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Appraisal of Elastic Follow Up

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SUMMARY

Elastic computations are widely used in structural analysis, and their results are used when material behaviour is non elastic. The current practice is the partition of the computed stress between primary and secondary stress. The basic characteristic of primary stress is that it is not self limiting. On the contrary the basic characteristic of a secondary stress is that it is self limiting, and failure from one application of the stress is not to be expected. Il must be emphasized that self limitation is not sufficient and that it is also necessary that strains are small enough to avoid any material disorder. Unfortunately, elastic computations do not give real strain distribution and computed strain in highly stressed areas can be magnified under conditions of plastic flow by reason of the follow up elasticity of most lowly stressed areas. If temperature is high enough, an undesirable amount of creep occurs in areas of reduced strength and failure can happen. In creep range, to avoid elastic follow up, the most important part of elasticaly computed stress is considered as primary. This practice is over conservative, and the aim of this paper is to provide indications to choise what fraction of a self limiting stress can be considered as secondary.

At first, considerations are given to a simple structure which could be called "creep relaxation tensile test". A bar (with constant cross section) is loaded by an elastic spring in order to obtain a given elongation of the assembly. The stress evolution is studied. Then the creep damage is computed, and compared to the damage corresponding to the damage corresponding to the elastic computed stress. This comparison give the fraction of the self limiting stress which must be considered as primary.This involve the structural parameter Θ which is the initial value of the ratio of elastic energy to dissipating power.

Extension of the rule is made with the help of KACHANOV approximation. As a conclusion a procedure is described which determine what fraction of a self limiting stress must be considered as primary. It must be pointed out that the proposed method can be considered as a generalization of the "reference stress concept" where relaxation tests are used in lieu of creep tests.

1. Scope

In designing mechanical structures, elastic analysis is used by drawing a distinction between primary and secondary stresses. This distinction is very difficult to make when slightly loaded parts of the structure act as a spring on the heavily stressed parts, where the bulk of the deformations are concentrated. If the temperature is fairly high, an undesirable accumulation of creep occurs in the weak zones, and this may result in fracture. This mechanism is the so-called "elastic follow -up" effect. This paper is intended to provide a guide to appraise this effect and hence to achieve a better classification of stresses among secondary and primary stresses, when creep is significant. Since this is a compact paper, further details can be obtained by referring to [14].

2. Primary stress, secondary stress and elastic follow-up

Mechanical structures are most often built of materials liable to display plastic deformation or creep deformation. However, dimensioning is which material behavior is assumed to be elastic and linear. This is justified both by the often excessive cost of computations which take account of inelastic behavior of the material, and by the difficulty of selecting the right equation representing the material. Although the calculations were carried out assuming elastic behavior, proper use of the results obtained makes it necessary to account for the real behavior of the material. It is routine practice to break down the calculated stress into several parts [1,2,3,4,5]. The subdivision into primary stresses and secondary stresses is discussed here. Based on the good definition given in [6], it may be written that the basic characteristic of a primary stress is that it is not self-limiting, whereas a secondary stress is self-limiting.

To distinguish between two types of stress, the accent is placed on the possibility of self-limitation (by small deformations).

Unfortunately, it would be a mistake to restrict oneself to this condition. It must be emphasized that the self-limiting condition is not sufficient and that it is also necessary for the deformations to be sufficiently small to avoid any disorder in the material. This risk was felt since 1955 [7] concerning piping systems subjected to cyclic temperature variations, and it is possible to find a classic illustration in [8]. Since then, this risk has attracted considerable attention in the design of elevated temperature vessels, in which creep is liable to occur, especially in nuclear power plants [9].

