6. International conference on structural mechanics in reactor technology. 6. SMIRT.
Paris, France,August 17 - 21, 1981.
CEA - CONF 5964

- Elastic plastic buckling of elliptical vessel heads

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S U M M A R Y

The risks of buckling of dished vessel head increase when the vessel is thin walled. This paper gives the last results obtained in Saclay on experimental tests of 3 elliptical heads and compares all the results with some empirical formula dealing with elastic and plastic buckling.

1 - INTRODUCTION

Buckling of vessel heads must be a preoccupation of pressure vessels designers. The risks of buckling increase greatly when the vessel is large enough and thin walled. In the PWR type of reactors, the internal pressure makes that the walls are very thick and eliminate all buckling probabilities. We already mention that in the pool type architecture adopted in France for FBR, the main vessel is very large and relatively thin ($t/d \sim 1{,}25.10^{-3}$). So it is necessary to work in two coherent directions :

- building efficient computer programs dealing with elastic-plastic buckling of various structures ;

- testing representative specimen to check the validity of these programs and also to give safe rules to designers who don't have any access to the expansive way of sophisticated computing.

2 - DEFINITION OF THE BUCKLING PRESSURE

It can be noted that computation and experimental tests give very useful informations to each other. Bushnell pointed out [1] that the critical buckling pressures he calculated were about 20 % lower than the ones we measured, at Saclay, on the first set of 18 elliptical vessel heads [2]. He said that this permanent discrepancy could be due to the fact that a fold begins to form before it can be seen by means of a pressure fall for instance. That gives us opportunity to say that performing a valuable experiment is not easier than determining an adequate mathematical model. Effectively it does'nt exist a conventional definition of the critical buckling load yet. It seems that for theoreticians it is the load at which the very beginning of a fold appears. Unfortunately it is very difficult to test it. We don't think that even a rotating probe could show it, for the first fold forms rather fast in a thin dished head and it is impossible to know its location before seeing it. So the probe could be elsewhere on the periphery of the head at the critical moment. That rotating probe could be more efficient with thicker heads where the folds formation is slower and equaly distributed all around the knuckle region.

3 - NEW EXPERIMENT

3 new elliptical heads have been added to the previous set of 18. The characteristics and results of these experiment are on the Table I.

These 3 heads have not been cut so the actual characteristics of material are not yet known. The thickness has been measured by ultrasonic method.

We postpone further measurement on these 3 heads when we complete the serial with 7 more elliptical heads waiting for tests. Figure 1 shows the aspect of these 3 tested heads. Figure 2 gives an example of the recorded curve of the polar deflection corresponding to pressure.

4 - DESIGN FORMULA

As we mentioned before, we are concerned by the study of as simple as possible formula allowing to know the buckling pressure as a function of geometry and material characteristics of dished heads.

The calculations made at Saclay using the CEASEMT system showed that the elastic buckling pressure of an ellipsoidal head could be calculated by equation (1) :

$$Pe = Ke \frac{\pi^2 E}{1 - \nu^2} \frac{t^2 b^2}{a^4} \qquad \text{eq. (1)}$$

Where
- E : Young modulus
- ν : Poisson coefficient
- t : thickness
- a : half the major axis
- b : half the minor axis
- Ke : coefficient shown in Figure 3 where it can be seen that $Ke = 1$ when $a/b \geqslant 2$.

We also found that the whole experimental results can be represented by formula (2) dealing with plastic buckling :

$$Pc = Kc\, \sigma_f \frac{t^{5/3} b^{4/3}}{a^3} \qquad \text{eq. (2)}$$

where
- σ_f : .5 $(\sigma_u + \sigma_y)$, flow stress
- σ_u : ultimate strength
- σ_y : yield strength (.2 per cent offset)
- t : thickness in the knuckle region
- a : half the major axis
- b : half the minor axis
- Kc : coefficient calculated by equation (3)

$$Kc = 88 \left(1 - 2 \frac{b^2}{a^2}\right)^{-2/3} \qquad \text{eq. (3)}$$

Eq.-(2)-(3) fairly accurately ($\pm$ 10 per cent) represent all the results except for head n° 22. That can be observed in table II. See also figure 4.

