TRN IL8202408

WIS-82/1/Jan-ph

Are Quarks and Leptons Composite?*

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Lectures delivered at the Canadian Summerschool of Theoretical Physics, Banff, Alberta, August 1981

*Supported in part by the US-Israel Binational Science Foundation, Grant No. 2425.

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1. Introduction and Outline

All of matter is made of atoms. Atoms are made of nuclei and electrons. Nuclei are made of hadrons. Hadrons are made of quarks. What are quarks (and leptons) made of?

This is the most naive and childish way of introducing our difficult subject. At present, we certainly do not know the constituents of the quarks and leptons. We do not even know if quarks and leptons are composite. However, it is clear that such a possibility exists and should be seriously studied.

These notes are devoted to the topic of composite quarks and leptons. Our discussion evolves along the following logical steps:

(i) We accept the standard model as a valid theory of quarks, leptons and their interactions.

(ii) We then discuss the various open problems which remain, even if the standard model is accepted. Each of these problems leads us to conclude that there is deeper physics beyond the standard model.

(iii) Several possible solutions exist for each of the open problems. Grand Unification, Technicolor, Horizontal Symmetries, Supersymmetry, Composite quarks and leptons, are some of the possibilities. Each has its pros and cons.

(iv) Among the different possible paths, we concentrate on the possibility of composite quarks and leptons. Even before we consider a specific model, a variety of difficulties and constraints become evident. A list of requirements for a realistic model should be prepared.

(v) Several general variations should be considered. Should the model be left-right symmetric? Can gauge bosons be composite? Do we allow new types of interactions? At this stage only prejudice and personal taste can guide us, as long as no experimental tests are available.

(vi) Given a selected list of requirements, the search for a simple realistic model can be launched. We find a simple model which has surprisingly many good features but needless to say, many difficulties.

(vii) Now that we have an explicit model, we should study it in detail. Even if the model is wrong, one learns a great deal from its properties. A list of successes and difficulties should be prepared.

(viii) Finally, it is important to distil from the discussion of the model, some ideas which may remain correct even if the specific model is not. Such ideas may be utilized in future attempts.

In the following sections we pursue the above program, starting with generalities and gradually concentrating our attention on the specific example of the rishon model.

2. The Standard Model

We start by constructing the standard model Lagrangian. The gauge group is $SU(3)_C \times SU(2) \times U(1)$. We assume three generations of quarks and leptons. The Lagrangian can be written in five easy steps:

(i) Write the kinetic terms for the quarks and leptons (no mass terms).

(ii) Add gluons. Write the gluon-quark couplings and the $F_{\mu\nu}F^{\mu\nu}$ term for the color interactions.

(iii) Add the photon, its couplings to charged fermions and its $F_{\mu\nu}F^{\mu\nu}$ term.

(iv) Add W^+, W^-, Z, their couplings to fermions and their $F_{\mu\nu}F^{\mu\nu}$ term.

By now we have a Lagrangian which is exactly gauge invariant under the local gauge group of $SU(3)_C \times SU(2) \times U(1)$. All fermions and bosons are massless. There are only three arbitrary parameters, representing the coupling constants of the three gauge groups.

(v) Add the scalar Higgs particles, their Yukawa couplings to fermions, their couplings to W and Z and their full Higgs potential. At this stage, we already have a large number of arbitrary parameters. When the symmetry is spontaneously broken, these parameters will determine the masses and mixing angles of the theory. The total number of free parameter is somewhere between 15 and 30, depending on whether neutrinos are massless, the Higgs sector, etc.

Experimentally, many parts of this Lagrangian are still untested. In particular, the W and Z have not been seen and no direct or indirect evidence for scalar particles is available.

However, we shall accept the standard model as valid. By this we mean that the model is, at worst, a good low-energy approximation of some deeper theory which will be revealed at higher energies.

There is plenty of room for modifications and extensions of the standard model, representing physics at higher energy scales. Before discussing these, we now turn to a few features of the model itself which may be useful later as a basis for analogies.

3. Chiral Symmetry and Its Breaking

The Lagrangian of the Standard Model possesses a chiral symmetry as long as no fermion mass terms are introduced. If we have N massless quark-flavours, we have an $SU(N)_L \times SU(N)_R \times U(1)$ chiral symmetry. Even when quark masses are generated by the usual Higgs mechanism, some of the quarks (at least u and d) remain approximately massless and we still have an approximate chiral $SU(2)_L \times SU(2)_R \times U(1)$ symmetry. When the quarks form composite hadrons, this chiral symmetry may or may not be broken, apriori. If it is not broken, all composite baryons must be massless (or appear as parity doublets). If the symmetry is broken, a massless Goldstone particle must appear and the composite baryons have no reason to be massless.