Hence it is important to restrict damage by creep and therefore the values of strain. Unfortunately, elastic calculations cannot provide the real distribution of strains and the calculated strains may be magnified in highly stressed regions by plastic or viscous flow due to the elasticity of slightly stressed regions (elastic follow-up). In other words, slightly stressed parts of the structure may act as a spring that loads the highly stressed parts, where strain is concentrated. This ignorance of the real strain distribution leads, by precaution, to the classification of self-limiting stresses as primary stresses, for which it cannot be proved that relaxation cannot be obtained by small deformations (and without significant creep damage). This practice is pessimistic, and hence safe, but it penalizes the results heavily and certainly leads to a useless increase in component costs. Thus it is wise to examine whether an appraisal of elastic follow-up can be recommended. Since it is natural to proceed from the simple to the complex, a typical illustration of the concepts of primary stress, secondary stress and elastic follow-up will first be presented.

3. Illustration by a typical case

3.1 Two methods for stretching a bar

The simplest structure is a straight bar that is stretched. As an order of magnitude, a one-meter long bar with a constant cross-section of 1 mm^2 can be considered, in which the material is a mild steel with a yield stress of 250 MPa and an ultimate strength of 480 MPa (Young's modulus 200,000 MPa). (See Figure 1). An attempt will be made initially to establish an elastically calculated stress of 1000 MPa in two identical bars, using two different processes :

- suspension of a dead weight (weighing 102 kg),
- elongation of the bar by 5 mm using a bolt/nut system.

If the stress is calculated elastically, the value of 1000 MPa is obtained. However, the two bars behave in widely differing manners : the first breaks long before the 102 kg weight has been placed on the tray, while the second does not break at all. This difference is easily explained. In the first bar, the stress results from the equilibrium equations, and the deformations that it can undergo cannot reduce this stress. This stress is not thus self-limiting, and when the yield stress is exceeded, considerable deformation is observed, followed by fracture. In the case of the first bar, this is clearly a case of primary stress, which can be seen to cause serious disorders and even fracture if its value is high.

In the second bar, this stress can only exist to obtain an imposed elongation elastically. The action of this stress can cause plastic deformation which, helping to obtain the imposed elongation, reduces the stress required. This stress is thus self-limiting, and small plastic deformations that it can cause suffice to meet the conditions for the appearance of the stress. This stress is thus totally secondary, and it can be seen that its application does not cause disorders (at least in one application as in the case discussed above).

3.2 Illustration of elastic follow-up

Other means are available for attempting to establish an elastically calculated stress of 1000 MPa in a bar. An identical bar to the foregoing can be loaded using a spring of stiffness K, which is elongated by a length u, given by the condition $K(u - 5) = 1000$ N (to obtain this elastic stress of 1000 MPa). This loading process displays intermediate characteristics between those of the above cases. Hence the behavior of the bar falls between the two foregoing behaviors. It can moreover be shown easily (like the two foregoing cases) on the stress/strain diagram on which the characteristic curve of the material is plotted (see Figure 2).

It is obvious that the point representing the "elastic stress" of 1000 MPa is point A. The characteristic line of the spring must pass through this point, and its slope is given by the flexibility. Depending on the spring's flexibility, it clearly emerges that the behavior varies between that of the two first bars. This is a clear case of elastic follow-up in the elastic and plastic strain region. Hence when examining elastic follow-up in the region in which creep is significant, it is wise to consider the same structure, a bar stretched by a spring.

4. Typical case of a bar stretched by a spring

4.1 Definition of the case considered

The structure investigated is shown in Figure 3, and consists of a cylindrical bar of length L and cross-sectional area A. This bar is coupled to a perfectly elastic spring with stiffness K. The assembly is elongated by a length ΔL such that the initial stress in the bar is equal to S_0 if the behavior of the bar is elastic.

The bar material may creep according to an equation of the Norton type :

$$\frac{d\varepsilon^c}{dt} = B(\tau)\,\sigma^n$$

where ε^c is the creep deformation, σ the stress applied, and B a time function τ. In the following discussion, the time scale is changed and astronomic time τ is replaced by natural time :

$$t = \int_0^{\tau} \frac{B(\tau)}{B(o)}\, d\tau \qquad \text{where} \qquad \frac{d\varepsilon^c}{dt} = B_0\sigma^n$$

where B_0 is a constant.