The equations giving plastic buckling pressure can be rewritten in the form of the French CODAP (Code Français de Construction des Appareils à Pression) :

$$P = \frac{2f}{C} \frac{t}{D} \qquad \text{eq. (4)}$$

where $f \leqslant \sigma_y/1,6$ and $f \leqslant \sigma_u/3$ eq. (5)

t = thickness

D = diameter

C is a coefficient given by curves of figure 5. The calculation of C, made in the spirit of design criterium, includes a safety coefficient. A comparison of P and tests is shown in table III.

It is sure that for most of the metallic materials the buckling always occurs in the plastic region. That is much less obvious when the head is built with plastics. From formula (1) to (3) we calculated a curve (figure 6) showing the boundary between elastic and plastic risks.

5 - CONCLUSIONS

If the material shows plastic behaviour, no sure buckling theory exists. That implies that a method for calculating elasto-plastic buckling cannot be based on the laws of mechanics and requires an assumption. This assumption and the calculation method must be validated by reference to experiment.

It is suitable to adopt a definition of the buckling pressure which can be a reference for both theoreticians and experimenters. It means that the definition must refer to measurable phenomena in spite of the beauty of more intellectual concepts.

It has been shown that it was possible to fit an empirical formula giving plastic buckling pressure, in good agreement with experimental results.

It is hoped that new experimental results about elliptical and torispherical heads will be obtained by the end of the year.

REFERENCES

[1] BUSHNELL, D., "Elastic-Plastic Buckling of Internally Pressurized Ellipsoidal Pressure Vessel Heads".
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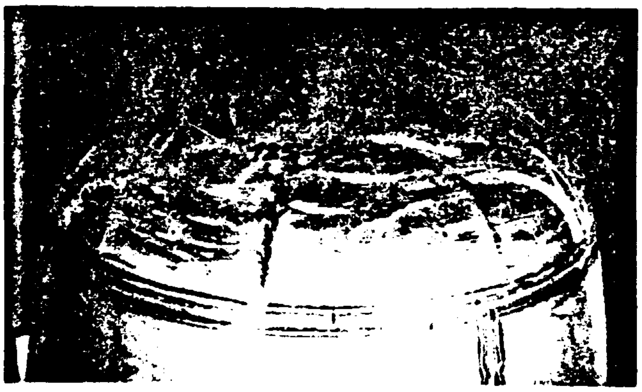
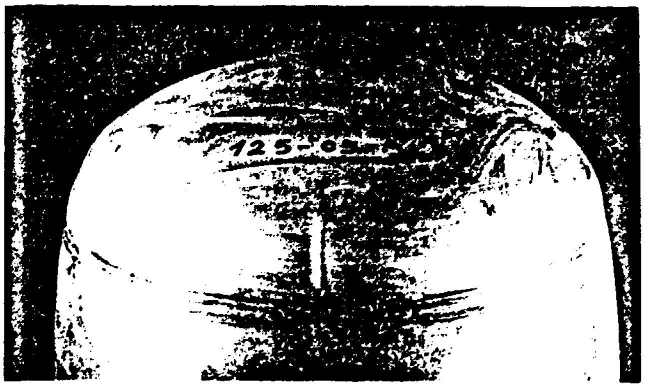
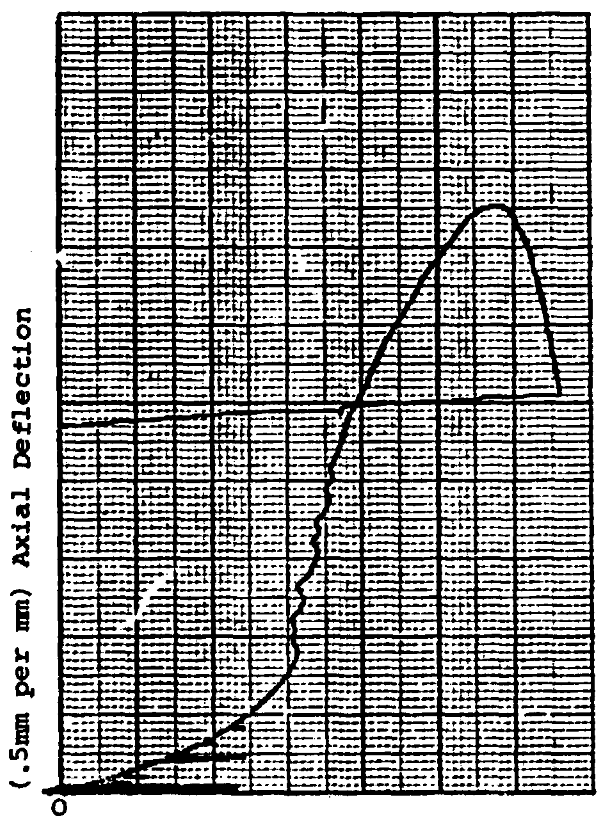
[2] ALIX, M. et ROCHE, R. L., "Essais de flambage de fonds elliptiques sous pression interne"
Note CEA-N-2075, Janvier 1979.