A complete dynamical understanding of QCD should enable us to calculate the forces and to decide whether a $q\bar{q}$ condensate forms, obtaining a vacuum expectation value and breaking the chiral symmetry. So far, no such understanding is available. We have arguments that, in the limit $N_C \to \infty$, the chiral symmetry must be broken[1]. We also have other plausibility arguments, but no real proof, leading to the same conclusion. On the other hand, we know what happens phenomenologically - the baryons do have masses and a Goldstone particle (the pion) exists. Hence, the approximate chiral $SU(2)_L xSU(2)_R$ symmetry is spontaneously broken by a $q\bar{q}$ condensate and the remaining unbroken symmetry is the "diagonal" isospin group SU(2).

Since we do not fully understand the breaking of the chiral symmetry in this case, we do not know how to generalize it to more complicated cases (more flavours; two different color groups; several types of fermions; etc.). The detailed pattern of chiral symmetry-breaking will become one of the central issues of any composite model of quarks and leptons.

4. Left-Right Symmetry and Parity Violation

The Lagrangian of the standard model preserves the local gauge symmetries of $SU(3)_C xSU(2)xU(1)$. Only one type of symmetry is explicitly broken: Parity (and consequently - charge conjugation). The classification of left- and right-handed quarks is not parity conserving and a variety of weak interaction terms breaks parity explicitly. It is not clear at all why two of the gauge interactions (color and electromagnetism) conserve Parity and remain unbroken while the weak interactions break Parity explicitly and end up with a spontaneously broken gauge symmetry.

In this context, it is interesting to remember that we may slightly extend the standard model into a left-right symmetric theory without introducing any fundamental new ideas and without spoiling the agreement with experiment. All we have to do is replace SU(2) by $SU(2)_L xSU(2)_R$. We now have a full $SU(3)_C xSU(2)_L xSU(2)_R xU(1)$ gauge group. The gauge couplings of $SU(2)_L$ and $SU(2)_R$ are assumed equal (because of the left-right symmetry). All left-handed quarks and leptons are in (½,0) doublets while their right-handed counterparts are in (0,½). We have six weak gauge bosons: $W_L^{\pm}, W_R^{\pm}, Z_1, Z_2$. The Lagrangian itself is completely Parity invariant. The spontaneous symmetry-breaking which gives masses to all W and Z bosons, provides W_R and W_L with different masses, breaking Parity. If W_R is sufficiently heavier than W_L, all low energy predictions of the standard model are reproduced. All corrections to the ordinary predictions vanish in the limit $M(W_R) \to \infty$. In practice, the additional W and Z bosons may be as light as a few hundred GeV, without spoiling the agreement with experiment. They may also, of course, be extremely heavy and we have no predictions for their masses.

The U(1) factor in the standard model has no simple physical meaning. It is , of course, different for left-handed and right-handed fermions of the same flavour. On the other hand, the U(1) factor in $SU(2)_L xSU(2)_R xU(1)$ is identical for the left- and right-handed components of a given fermion. It is proportional to B-L, the difference between baryon and lepton member.

The left-right symmetric version of the standard model is therefore a perfectly reasonable extension of the model, introducing no new gauge coupling-constants, three new vector bosons, an important new symmetry (Parity) and possible unknown complicationsin the Higgs sector.

5. Neutrino Masses

Neutrinos are massless (or almost massless). They are the only massless (or almost massless) fermions in the standard model. They are also the only neutral fermions. It would be nice to correlate the two facts.

Whether or not the masslessness of the neutrino has anything to do with its charge neutrality, there must be a symmetry principle which tells us why neutrinos are so much lighter than other fermions. Such a symmetry argument is required, regardless of whether the neutrino is exactly massless or only approximately massless.

No convincing argument has, so far, been given for an exactly massless neutrino. An attractive possibility was to consider it as a "Goldstone fermion" of a supersymmetric theory[2]. That suggestion can be ruled out,[2] and no other argument has been suggested.

However, a very attractive symmetry-argument can be given for an extremely light (but not massless) neutrino. The argument has the further benefit of explaining why only electrically neutral particles can have such small masses.

The only fermions which can have a Majorana mass-term are neutral leptons (Q=0 color singlets). These are the neutrinos. If a left-handed and a right-handed neutrino exist and if one of them obtains a large Majorana mass, a 2x2 mass matrix emerges, of the form:[3]

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

where m is an ordinary (Dirac) fermion-mass and M is a Majorana mass determined by an energy scale beyond the standard model. The eigenvalues of this mass matrix are $M; m^2/M$. Hence - we have a heavy right-handed neutrino and an extremely light left-handed neutrino[3], lighter than ordinary fermions by a factor m/M. In the case of the electron and its neutrino - if $m \sim$MeV, $M \sim$TeV, we obtain: $m(\nu_{eL}) \sim$eV.

The new mass scale M may come from a left-right symmetric theory ($M \sim M(W_R)$). It could, alternatively, come from a GUT scale or from a horizontal symmetry or from any other new phenomena at energies well above 100 GeV.

