

FR 8200606

COMMISSARIAT A L'ENERGIE ATOMIQUE

DIVISION DE LA PHYSIQUE

**SERVICE DE PHYSIQUE THEORIQUE**

NEUTRON STAR EVOLUTION AND THE STRUCTURE  
OF MATTER AT HIGH DENSITY

BY

MADELEINE SOYEUR

Symposium on neutrons stars.  
Istanbul, Turquie, September 7 - 11, 1981.  
CEA - CONF 5927

CEN-SACLAY - 91191 GIF-sur-YVETTE CEDEX - FRANCE

# NEUTRON STAR EVOLUTION AND THE STRUCTURE OF MATTER AT HIGH DENSITY

Madeleine Soyeur

CEN de Saclay - Service de Physique Théorique  
F-91191 - Gif-sur-Yvette Cedex - France

**ABSTRACT :** The structure and properties of neutron stars are determined by the state of cold nuclear matter at high density. In order to investigate the behavior of matter inside neutron stars, observables sensitive to their internal structure have to be calculated and confronted to observations. The thermal radiation of neutron stars seems to be a good candidate to be such observable. It can be shown that the neutrino luminosity of neutron stars, responsible for their cooling in the early stages of their evolution is strongly dependent on possible phase transitions to superfluid nucleons, to pion condensation or to quark matter. The specific heat of matter is also not the same in the various phases expected at high density and is particularly sensitive to the nucleon superfluidity. At present, both the theoretical estimates and the observations of the thermal properties of neutron stars are still quite preliminary. In particular, large uncertainties due to possible reheating mechanisms and magnetic field effects make the theoretical interpretation of the steady radiation of pulsars quite difficult.

## I. INTRODUCTION

The recent discoveries of compact X-ray sources at the center of new supernova remnants (1) seem to reinforce the assumption that the gravitational collapse of massive stars provides an important mechanism for the formation of neutron stars as initially inferred from the observation of pulsars in the Crab and Vela supernovae. Other mechanisms for the formation of neutron stars are however presently discussed such as the collapse of mass accreting carbon-oxygen white dwarfs in binary systems (2). Our discussion will be limited to the evolution of neutron stars formed in the collapse of supernovae. We shall take as typical initial condition the neutron star remnant of the supernova explosion calculated recently by Brown, Bethe and Baym (3) and predicted to have a gravitational mass of  $\sim 1.4 M_{\odot}$  and a temperature of  $\sim 10$  MeV.

The question addressed in this paper is the subsequent evolution

of this hot neutron star as a function of the state of matter in the density range available in neutron stars.

An important simplification in the calculation of the evolution of neutron stars results from the property that the equations determining the thermal history of the star decouple from the equations determining the structure of the star. This effect has been examined recently by Baym (4) who showed that the contraction of a degenerate neutron star during its cooling has a negligible effect on its luminosity, of order  $\left(\frac{\pi T}{T_f}\right)^2$ , in which  $T$  is the internal temperature and  $T_f$  the Fermi temperature. This result indicates that except in the atmosphere and in the very early stages of the cooling, when matter is not highly degenerate, it is quite accurate to use the solution of the structure equations calculated at zero temperature and to compute the thermal evolution of neutron stars without taking into account the changes in their structure due to thermal effects.

Consequently, this paper is organized as follows. In Section II, we review the recent progress in nuclear equations of state and in the understanding of the high density phases of matter. The thermal evolution of neutron stars and its dependence on the state of matter at high density are discussed in Section III. In Section IV, we compare the theoretical predictions to the observational information presently available.