700-10-A

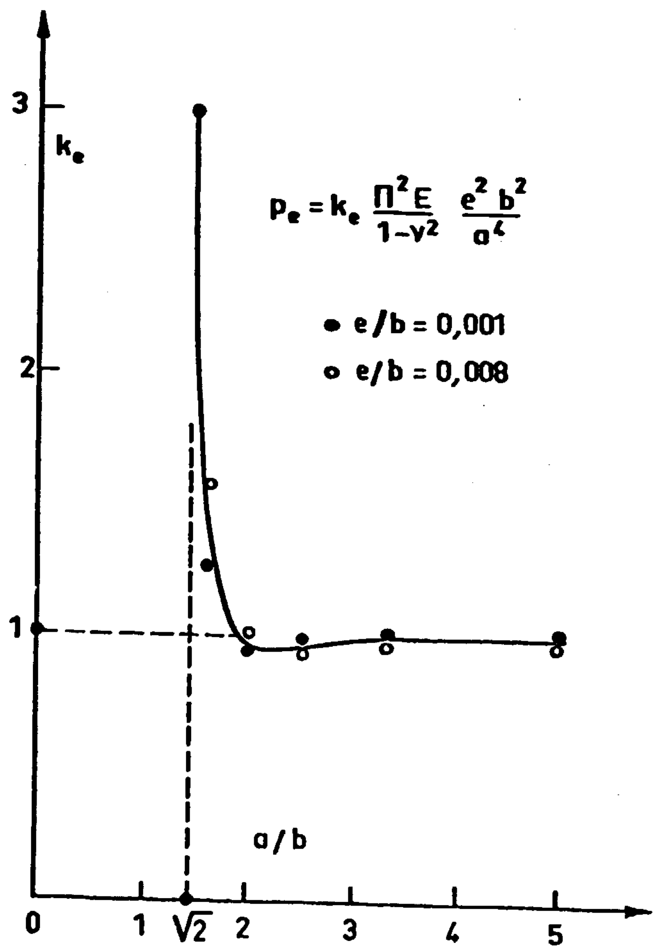


(0,025 MPa per mm) PRESSURE

(Curve recorded during the test)

