\\17\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\5\\ \hline 7\\18\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\4\\ \hline 11\\15\\ \hline\end{array} - \begin{array}{|c|}\hline 1\\5\\ \hline 12\\13\\ \hline\end{array}$$

$$\left. + \begin{array}{|c|}\hline 2\\4\\ \hline 10\\15\\ \hline\end{array} - \begin{array}{|c|}\hline 2\\6\\ \hline 10\\13\\ \hline\end{array} - \begin{array}{|c|}\hline 3\\4\\ \hline 10\\14\\ \hline\end{array} + \begin{array}{|c|}\hline 3\\5\\ \hline 10\\13\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\4\\ \hline 12\\14\\ \hline\end{array} + \begin{array}{|c|}\hline 1\\6\\ \hline 11\\13\\ \hline\end{array} \right\} \qquad \text{(B19)}$$

The method developed here for finding the wavefunctions are very convenient in the sense that you start from scratch and make your own phaseconventions etc. You also avoid the task of finding the right Clebsch Gordan coefficients for the different couplings, which is far from trivial in a case like this where three different groups are coupled together.

Appendix C

The construction of colourless $4q\bar{q}$ states.

Given the highest state in each 4q multiplet it is easy to find the right $4q\bar{q}$ states using usual spin Clebsch-Gordan coefficients and using that a $SU_C(3)$-singlet is obtained from $3 \times \bar{3}$ by $\sqrt{\frac{1}{3}}(r\bar{r}+w\bar{w}+b\bar{b})$, where r,w, and b in our case actually means 4q-states coupled to colour 3. At this stage we also break the $SU_F(3)$-symmetry assuming magic mixing. The flavour multiplets we get are thus not usual SU(3) weight-diagrams any more, but they are separated into multiplets with no hidden $s\bar{s}$-pair and multiplets with one hidden $s\bar{s}$-pair (see fig. 2.1).

Colour and spin step is easy to perform since we have no degeneracy in the $SU_C(3)$-triplet or in any spin weight-diagram.

A little problem arises when we have degeneracy in the 4q-flavour weight diagram. This occurs in the 15-plet ⊞ .

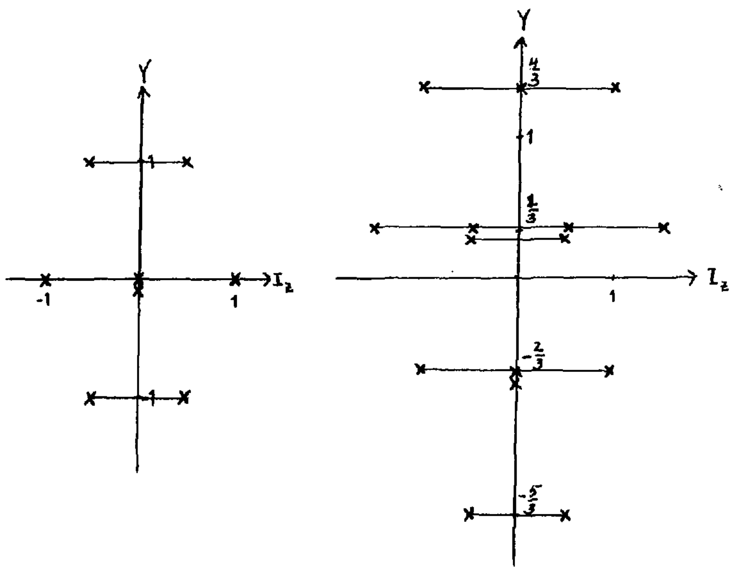


Fig. C1. SU(3) {8} and {15} weight-diagrams.

The problem is how to get the $|Y = \frac{1}{3}, I = \frac{1}{2}\rangle$ state when we only know the $|Y = \frac{4}{3}, I = 1\rangle$ state. This is the same problem as in the octet where most people are familiar with the way you find the $|\Lambda^0\rangle$ from $|p\rangle$ by demanding $\langle\Lambda^0|\Sigma^0\rangle = 0$. One finds $|\Lambda^0\rangle = \sqrt{\frac{1}{3}}\,[\sqrt{\frac{1}{2}}\, I_-U_-|p\rangle + \sqrt{2}V_+|p\rangle]$