It is assumed that the material is effectively damaged when its elongation reaches a value e (about 1%).

4.2 Elements of an energy balance

If the bar/spring assembly is in a state of tension such that the stress in the bar is equal to S, the elastic energy F of the system is the sum of the elastic energy of the bar and that of the spring :

$$F = \frac{S^2}{2} AL + \frac{(AS)^2}{2K} = AL\left(\frac{1}{E} + \frac{A}{KL}\right)\frac{S^2}{2}$$

At the initial instant at which $S = S_0$ we had :

$$F_0 = AL\left(\frac{1}{E} + \frac{A}{KL}\right)\frac{{S_0}^2}{2} \qquad (1)$$

Hence :

$$\frac{F}{F_0} = \left(\frac{S}{S_0}\right)^2 \qquad (2)$$

In the state considered, creep occurs in the bar and results in a dissipated power equal to :

$$D = ALS\,\frac{d\varepsilon^c}{dt} = ALBS^{n+1}$$

at the initial instant $S = S_0$ the dissipation power was :

$$D_0 = AL\ B_0\ S_0^{\ n+1} \tag{3}$$

hence :

$$\frac{D}{D_0} = \left(\frac{S}{S_0}\right)^{n+1} \tag{4}$$

4.3 Stress variation

It goes without saying that the rate of decrease of the elastic energy is equal to the power dissipated :

$$\frac{dF}{dt} = -D$$

using equations (1), (2), (3) and (4), this becomes :

$$2\frac{d(S/S_0)}{d(t/\theta)} = \left(\frac{S}{S_0}\right)^n$$

where θ is the structural parameter which is the initial value of the ratio of elastic energy to power dissipated :

$$\theta = \frac{F_0}{D_0} \tag{5}$$

Integration gives the variation in stress as a function of natural time t :

$$\frac{S}{S_0} = \left[\frac{n-1}{2}\ \frac{t}{\theta} + 1\right]^{-\frac{1}{n+1}} \tag{6}$$

Where S_0 is the initial value of the stress. Figure 4 gives the plot of this variation.

4.4 Cumulative damage - appraisal of the primary part of relaxable stress

The elongation is easy to calculate, and divided by the allowable elongation, it gives the cumulative damage :

$$U = \frac{\varepsilon^c}{e} = \frac{2\ \theta\ B_0 S_0^{\ n}}{e}\left[1 - \left(\frac{n-1}{2}\ \frac{T}{\theta} + 1\right)^{-\frac{1}{n-1}}\right] \tag{7}$$

where T is the duration of application of the imposed elongation. If application time is long, the damage tends towards a limit equal to :

$$U = \frac{2\theta}{e}\ \left(\frac{d\varepsilon^c}{dt}\right)_0 \tag{8}$$

where :

$$\left(\frac{d\varepsilon^c}{dt}\right)_0 = B_0 S_0^{\ n}$$

is the initial rate of creep .

In the case at hand, it must be considered that the primary part of the relaxable stress S_0 is a constant stress P which would produce the same damage if it was applied for the same period :

$$B_0 P^n T = U$$

$$\frac{P}{S_0} = \left\{ \frac{1 - \left(\frac{n-1}{2}\frac{T}{\theta} + 1\right)^{-\frac{1}{n-1}}}{T/2\theta} \right\}^{1/n}$$

This stress depends on the application time T (P decreases with increasing T). In practice, this is acceptable because it varies slowly. Since only long-term applications are considered here, durations of 1000 to 5000 hours can be considered as sufficiently sure. However, if possible, the intended use of the structure can be taken into account in selecting T. Figure 5 gives an illustration of the variation in P.

5. Other structures

A structure that is very sensitive to elastic follow-up is a bar of variable cross-section subjected to a given elongation (Figure 6). During creep, the strain accumulates in the small-section zones.