ELLIPTICAL HEAD NO. 100-10-A

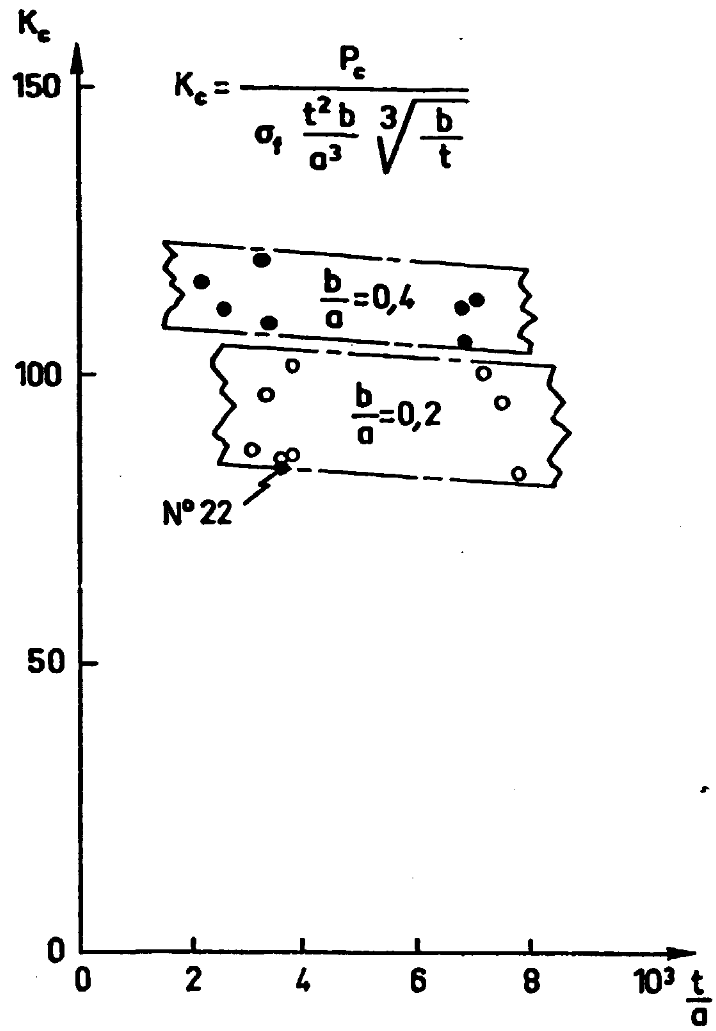
Fig. 2



ELLIPTICAL HEADS

Elastic buckling

Fig. 3



ELLIPTICAL HEADS

Plastic buckling

Fig. 4

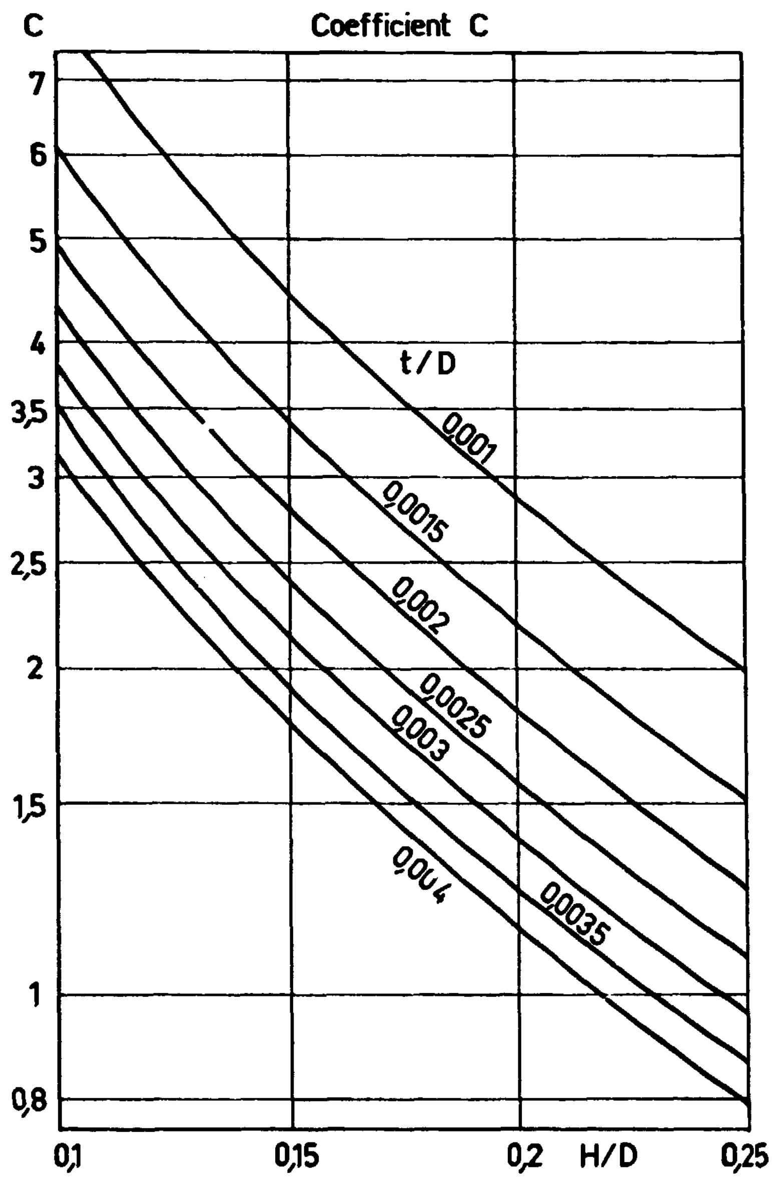