$$= \sqrt{\frac{1}{3}}\,[\,|\Sigma^0\rangle + 2\,|x\rangle]$$

where $|x\rangle$ is a U-spin 1 state. The Clebsch-Gordan coefficient in this linear combination is determined by the representation. In an analog fashion one constructs the wavefunction of $|Y = \frac{4}{3}, I = 1\rangle$ in the 15-plet using one of the symmetries contained in for instance  . By doing this in the same way as for the {8} one finds

$$\left|Y = \frac{1}{3}, I = I_Z = \frac{1}{2}\right\rangle = \sqrt{\frac{1}{8}}\left[\left|Y = \frac{1}{3}, I = \frac{3}{2}, I_Z = \frac{1}{2}\right\rangle + \sqrt{3}V_+\left|Y = \frac{4}{3}, I = I_Z = 1\right\rangle\right] \tag{C1}$$

$$\left|Y = -\frac{2}{3}, I = 0\right\rangle = \sqrt{\frac{1}{2}}\left[\left|Y = -\frac{2}{3}, I = 1, I_Z = 0\right\rangle + \frac{1}{2}V_+V_+\left|Y = \frac{4}{3}, I = I_Z = 1\right\rangle\right] \tag{C2}$$

All states can in this way be constructed by means of step-operators if one knows the highest state. The symmetry of the highest state is preserved using the step-operators, and all states constructed in this way will therefore have the right overall antisymmetry.

Appendix D

On the colour-magnetic interaction

The colour-magnetic interaction defined over colour spin space can be written as (see eq. (1.4))

$$H_{CM} = -\sum_{i<j} c_{ij}\, \lambda_i^a\, \vec{\sigma}_i\, \lambda_j^a\, \vec{\sigma}_j \qquad \text{(D1)}$$

where $\vec{\sigma}$ are the Pauli spin matrices and λ^a are the Gell-Mann SU(3) matrices. The summation is over all quark pairs. Summation over the index a is also understood. The usual way to handle this operator is to express it by the Casimir-operators of $SU_{CS}(6)$, $SU_C(3)$ and $SU_S(2)$ [8,9,10]. In this way it is not necessary to know the exact expressions for the wavefunction; it is only necessary to know the representations in which a given quark-pair sits. If we have more than 2 quarks the usual technique is then to use crossing matrices [9,10]. This works well in the flavour symmetric case when all the c_{ij}'s are equal. It is however hard to do this when the c_{ij}'s depend on the flavour of the quarks. One way to do this is then the straightforward method where we compute the action of the operator and put the right coefficient in front corresponding to what kind of flavours we have in the pair. Using this method we have to have exact expressions for the wavefunctions (see appendix B).

For instance, if we in a given quark-pair have $q(r,\uparrow,s)q(w,\downarrow,u)$ we know that

$$c_{ij}\, \lambda_i^a\, \vec{\sigma}_i\, \lambda_j^a\, \vec{\sigma}_j\, |qq\rangle = c_{us} \lambda_i^a\, \vec{\sigma}_i\, \lambda_j^a\, \vec{\sigma}_j\, |qq\rangle \qquad \text{(D2)}$$

To proceed we do the usual trick and write

$$\lambda_i^a\, \lambda_j^a = \sum_a \tfrac{1}{2}[(\lambda_i^a + \lambda_j^a)^2 - {\lambda_i^a}^2 - {\lambda_j^a}^2] \qquad \text{(D3)}$$

Because both particles i and j are in a colour-triplet $\sum_a {\lambda_i^a}^2 = \sum_a {\lambda_j^a}^2 = \frac{16}{3}$. The value of $\sum_a (\lambda_i^a + \lambda_j^a)^2$ is $\frac{16}{3}$ if the two quarks are coupled to pure $\bar{3}$, and $\frac{40}{3}$ if coupled to pure 6. This gives from (D3):

$$\lambda_i^a\, \lambda_j^a = \begin{cases} -\frac{8}{3} & \text{if total colour } \bar{3} \\ +\frac{4}{3} & \text{-- " --} \quad 6 \end{cases} \qquad \text{(D4)}$$

The same trick is done for the spin and we get

$$\vec{S}_i \cdot \vec{S}_j = \tfrac{1}{2}[(\vec{S}_i + \vec{S}_j)^2 - \tfrac{3}{2}] = \begin{cases} \tfrac{1}{4} & \text{if total spin is } 1 \\ -\tfrac{3}{4} & \text{- " -} \quad 0 \end{cases} \tag{D5}$$

Since the quark pair in our example belongs neither to a pure irreducible colour or spin representation it must be written as a linear-combination of such.