Analysis of the mechanism yields identical results to the typical case examined above. This is due to the fact that in all these structures, the stress field, determined by equilibrium equations alone, always remain similar.

For any structure, this property of the similarity of stress fields is not guaranteed. Fortunately, the practice of relaxation estimates very often uses Kachanov's hypothesis, which consists precisely of assuming that the stress field remains relatively unchanged during deformation. This hypothesis was examined in [12]. Kachanov's approximation fails to take account of stress redistribution due to inelastic deformation. Except in the case of isostatic structures, this redistribution takes place and leads to slower relaxation than that calculated. The difference is greater if the structure is more redundant (more hyperstatic). But this cannot be considered as a drawback for elastic follow-up, because the latter is liable to occur more in isostatic structures. As an illustration, Figure 7 shows two typical cases in which Kachanov's approximation is valid or very poor, and where it can be seen that elastic follow-up, which is liable to occur in the first case, is impossible in the second.

As a rule, elastic follow-up is mainly to be feared in structures (approaching isostatism) in which Kachanov's approximation is correct or very close, and it is virtually impossible where the approximation is inapplicable (kinematically determined structures). Hence it seems perfectly reasonable to adopt Kachanov's approximation. Given this validity, the entire method discussed can be employed. For creep, the structural parameter θ is always the initial value of the ratio of elastic energy to power dissipation, and changes in the stress field are given by equation (6). The damage can be calculated in the same way as above, noting that it may be different from one point to another, equation (7). As for the primary fraction, it is always given by equation (9).

6. Summary of the elastic follow-up appraisal method

This generalization serves to suggest a few guidelines to appraise elastic follow-up.

Il plastic deformations can be ignored, it is necessary to :

- carry out an elastic calculation,
- extract from it the share of stress that is relaxable (subtract a share that is necessary to satisfy the equilibrium equations),
- deduct the stress concentrations,
- with the remainder (relaxable except the concentrations), calculate :
  - the elastic energy of the structure F_0,
  - the power dissipation by creep D_0 :

$$D = \iiint B_0 \sigma^{*n+1} \, dv \qquad \sigma^* = \frac{3}{2} s_{ij} s_{ij}$$

- derive the structural parameter $\theta = F_0/D_0$,
- the damage can be appraised by an equation like (8),
- the portion to be classified as primary can be estimated by equation (9), which implies setting a time of application of the stress (a fraction of the service life, for example, if several loadings must be considered).

7. Conclusions

The first and most important conclusion is that practical rules for appraising elastic follow-up can be proposed. It should be noted that these rules use equations characterizing the behavior of the material (Norton's Law, temperature effect, time and assessment of damage). As these equations are established in widely differing circumstances of cases in which elastic follow-up occurs, they need to be validated or improved. To achieve this, it is recommended to use the bar/spring structure discussed in Section 4.

A second conclusion is concerned with a type of test designed to provide a better knowledge of the material. Finally, the similarity of the equations selected for this very simple case and the real structures in which elastic follow-up is liable to occur, elicits the recommendation of combined tests of a structural maquette and a corresponding bar/spring element. The same material should be used, and not necessarily that of the intended structure (lead and tin allow accelerated tests at low temperature). It is also necessary to use the same characteristic values of S_0 in both tests, and especially the same structural parameter θ.

This is the third conclusion concerning experimental verifications of the method, and more specifically a comparison of the behaviors of two extremely different structures which should exhibit an effective similarity. Il may be noted that the method suggested can be considered as an extension of the reference stress method, in which one refers not to a conventional creep test, but to a creep/relaxation test on an assembly similar to the one considered in Section 4.