## II. THE STRUCTURE OF NEUTRON STARS

The structure of a static, spherically symmetric, general relativistic neutron star at zero temperature is determined by the equations of hydrostatic equilibrium :

$$\frac{dP}{dr} = \frac{-G\left(\rho + \frac{P}{c^2}\right)\left(m + 4\pi r^3 \frac{P}{c^2}\right)}{r\left(r - \frac{2m}{c^2} G\right)}, \quad (1)$$

(Tolman-Oppenheimer-Volkoff equation)

$$\frac{dm}{dr} = 4\pi \rho r^2, \quad (2)$$

(Mass equation)

$$\frac{d\phi}{dr} = \frac{dP}{dr} (\rho c^2 + P)^{-1}, \quad (3)$$

and the equation of state :

$$P = P(\rho), \quad (4)$$

in which  $G$  is the gravitational coupling constant,  $\rho(r)$  the matter-

energy density,  $P(r)$  the pressure,  $m(r)$  the gravitational mass interior to the radius  $r$  and  $\phi(r)$  the gravitational potential ( $g_{00} = e^{2\phi}$ ).

The structure of stable neutron stars, solutions of eqs.(1)-(4), depends on the equation of state (4) which gives the pressure by which matter can counterbalance the gravitational pressure in the density range available in neutron stars. This equation of state is quite uncertain at and beyond nuclear matter density ( $\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$ ) and, therefore, neutron star parameters are not very precisely predicted. For masses of the order of  $1.4 M_\odot$ , the calculated radii vary from  $\sim 9$  to  $\sim 16$  km and central densities from somewhat less than twice nuclear matter density till  $\sim 7\rho_0$  (5,6).

Recent progress in the equation of state of nuclear and neutron matter has come from the work of Lagaris and Pandharipande (7-9) applied to neutron stars by Friedman and Pandharipande (10). First, Lagaris and Pandharipande (7) obtained a phenomenological two nucleon interaction operator by fitting the nucleon-nucleon phase shifts up to 425 MeV in S,P,D and F waves and the deuteron properties. Compared to previous two nucleon interaction operators used in many-body calculations, this new operator includes six additional terms quadratic in the total angular momentum of the two nucleons  $\bar{L}$  and in the spin-orbit operator  $\bar{L} \cdot \bar{S}$  ( $\bar{S}$  = total spin of the two nucleons). The additional terms containing  $L^2$  and  $(\bar{L} \cdot \bar{S})^2$  operators are relatively weak but important to fit the D and F wave phase shifts. The above two nucleon interaction operator is then used in variational calculations of nuclear matter (8). The equilibrium density is found to be  $\sim$  twice the empirical value and the binding energy per nucleon is too large by about 10 percent. In the next step, they assume that the difference between the calculated and empirical values of the equilibrium energy and density of nuclear matter is entirely due to three nucleon interactions. Consequently, they add phenomenological three nucleon interaction contributions adjusting their parameters to reproduce the empirical equilibrium values of the energy, density and compressibility (8). The properties of asymmetric nuclear matter (9) and of neutron matter (10) are then studied with this hamiltonian.

The structure of neutron stars is calculated (10) from eqs.(1)-(4) with the equation of state of pure neutron matter supplemented in the inner crust by the equation of state of Negele and Vautherin (11) and in the outer crust by the equations of state of Baym, Pethick and Sutherland (12) and of Feynman, Metropolis and Teller (13). For a  $1.4 M_\odot$  neutron star, the central density is of the order of 4.5 times nuclear matter density and the radius is  $\sim 10.5$  km. The predictions of Friedman and

Pandharipande (10) are quite close to the previous calculation (14) of Bethe and Johnson (Model 5) for neutron stars of masses smaller than  $1.5 M_{\odot}$ . Friedman and Pandharipande (10) find however a maximum neutron star mass of the order of  $\sim 1.95 M_{\odot}$ , substantially larger than Bethe and Johnson (14). The work of Friedman and Pandharipande suggests therefore that the most realistic calculations of the cooling of  $1.4 M_{\odot}$  neutron stars presently available are those done with the Bethe-Johnson models. One should recall however that the above equation of state is expected to be valid mostly around nuclear matter density and that the high density behavior of matter remains quite uncertain.

At those high densities, various phase transitions of matter have been envisaged (15). It seems at present that two of these possible phases of matter could substantially modify the evolution of neutron stars and therefore be detectable through their thermal radiation: the presence of a pion condensate and/or the existence of a quark matter core. Recent progress concerns mainly pion condensation.