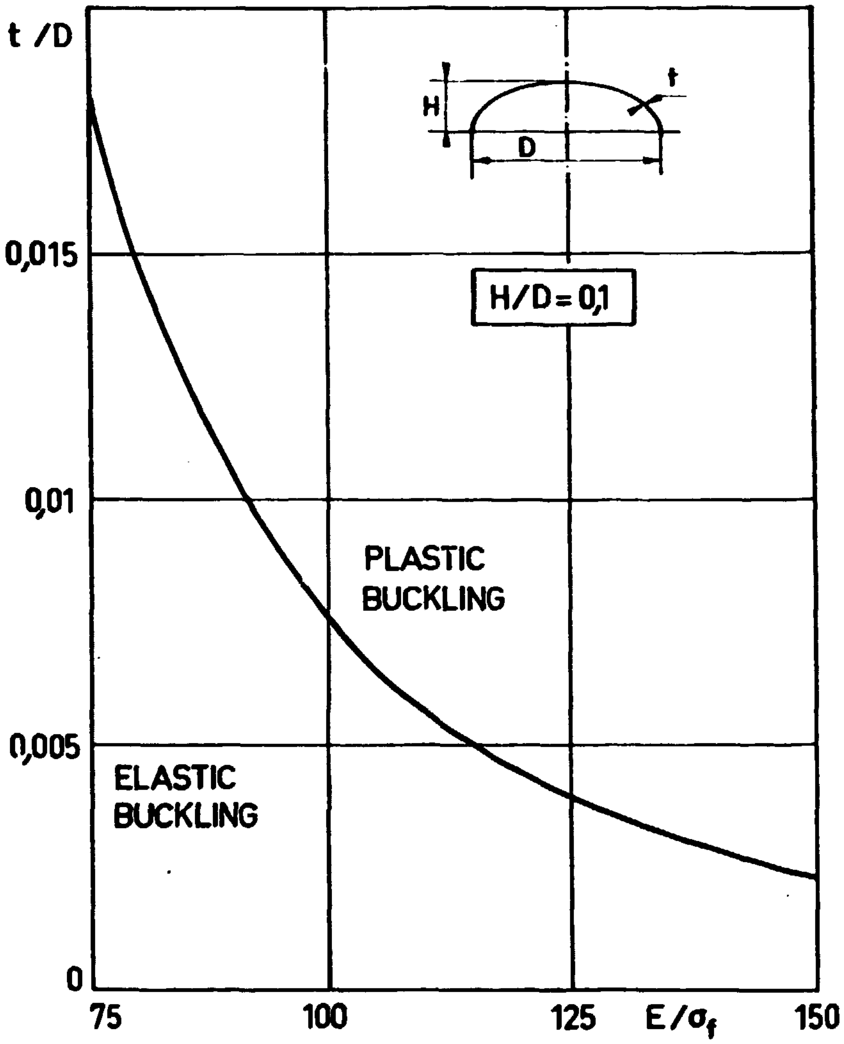

C
Coefficient C
7
6
5
4
3,5
3
2,5
2
1,5
1
0,8
t/D
0,001
0,0015
0,002
0,0025
0,003
0,0035
0,004
0,1
0,15
0,2
H/D
0,25

Fig. 5



BUCKLING OF ELLIPTICAL HEADS
Type of buckling (function of t/D & E/σ_f)