For $q \otimes q$ we have the possibilities

$$\square \times \square = \square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix} \tag{D6}$$

which in the actual groups reads

$$\begin{aligned} SU(2)&: \quad 2 \times 2 = 3 + 1 \\ SU(3)&: \quad 3 \times 3 = 6 + \bar{3} \end{aligned} \tag{D7}$$

The SU(2) case is well known, and we have

$$\square\square : \quad \begin{aligned} |11\rangle &= |\uparrow\uparrow\rangle \\ |10\rangle &= \sqrt{\tfrac{1}{2}}[\,|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle] \\ |1\text{-}1\rangle &= |\downarrow\downarrow\rangle \end{aligned} \tag{D8}$$

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} : \quad |00\rangle = \sqrt{\tfrac{1}{2}}[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] \tag{D9}$$

The SU(3) case is also familiar, but depends a little on different phaseconventions. We get (notation: $|Y, II_Z\rangle^R$, where R = representation)

$$\square\square : \quad \begin{aligned} |\tfrac{2}{3}, 11\rangle^6 &= |rr\rangle \\ |\tfrac{2}{3}, 10\rangle^6 &= \sqrt{\tfrac{1}{2}}[\,|rw\rangle + |wr\rangle] \\ |\tfrac{2}{3}, 1\text{-}1\rangle^6 &= |ww\rangle \\ |-\tfrac{1}{3}, \tfrac{1}{2}\tfrac{1}{2}\rangle^6 &= \sqrt{\tfrac{1}{2}}[\,|rb\rangle + |br\rangle] \\ |-\tfrac{1}{3}, \tfrac{1}{2}\text{-}\tfrac{1}{2}\rangle^6 &= \sqrt{\tfrac{1}{2}}[\,|wb\rangle + |bw\rangle] \\ |-\tfrac{4}{3}, 00\rangle^6 &= |bb\rangle \end{aligned} \tag{D10}$$

$$\begin{smallmatrix}\square\\\square\end{smallmatrix}: \quad \left|\tfrac{2}{3}, 00\right\rangle^{\bar{3}} = \sqrt{\tfrac{1}{2}}(|wb\rangle - |bw\rangle)$$

$$\left|-\tfrac{1}{3}, \tfrac{1}{2}\tfrac{1}{2}\right\rangle^{\bar{3}} = \sqrt{\tfrac{1}{2}}(|rb\rangle - |br\rangle)$$

$$\left|-\tfrac{1}{3}, \tfrac{1}{2}\tfrac{1}{2}\right\rangle^{\bar{3}} = \sqrt{\tfrac{1}{2}}(|wb\rangle - |bw\rangle) \qquad \text{(D11)}$$

According to (D8-D9) we see that

$$|\uparrow\downarrow\rangle = \sqrt{\tfrac{1}{2}}(|10\rangle + |00\rangle) \qquad \text{(D12)}$$

which gives from (D5)

$$\vec{S}_i \cdot \vec{S}_j |\uparrow\downarrow\rangle = \sqrt{\tfrac{1}{2}}(\tfrac{1}{4}|10\rangle - \tfrac{3}{4}|00\rangle)$$

$$= -\tfrac{1}{4}|\uparrow\downarrow\rangle + \tfrac{1}{2}|\downarrow\uparrow\rangle \qquad \text{(D13)}$$

Similarly we get from (D10-D11)

$$|rw\rangle = \sqrt{\tfrac{1}{2}}\left(\left|\tfrac{2}{3}, 00\right\rangle^{\bar{3}} + \left|\tfrac{2}{3}, 10\right\rangle^{6}\right) \qquad \text{(D14)}$$

which gives from (D4)

$$\lambda_i^a \lambda_j^a |rw\rangle = -\tfrac{2}{3}|rw\rangle + 2|wr\rangle \qquad \text{(D15)}$$

Eq. (D13) and (D15) taken together gives thus

$$\lambda_i^a \vec{S}_i \lambda_j^a \vec{S}_j |r\uparrow, w\downarrow\rangle \qquad \text{(D16)}$$

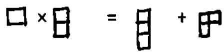
$$= \tfrac{1}{6}|r\uparrow,w\uparrow\rangle - \tfrac{1}{3}|r\downarrow,w\uparrow\rangle - \tfrac{1}{2}|w\uparrow,r\downarrow\rangle + |w\downarrow,r\uparrow\rangle$$