The phenomenon of pion condensation is the appearance, at some threshold density, of pions in the ground state of nuclear matter. Because of their bosonic character, the pions will occupy the lowest mode and form a condensate. It was first suggested by Migdal (16) and Sawyer and Scalapino (17) that it would be energetically favorable for pions to condense in a state of finite momentum due to the attractive pion-nucleon p-wave interaction, the condensation in a state of zero momentum being inhibited by the repulsive  $\pi^-$ -neutron s-wave interaction (15). The pion spectrum which defines the condensation conditions is therefore largely determined by the pion coupling to the nucleon particle-hole and isobar-hole configurations as shown in Fig.1. An important role is played in the calculation of the graphs of Fig.1 by the

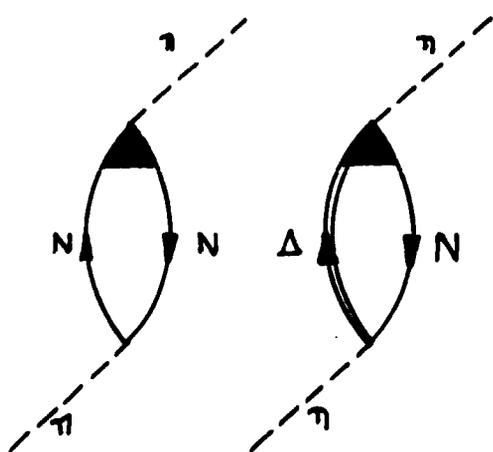


Figure 1

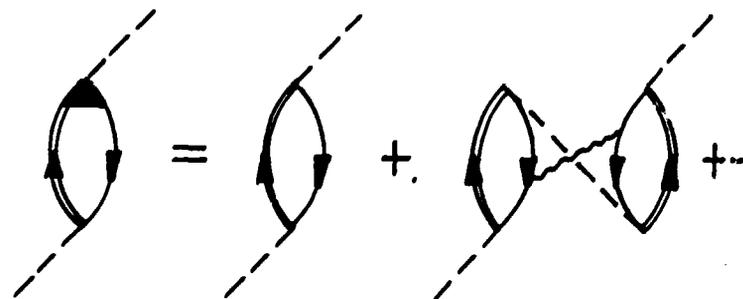


Figure 2

renormalization of the  $\pi NN$  and  $\pi \Delta N$  vertices (in black in the figure) due to the effects of the nuclear short range correlations between nucleons and isobars in repeated scatterings as indicated schematically by a wavy line in Fig.2. The strength of this short range repulsion is parametrized by the Landau-Migdal Fermi liquid parameter  $g'$  (18). The recent progress in the determination of the pion condensation threshold conditions has come from constraints on  $g'$  derived from properties of finite nuclei. If the present interpretation of the observed Gamow-Teller and magnetic multipole transitions is correct, values of  $g'$  between 0.6 and 0.7 are favored by experiment (19). These values are also consistent with very accurate scattering experiments which show no indication of precritical behavior (19). What are the implications for pion condensation? If  $g'=0.6$ , the threshold density for pion condensation is expected to be  $\sim 3\rho_0$  ( $\rho_0$  = nuclear matter density) ; if  $g'=0.7$ , pion condensation should occur at  $\sim 5\rho_0$  (19). We see that these threshold densities for pion condensation are quite close to the central density of  $4.5\rho_0$  predicted for a  $1.4M_\odot$  neutron star using the Friedman-Pandharipande equation of state (10).

For a review of quark matter calculations, we refer to Baym (15). The important problem of the nature and of the critical density of the transition from nucleon to quark matter remains very much unexplored (see however ref.20) and most calculations simply assume the existence of an interacting quark gas in which quark-gluon interactions are treated perturbatively (15,21).

Finally, the neutrons in the inner crust of neutron stars and the neutrons and protons of the interior are expected to become superfluid at sufficiently low temperature due to pairing interactions in the  $^1S_0$  channel for densities somewhat lower than  $\rho_0$  and in the  $^3P_2$  channel at higher densities (22). The corresponding pairing gaps are shown in Fig.3.