Fig. 6

TABLE 1

GEOMETRY AND BEHAVIOUR OF VESSEL HEADS SUBJECTED TO INTERNAL PRESSURE

ELLIPTICAL HEADS			
MATERIAL : Carbon steal E26 (σ_f = 340 Mpa)			
DIAMETER D : 503 mm	Ferrule height : 500 mm		
HEAD NR	100-10-A	125-05-A	125-05-B
Height H (mm)	100	125	125
Thickness t (mm) (knuckle region)	0,85	0,46	0,48
FIRST FOLD (Buckling) Deflection at Apex (mm) Pressure P_{exp} (MPa) Calculated Pressure $P_{cal.}$ (x) P_{exp} / $P_{cal.}$	 9 0,78 0,85 0,92	 2,7 0,60 0,505 1,19	 3,35 0,565 0,54 1,05
AT THE END OF THE EXPERIMENT Number of folds (+ almost invisible ones) Pressure (MPa) Ferrule max-diameter (mm)	 11 (+ 1) 1,65 601	 8 (+ 3) 0,775 552	 11 (+ 2) 0,775 562

(x) Calculated values (see formula (2) (3)).

TABLE II

head	H (mm)	σ_y	(MPa)	σ_u	t (mm)	P_{buckle} (MPa)	$P_{cal.}$ (MPa)	$\frac{P_{buckle}}{P_{cal}}$
1	100	(280)	(1)	(330)	0,64	0,48	0,49	0,98
2	100	310		340	0,85	0,80	0,83	0,96
3	100	300		340	1,74	2,50	2,66	0,94
4	50	280		340	0,85	0,27	0,26	1,04
5	50	260		330	0,95	0,325	0,30	1,08
6	50	240		300	1,95	0,80	0,90	0,89
11	100							
12	100	480		690	0,82	1,50	1,41	1,06
13	100	520		700	1,70	4,90	4,97	0,99
14	50	380		680	0,78	0,36	0,38	0,95
15	50	470		770	0,95	0,575	0,62	0,93
16	50	570		750	1,81	2,10	1,93	1,09
21	100	130		230	0,55	0,23	0,22	1,05
22	100	180		230	0,89	0,42	0,57	0,74
23	100	170		240	1,78	1,80	1,80	1,00
24	50							
25	50	170		230	0,89	0,165	0,18	0,92
26	50	150		200	1,88	0,56	0,55	1,02

$P_{cal.}$ Calculated values (formula (2) (3)).

P_{buckle} Experimental values of critical buckling pressure.

(1) Means of σ_y and σ_u values for heads 2 to 6.

TABLE III

ELLIPTICAL HEADS

N°	e/D	H/D	(C)	P	(MPa)	$P_{exp.}$	$P_{exp.}/P$
			CARBON STEEL (f = 100)				
1	0,0013	0,2	(2,43)	0,11		0,48	4,5
2	0,0017	0,2	(2,03)	0,17		0,80	4,8
3	0,0035	0,2	(1,25)	0,56		2,5	4,5
4	0,0017	0,1	(5,59)	0,061		0,27	4,4
5	0,0019	0,1	(5,19)	0,073		0,325	4,4
6	0,0039	0,1	(3,22)	0,24		0,80	3,3
			AUSTENITIC STEEL (f = 220)				
12	0,0016	0,2	(2,11)	0,33		1,5	4,5
13	0,0034	0,2	(1,28)	1,17		4,9	4,2
14	0,0016	0,1	(5,82)	0,12		0,36	3,0
15	0,0019	0,1	(5,19)	0,16		0,575	3,6
16	0,0036	0,1	(3,39)	0,47		2,1	4,5
			ALUMINIUM ALLOY (f = 67)				
21	0,0011	0,2	(2,71)	0,054		0,23	4,2
23	0,0036	0,2	(1,23)	0,39		1,80	4,6
25	0,0018	0,1	(5,38)	0,045		0,165	3,7
26	0,0038	0,1	(3,27)	0,16		0,56	3,6

P : calculated pressure $P = \frac{2f}{C} \frac{t}{D}$

C : form coefficient (see figure 5)

$P_{exp.}$: experimental values of buckling pressures