Similar calculations have to be done for the other 35 possibilities. The result can be gathered in the following expression (using the notation $\Lambda_i \Lambda_j \equiv \lambda_i^a \vec{\sigma}_i \lambda_j^a \vec{\sigma}_j$)

$$\Lambda_i\Lambda_j \begin{bmatrix} |C_1S_1, C_2S_2\rangle \\ |C_1S_2, C_2S_1\rangle \\ |C_2S_1, C_1S_2\rangle \\ |C_2S_2, C_1S_1\rangle \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{4}{3} & -2 & 4 \\ -\frac{4}{3} & \frac{2}{3} & 4 & -2 \\ -2 & 4 & \frac{2}{3} & -\frac{4}{3} \\ 4 & -2 & -\frac{4}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} |C_1S_1, C_2S_2\rangle \\ |C_1S_2, C_2S_1\rangle \\ |C_2S_1, C_1S_2\rangle \\ |C_2S_2, C_1S_1\rangle \end{bmatrix}$$

where C_1, C_2 = r,w or b and S_1, S_2 = $\uparrow$ or $\downarrow$. (D17)

The equation (D17) covers all the possibilities for the qq pairs. In a $4q\bar{q}$ state we must also include the possible $q\bar{q}$ interactions. The possible colour-representations for $q\bar{q}$ is given by

 (D18)

In the same way as for the qq system we must thus write down the {8}-states and the {1} state and invert these expressions to get a particular $q\bar{q}$-pair expressed as a linear combination of a pure {8} and a pure {1} state. It is however only the $r\bar{r}$, $w\bar{w}$ and $b\bar{b}$ pairs which is such a linear combination. All the others are pure {8} states. The spin coupling is done in the same way as for qq pairs and we get

$$\Lambda_i\Lambda_j^{*)} \begin{bmatrix} |rS_1\bar{r}S_2\rangle \\ |rS_2\bar{r}S_1\rangle \\ |wS_1\bar{w}S_2\rangle \\ |wS_2\bar{w}S_1\rangle \\ |bS_1\bar{b}S_2\rangle \\ |bS_2\bar{b}S_1\rangle \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & -\frac{8}{3} & 2 & -4 & 2 & -4 \\ -\frac{8}{3} & \frac{4}{3} & -4 & 2 & -4 & 2 \\ 2 & -4 & \frac{4}{3} & -\frac{8}{3} & 2 & -4 \\ -4 & 2 & -\frac{8}{3} & \frac{4}{3} & -4 & 2 \\ 2 & -4 & 2 & -4 & \frac{4}{3} & -\frac{8}{3} \\ -4 & 2 & -4 & 2 & -\frac{8}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} |rS_1\bar{r}S_2\rangle \\ |rS_2\bar{r}S_1\rangle \\ |wS_1\bar{w}S_2\rangle \\ |wS_2\bar{w}S_1\rangle \\ |bS_1\bar{b}S_2\rangle \\ |bS_2\bar{b}S_1\rangle \end{bmatrix}$$

where S_1, S_2 = ↑ or ↓. (D19)

$$\Lambda_i\Lambda_j \begin{bmatrix} |C_1S_1\ \bar{C}_2S_2\rangle \\ |C_1S_2\ \bar{C}_2S_1\rangle \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{4}{3} \\ \frac{4}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} |C_1S_1\bar{C}_2S_2\rangle \\ |C_1S_2\ \bar{C}_2S_1\rangle \end{bmatrix}$$

where C_1 = r,w or b and $\bar{C}_2 = \bar{r}$, $\bar{w}$ or $\bar{b}$ but $C_1 \neq C_2$ and S_1, S_2 = ↑ or ↓. (D20)

It is now of course a formidable task to do the calculations this way by hand. The wavefunctions will for example often contain more than

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*) When having a $\bar{q}$ we must change $\lambda_j \to -\lambda_j^*$ and $\sigma_j \to -\sigma_j^*$ in (D1).

1000 terms *) of the form $a_i qqqq\bar{q}$, and using the operator (D1) on such a state implies using it on 10 different pairs of quarks. From every pair of quarks we get ~ 4 new terms according to (D17), (D19) and (D20). This gives easily ~ 40 000 terms which must then be collected to see if cancellations appear etc. As this is not enough, these states are usually not eigenstates for the operator, so these 40 000 terms doesn't reduce to the 1000 terms we started with, but to some linear combination of several wavefunctions. This clearly motivates the use of the algebraic manipulation program SCHOONSCHIP[17] originally written by Veltman in 1965. The use of SCHOONSCHIP in multiquark calculations is described elsewhere[26].