Recently, the  $^3P_2$  superfluidity gap in the presence of a pion condensate has been studied by Takatsuka, Tamagaki and Fukawa (23). They found that the  $^3P_2$  pairing is essentially unaffected by *neutral* pion condensation but substantially reduced in the presence of a *charged* pion condensate.

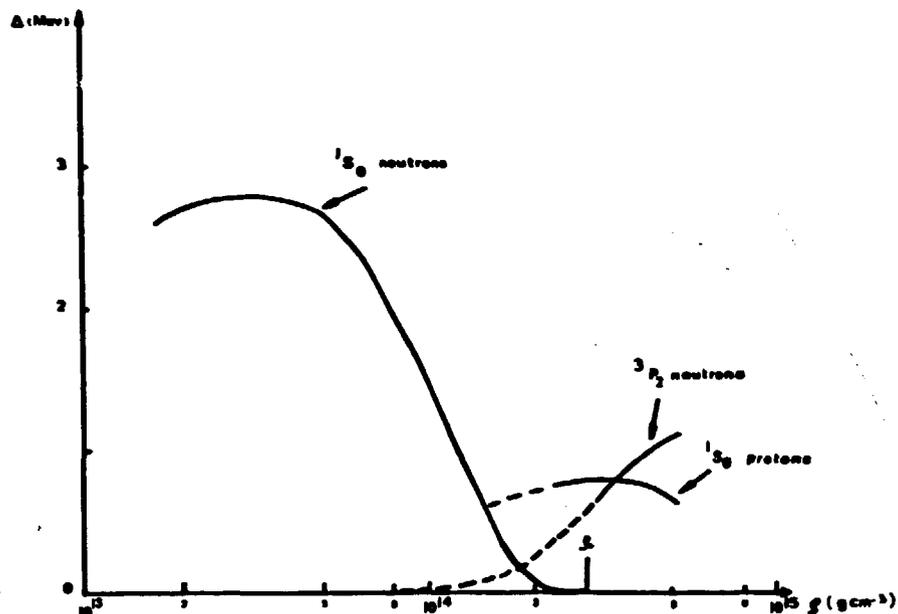


Figure 3 (from ref.22)

### III. THERMAL EVOLUTION OF NEUTRON STARS

As we mention in the introduction, neutron stars formed in the collapse of supernovae are expected to have initial temperatures of  $\sim 10^{11}$  K (3). Recently, in a paper entitled "The coldest neutron star" (24), Feinberg speculated that the instability of baryons against decay into leptons predicted by the present grand unified theories of strong, electromagnetic and weak interactions (25) maintain the surface of old neutron stars at a minimum temperature of 100 K. The problem is to understand what happens in between those limits.

As Bahcall and Wolf (26) pointed out in 1965, the luminosity of neutron stars consists of two terms: the neutrino luminosity ( $L_\nu$ ) dominant at high temperatures ( $T \gtrsim 2 \times 10^8$  K) and the photon luminosity ( $L_\gamma$ ) dominant at lower temperatures ( $T < 10^8$  K). The mean free path of the thermal neutrinos is of the order of the star radius and therefore they are radiated by the whole volume of the star; the photons are radiated only by the surface.

The thermal evolution of neutron stars is determined by the equation of energy balance,

$$\frac{d}{dr} [L e^{2\phi}] = \frac{-4\pi r^2}{\left(1 - \frac{2Gm}{rc^2}\right)^{1/2}} e^\phi c_v \frac{dT}{dr}, \quad (5)$$

the neutrino luminosity gradient,

$$\frac{d}{dr} [L_{\nu} e^{2\phi}] = \frac{4\pi r^2}{\left(1 - \frac{2Gm}{rc^2}\right)^{1/2}} e^{2\phi} \epsilon^{\nu} \quad , \quad (6)$$

and the heat transfer equation,

$$\frac{d}{dr} [T e^{\phi}] = \frac{-3\kappa\rho}{16\sigma T^3} \frac{L_{\gamma} e^{\phi}}{4\pi r^2 \left(1 - \frac{2Gm}{rc^2}\right)^{1/2}} \quad , \quad (7)$$

in which  $L = L_{\nu} + L_{\gamma}$  is the total luminosity,  $C_{\nu}$  the volume specific heat,  $\epsilon^{\nu}$  the neutrino emissivity per unit volume,  $\kappa$  the total opacity and  $\sigma$  the Stefan's constant.