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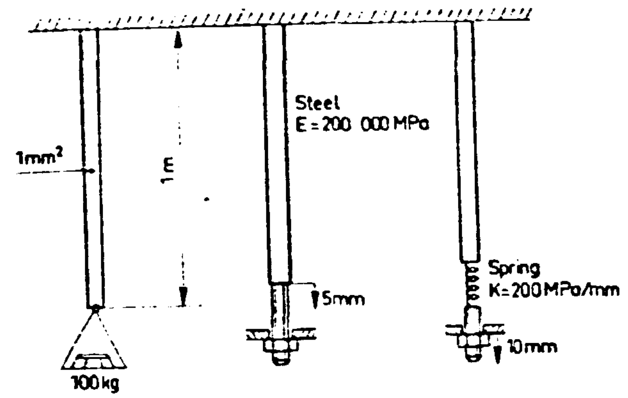


Fig.1

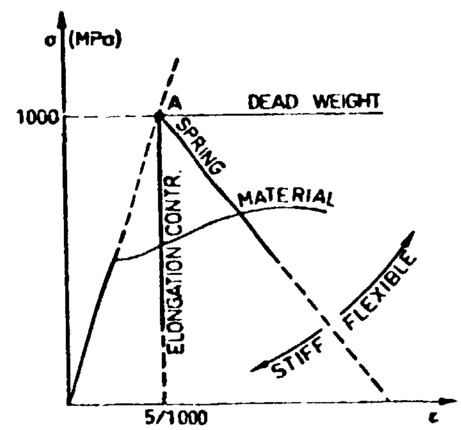


Fig. 2

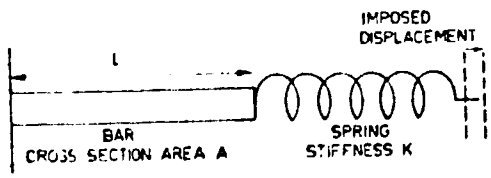


Fig 3

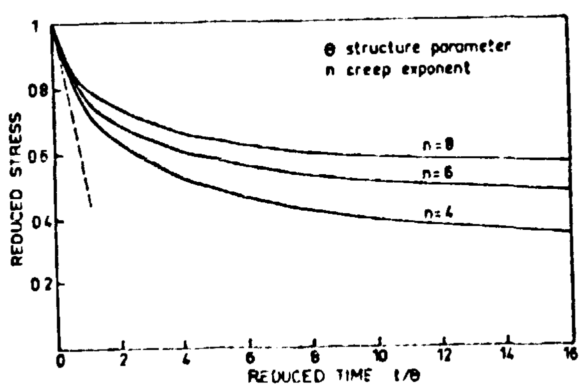


Fig 4

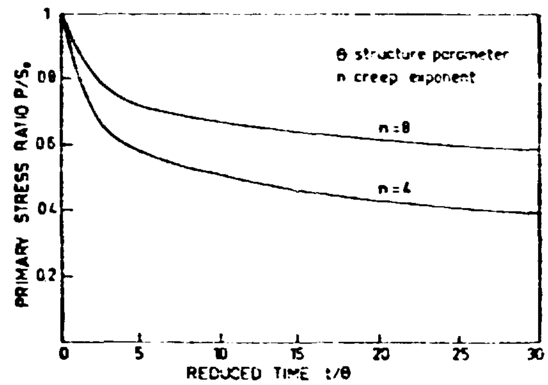
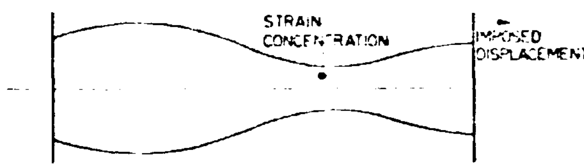


Fig 5



BAR WITH VARIABLE CROSS SECTION

Fig 6

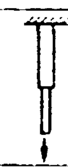
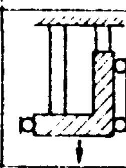
CASE	STATICAL DETERMINED	KINEMATIC DETERMINED
EXEMPLE		
KACHANOV APPOX.	EXACT	INVALID
ELASTIC FOLLOW UP	YES	NO

Fig 7

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