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*) In the actual calculations the longest wavefunction contained 8064 terms.

Appendix E

Why antisymmetrize all 4q states?

There might be some questions concerned with the need to totally antisymmetrize the 4q wavefunctions when we actually break flavour symmetry later on. The answer to this is that it is not necessary to antisymmetrize between the u (or d) and the s-quark when we break $SU_F(3)$, but that it is convenient to do it in order to make the treatment more systematic. Indeed it increases the number of terms in the wave functions in most cases, but this matters little as long as we have to use SCHOONSCHIP anyhow.

In this context let us elaborate a little on the validity of antisymmetrizing between all four quarks. This discussion will be based on the presentation of Dirac's[23] ideas on the subject as given by Hussar et al.[24] in a recent article.

Consider the permutation group of four objects: S_4. The elements of S_4 can be divided into five equivalence classes corresponding to the five different partitions it is possible to make. These are given in table (E1) (See also Hamermesh[25] page 25).

Partition [λ]	Elements in corresponding equivalence class
[1111]	e = (1)(2)(3)(4)
[211]	(12), (13), (14), (23), (24), (34)
[31]	(123), (132), (124), (142), (134), (143), (234), (243)
[4]	(1234), (1243), (1324), (1342), (1432), (1423)
[22]	(12)(34), (13)(24), (14)(23)

Doing as Dirac in his eq. (13) of sec. 55 and Hussar and Kim in their eq. (3)*), we define the following operators

$$
\begin{aligned}
X_1 &= e \\
X_2 &= [(12) + (13) + (14) + (23) + (24) + (34)]/6 \\
X_3 &= [(123) + (132) + (124) + (142) + (134) + (143) + (234) + (243)]/8 \\
X_4 &= [(1234) + (1243) + (1324) + (1342) + (1432) + (1423)]/6 \\
X_5 &= [(12)(34) + (13)(24) + (14)(23)]/3 \qquad \text{(E1)}
\end{aligned}
$$

----

*) Hussar et al.[24] worked out the 3-body system.

It can be shown that permutations belonging to different classes commute, and therefore

$$[X_i, X_j] = 0 \tag{E2}$$

It can further be shown that

$$[X_i, P_j] = 0 \text{ for any } P_j \in S_4 \tag{E3}$$

The X_i's can be used to express different symmetrizing operators, for instance

$$B_a = \frac{1}{24}[X_1 - 6X_2 + 8X_3 - 6X_4 + 3X_5] \tag{E4}$$

is the operator that completely antisymmetrizes a given wavefunction, and

$$B_s = \frac{1}{24}[X_1 + 6X_2 + 8X_3 + 6X_4 + 3X_5] \tag{E5}$$

is the total symmetrizer.

It is easy to show that

$$B_{a(s)} = B^2_{a(s)} = B^3_{a(s)} \tag{E6}$$

so that the only eigenvalues of this operators is either 0 or 1. This means for instance that B_a applied to a state will give zero if the state has a certain symmetry which is <u>not</u> total antisymmetry, and it will give one if the state is totally antisymmetric. From (E6) we see that the operators $B_{a(s)}$ are projection operators.

From (E2), (E4) and (E5) it follows that

$$[B_a, B_s] = 0 \tag{E7}$$

This doesn't mean that a certain state can be both antisymmetric and totally symmetric (which is of course impossible), but it means that B_a and B_s can be simultaneously diagonalized. A totally antisymmetric state will have the eigenvalues $B_a = 1$ and $B_s = 0$, a mixed symmetric state will have $B_a = B_s = 0$ etc.

Assume now that our interaction Hamilton H_I is invariant under any permution of the particles, i.e.

$$[P_j, H_I] = [X_j, H_I] = 0 \tag{E8}$$

Then we must have

$$[B_a, H_I] = [B_s, H_I] = 0 \tag{E9}$$

also. Because we know in our case that the wavefunction must be anti-symmetric under exchange of identical particles (Pauli-principle), this partial antisymmetry can be included in the total antisymmetry, and it will be conveniant to use totally antisymmetric states.

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