A major simplification in the solution of the above equations comes from the high thermal conductivity of the degenerate electrons inside neutron stars. This implies that the star can be divided into two regions: the "isothermal" core where  $T e^{\phi} = \text{constant}$  and the mantle in which there exists a steep temperature gradient. The density at which the mantle begins is chosen to be  $10^{10} \text{ g cm}^{-3}$  (27,28). However, it has been pointed out recently by Ray (29) and confirmed by numerical calculations (30) that thermal conduction times could be quite large in the inner crust ( $10^{10} < \rho < 2.4 \times 10^{14} \text{ g cm}^{-3}$ ) of neutron stars calculated with stiff equations of state, i.e. for a crust thickness of  $\sim 5-6 \text{ km}$  ( $M = 1.3 M_{\odot}$ ). This effect is much less important and can be neglected for cooling times  $\gtrsim 100$  years for softer equations of state which we believe are much more realistic.

In order to solve eqs.(5)-(7), it is necessary to know the specific heat, the neutrino emissivity, the photon luminosity and the opacity of neutron stars.

The heat content of a  $1.4 M_{\odot}$  neutron star comes mostly from its interior. The volume specific heats of the non relativistic degenerate neutrons and protons are given by (31)

$$C_n(T_9) = (1.64 \times 10^{20}) \frac{m_n^*}{m_n} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}} T_9 \text{ erg cm}^{-3} \text{ K}^{-1} \quad , \quad (8)$$

$$C_p(T_9) = (4.1 \times 10^{19}) \frac{m_p^*}{m_p} \left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}} T_9 \text{ erg cm}^{-3} \text{ K}^{-1} \quad , \quad (9)$$

and for the relativistic degenerate electrons by (31)

$$C_e(T_9) = (3.72 \times 10^{18}) \left(\frac{\rho}{\rho_0}\right)^{\frac{4}{3}} T_9 \text{ erg cm}^{-3} \text{ K}^{-1} \quad , \quad (10)$$

in which  $T_9$  is the internal temperature in units  $10^9 \text{ K}$  and  $m_n^*$  and  $m_p^*$  are the neutron and proton effective masses. The neutron and proton specific heats (8)-(9) are strongly affected by the nucleon superfluidity at and

below the critical temperatures of  $\sim 5 \times 10^9$  K for the protons and  $\sim 10^9$  K for the neutrons. The most important effect is the exponential suppression of the neutron specific heat below  $10^9$  K; for  $T \lesssim 2 \times 10^8$  K, only the electron specific heat contributes significantly to the heat content of the star. The presence of quark matter or of a pion condensate does not seem to affect significantly the specific heat except for a possible lowering of the neutron superfluidity temperature in the presence of charged pion condensation (21,23).

The neutrino luminosity of neutron stars has two components : the neutrinos produced in the interior of the star and the neutrinos produced in the crust ( $\rho < 2.4 \times 10^{14}$  g cm<sup>-3</sup>).

In the absence of a quark core or a pion condensate, the dominant contributions to the interior neutrino luminosity are the modified URCA process



and neutrino pair bremsstrahlung arising from nucleon-nucleon scattering



The neutrino emissivities for the processes (11)-(13) were recalculated by Maxwell and Friman (18bis) in the Weinberg-Salam model. They found

$$\epsilon_{URCA}^{\nu}(T_9) = (1.8 \times 10^{21}) \left(\frac{m_p^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) \left(\frac{\rho}{\rho_0}\right)^{2/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}, \quad (14)$$

$$\epsilon_{nn}^{\nu}(T_9) = (4.4 \times 10^{19}) \left(\frac{m_n^*}{m_n}\right)^4 \left(\frac{\rho}{\rho_0}\right)^{1/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}, \quad (15)$$

$$\epsilon_{np}^{\nu}(T_9) = (5.0 \times 10^{19}) \left(\frac{m_n^*}{m_n}\right)^2 \left(\frac{m_p^*}{m_p}\right)^2 \left(\frac{\rho}{\rho_0}\right)^{2/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}. \quad (16)$$

Below the superfluidity temperature for protons  $T_p$ , the emissivities are suppressed by a factor  $e^{-\Delta_p/kT}$  and below  $T_n$  by an extra factor  $e^{-\Delta_n/kT}$ . The basic reason why the neutrino producing reactions have the form (11)-(13) is that the single  $\beta$ -decay reactions are forbidden by momentum conservation so that an extra nucleon is needed to share the momentum. In the presence of a pion condensate or in a quark core, this is no longer the case. If pions condense in neutron stars, the pion induced  $\beta$ -decay reaction



initially suggested by Bahcall and Wolf (26) for free pions and later discussed by Maxwell et al (32) for pion condensates has a much larger phase space than reactions (11)-(13) and the corresponding emissivity (32)

$$\epsilon_{\pi}^{\nu}(T_9) = (6.9 \times 10^{25}) \frac{m_n^*}{m_n} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1} \quad , \quad (18)$$

is consequently also much larger. A very similar enhancement is found in the presence of quark matter when the quark-quark interactions are taken into account to first order in the quark-gluon coupling constant (21). The simple  $\beta$ -decay of the down quark into the up quark

$$d \rightarrow u + e^{-} + \bar{\nu}_e \quad (19)$$

can proceed and the corresponding neutrino emissivity is found to be (21)

$$\epsilon_{\text{quark}}^{\nu}(T_9) = (1.9 \times 10^{25}) \left( \frac{\rho}{\rho_0} \right) T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1} \quad . \quad (20)$$

The dominant contributions to the crust neutrino luminosity are the neutrino pair bremsstrahlung from electron-nucleus scattering

$$e^{-} + Z_A^N \rightarrow e^{-} + Z_A^N + \nu + \bar{\nu} \quad , \quad (21)$$

and the plasmon process

$$\Gamma \rightarrow \nu + \bar{\nu} \quad . \quad (22)$$

The neutrino emissivities for these two processes have been recalculated by Soyeur and Brown (33) in the Weinberg-Salam model. We have

$$\epsilon_B^{\nu}(T_9) = (6.0 \times 10^{19}) \left( \frac{\rho}{\rho_0} \right) T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1} \quad , \quad (23)$$

$$\epsilon_{p1}^{\nu}(T_9) = (8.3 \times 10^{14}) \left( \frac{\mu_{p1}}{kT} \right)^{\frac{15}{2}} e^{-\frac{\mu_{p1}}{kT}} T_9^9 \text{ erg cm}^{-3} \text{ s}^{-1} \quad , \quad (24)$$

in which  $\mu_{p1}$  is the plasma frequency derived from the electron chemical potential  $\mu_e$  by

$$\mu_{p1} = \sqrt{\frac{4\alpha}{3\pi}} \mu_e \quad . \quad (25)$$

Finally, the last contribution to the luminosity of neutron stars is the surface photon luminosity

$$L_{\gamma} = 4\pi \sigma R^2 T_e^4 \quad , \quad (26)$$

where  $R$  is the radius of the star and  $T_e$  the effective surface temperature (for the relation between  $T$  and  $T_e$  see refs.34 and 31).

For the opacity  $\kappa$  in the mantle, recent calculations (27,28,30) use the opacities from the LASL Astrophysical Opacity Library (35) and from Flowers and Itoh (36).

temperatures vary from  $\sim 10^6$  to  $\sim 3.2 \times 10^6$  K and for Vela expected to be  $\sim 20000$  years old, they vary from  $\sim 7 \times 10^5$  to  $\sim 2.5 \times 10^6$  K.

Finally, we should mention four supernovae for which temperature limits are available (40) : RCW86 (1796 years old), W28 ( $\sim 3400$  years old), G350.0-18 and G22.7-0.2 (both  $\sim 10^4$  years old). The upper limits for neutron star temperatures deduced from the observability limits are respectively  $T_e < 1.5 \times 10^6$  K for RCW86,  $T_e < 1.5 \times 10^6$  K for W28,  $T_e < 1.5 \times 10^6$  K for G350.0-18 and  $T_e < 1.8 \times 10^6$  K for G22.7-0.12. The calculated values for RCW86 range from  $\sim 0.9 \times 10^6$  to  $\sim 3 \times 10^6$  K and for W28 from  $\sim 0.8 \times 10^6$  to  $\sim 2.9 \times 10^6$  K. For G350.0-18 and G22.7-0.12, the calculated surface temperatures vary from  $\sim 7 \times 10^5$  to  $\sim 2.7 \times 10^6$  K.

We can conclude from the present data, that pion or quark cooling which would produce much colder neutron stars are at present not required by the observations which we recall do not identify the thermal radiation of pulsars.

One should keep in mind however the large uncertainties involved in the cooling calculations (31). The most serious problems are probably the treatment of the effects of the surface magnetic field and the choice of the parameters determining the superfluid phase transition for neutrons and protons such as their effective masses.

It is also not clear at all that the observed surface temperature upper limits can be interpreted simply in terms of the cooling processes described above. Various reheating mechanisms both of the crust and of the surface of neutron stars could produce surface temperatures in the range of the observed upper limits. Greenstein (44) suggested that a frictional heating between the superfluid and normal components of the rotating neutron star could result in surface temperatures of the order of  $10^6$  K for very long periods of time ( $\sim 10^6$  years) if the star is sufficiently massive. A second possible heating of the surface of pulsars might come from an important flux of  $\gamma$ -rays and relativistic particles on the magnetic polar-caps of the neutron star (see ref.40 and the references therein for a discussion of this point).

It is only by a consistent description of these various phenomena and a more extensive comparison of models with observational data that we might hope to have informations on the equation of state of neutron stars.

## REFERENCES

- (1) I.TUOHY and G.GARMIRE, *Astrophys.J.Lett.* 239 (1980) L107 ;  
P.C.GREGORY and G.C.FANLHAN, *Nature* 287 (1980) 805.
- (2) Recent references on this subject are for example K.NOMOTO, *Space Sci.Rev.* 27 (1980) 563 ; R.CANAL, J.ISERN and J.LABAY, *Space Sci. Rev.* 27 (1980) 595 ; see also E.P.J.VAN DEN HEUVEL, contribution to this Symposium.
- (3) G.E.BROWN, H.A.BETHE, G.BAYM, *Supernova Theory*, preprint (April 1981).
- (4) G.BAYM, *Stellar Thermal Expansion Effects on the Cooling of Neutron Stars*, University of Illinois preprint (1981).
- (5) W.D.ARNETT and R.L.BOWERS, *Neutron star structure. A Survey, A Report of the Nuclear and Astrophysics Group, The University of Texas at Austin, 1974.*
- (6) V.R.PANDHARIPANDE and R.A.SMITH, *Nucl.Phys.* A237 (1975) 507 ;  
V.R.PANDHARIPANDE, D.PINES and R.A.SMITH, *Astrophys.J.* 208 (1976) 550.
- (7) I.E.LAGARIS and V.R.PANDHARIPANDE, *Nucl.Phys.* A359 (1981) 331.
- (8) I.E.LAGARIS and V.R.PANDHARIPANDE, *Nucl.Phys.*A359 (1981) 349.
- (9) I.E.LAGARIS and V.R.PANDHARIPANDE, *Univ.of Illinois preprint ILL-(NU)-81-15, March 1981.*
- (10) B.FRIEDMAN and V.R.PANDHARIPANDE, *Nucl.Phys.* A361 (1981) 502.
- (11) J.W.NECELE and D.VAUTHERIN, *Nucl.Phys.* A207 (1973) 298.
- (12) G.BAYM, C.J.PETHICK and P.C.SUTHERLAND, *Astrophys.J.* 170 (1971) 229.
- (13) R.P.FEYNMAN, N.METROPOLIS and E.TELLER, *Phys.Rev.* 75 (1949) 1561.
- (14) H.A.BETHE and M.B.JOHNSON, *Nucl.Phys.* A230 (1974) 1.
- (15) G.BAYM, *Les Houches Summer School, Session XXX, 1977* (Ed. by R.BALIAN, M.RHO and G.RIPKA, 1978, North Holland) p.745 and references therein.
- (16) A.B.NICDAL, *Zh.Eksp.Theor.Fiz.* 61 (1971) 2209 ; *Sov.Phys.JETP* 34 (1972) 1184.
- (17) R.F.SAWYER and D.J.SCALAPINO, *Phys.Rev.D7* (1973) 953.
- (18) G.E.BROWN, S.-O.BÄCKMAN, E.OSET and W.WEISE, *Nucl.Phys.* A286 (1977) 91.
- (18bis) B.L.FRIMAN and O.V.MAXWELL, *Astrophys.J.* 232 (1979) 541.

- (19) W.WEISE, Comments Nucl.Part.Phys. 10 (1981) 109 and references therein ; G.E.BROWN and M.RHO, Nordita preprint -81/9 ; A.HARTING, W.WEISE, H.TOKI and A.RICHTER, preprint.
- (20) G.BAYM, Physica 96A (1979) 131.
- (21) N.IWAMOTO, Univ.of Illinois preprint (1980).
- (22) G.BAYM and C.PETHICK, Ann.Rev.Nucl.Sci. 25 (1975) 27 and references therein.
- (23) R.TAMAGAKI, T.TAKATSUKA and H.FRUKAWA, Prog.Theor.Phys. 64 (1980) 2107 ; T.TAKATSUKA and R.TAMAGAKI, Prog.Theor.Phys.(Lett.) 64 (1980) 2270.
- (24) G.FEINBERG, Phys.Rev. D23 (1981) 3075.
- (25) H.GEORGI and S.L.GLASHOW, Phys.Rev.Lett. 32 (1974) 438.
- (26) J.N.BAHCALL and R.A.WOLF, Phys.Rev. 140B (1965) 1452.
- (27) G.GLEN and P.G.SUTHERLAND, Astrophys.J. 239 (1980) 671.
- (28) K.A.VAN RIPER and D.Q.LAMB, Astrophys.J.Lett. 244 (1981) L13.
- (29) A.RAY, Nucl.Phys. A356 (1981) 523.
- (30) K.NOMOTO and S.TSURUTA, Astrophys.J.Lett. (1981) in press.
- (31) O.V.MAXWELL, Astrophys.J. 231 (1979) 201.
- (32) O.MAXWELL, G.E.BROWN, D.K.CAMPBELL, R.F.DASHEN and J.T.NANASSAH, Astrophys.J. 216 (1977) 77.
- (33) M.SOYEUR and G.E.BROWN, Nucl.Phys. A324 (1979) 464.
- (34) S.TSURUTA, Phys.Rep. 56 (1979) 237.
- (35) W.F.HUEBNER, A.L.MERTS, N.H.MAGEE and M.F.ARG0, Astrophysical Opacity Library, Report # UC-34b, 1977, Los Alamos Scientific Lab., unpublished.
- (36) E.G.FLOWERS and N.ITOH, Astrophys.J. 206 (1976) 218.
- (37) F.R.HARNDEN (private communication).
- (38) F.R.HARNDEN et al., Bull.Am.Astr.Soc. 11 (1979) 424.
- (39) S.S.MURAY et al., Astrophys.J.Lett. 234 (1979) L69.
- (40) D.J.HELFAHD, G.A.CHANAN and R.NOVIK, Nature 283 (1980) 337.
- (41) J.P.PYE et al., M.N.R.A.S. (1980) in press.
- (42) O.V.MAXWELL and M.SOYEUR, Contribution to the 8<sup>th</sup> Int. Conf.on High En.Phys.and Nucl.Structure, Vancouver, August 13-18, 1979.
- (43) D.HELFAHD, private communication.
- (44) G.GREENSTEIN, Astrophys.J. 200 (1975) 281.