The above simple idea is, in our opinion, the most attractive explanation for the small mass of the neutrino which, at the same time, explains why all other fermion masses are at a different scale. We therefore believe that a light neutrino is, theoretically, preferrable to a massless neutrino. This, in turn, provides a further argument for considering left-right symmetric theories in which both left-handed and right-handed neutrinos must exist.

6. The Scalar Particles of the Standard Model

The scalar particles of the standard $SU(3)_C \times SU(2) \times U(1)$ model provide the quarks, leptons and weak bosons with their masses. Only SU(2) doublets can contribute to fermion masses. The success of the Weinberg mass relation ($M_W/M_Z = \cos\theta_W$) indicates that, to a good approximation, only SU(2) doublets contribute to the boson masses. The number of different scalar doublets is unknown. It may or may not be related to the number of generations. If a new quantum number which distinguishes among generations is found, different scalar fields possessing different eigenvalues of the new quantum number, may contribute to masses in different generations.

In the left-right symmetric extension of the standard model, the only scalars which contribute to fermion masses are in the $(\frac{1}{2},\frac{1}{2})_0$, where the subscript denotes the B-L value in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Scalars in the $(\frac{1}{2},\frac{1}{2})_0$ also contribute to the masses of W_L and W_R but cannot break the left-right symmetry. The minimal system of scalar particles which can do that[4] involves a $(1,0)_2 + (0,1)_2$ multiplet (and its conjugate). If only the neutral component in $(0,1)_2$ obtains a vacuum expectation value, we obtain $M(W_R) \neq M(W_L)$. W_L and Z_1 still satisfy the Weinberg mass relation since, under $SU(2)_L$, only scalar doublets contribute to their masses.

Thus, the Higgs sector of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ requires two classes of scalar fields to obtain vacuum expectation values: ϕ_i in $(\frac{1}{2},\frac{1}{2})_0$ and Δ_R in

$(0,1)_2$. In each class we may have one or several scalars. The ϕ-fields are entirely responsible for the fermion masses as well as for the W_L and Z_1 masses. The Δ_R-fields break parity, charge conjugation and B-L, contribute most of the W_R and Z_2 masses and induce a Majorana mass for right-handed neutrinos, leading to extremely light left-handed neutrinos, as explained in section 5. It is remarkable that one scalar field can do all of these things!

7. Residual Interactions and Fundamental Interactions

Hadronic forces are presently believed to be residual color forces, operating among colorless objects. The role played by the hadronic (or nuclear) forces is not anymore that of a fundamental force. They are treated on the same footing as Van-der-Waals forces, i.e. a complicated residue of a more fundamental force. In both cases we consider a composite system (Atom or Hadron) which is neutral under a fundamental interaction (electromagnetism or color) but contains constituents which are not neutral (nucleus and electrons or quarks). In both cases the residual force vanishes at long distances and it manifests itself as a complicated short-range interaction. The details of the distance dependence in the two cases are, of course, very different because of the different nature of the two gauge groups.

If there is further substructure inside quarks and leptons, we may face a situation in which one (or more) of the three fundamental forces (color, weak, electromagnetic) turns out to be a residue of a more fundamental force, under which quarks and leptons are neutral, but their constituents are not. It is difficult to consider all possible scenarios in such a speculative direction. However, we note that the weak interactions are the only remaining short range fundamental interaction (after the "demotion" of the strong hadronic force into the status of a residual force). Hence, if one of the three forces of the standard model is residual, the leading candidate is the weak force.

At this stage we must note that the SU(2)xU(1) theory does not really unify the weak and electromagnetic interactions. We still have two independent couplings. What it does is mix the electromagnetic current and the neutral weak current, within the framework of a beautiful self-consistent theory. However, they are mixed, not unified. It is perfectly possible that at a deeper level of the structure of matter one of them will remain as a fundamental interaction while the other will prove to be a residual force.

8. The Structure of One Generation

Each generation of quarks and leptons contains the following left-handed states, arranged in descending order of their electric charges:

	Color	$SU(2)_L \times SU(2)_R$	I_{3L}	I_{3R}	B-L	$Q=\frac{1}{2}(B-L)+(I_{3L}+I_{3R})$
e^+_L	1	(0, ½)	0	½	1	1
u_L	3	(½, 0)	½	0	$\frac{1}{3}$	$\frac{2}{3}$
$\bar{d}_L$	$\bar{3}$	(0, ½)	0	½	$-\frac{1}{3}$	$\frac{1}{3}$
ν_{eL}	1	(½, 0)	½	0	-1	0
$\bar{\nu}_{eL}$	1	(0, ½)	0	-½	1	0
d_L	3	(½, 0)	-½	0	$\frac{1}{3}$	$-\frac{1}{3}$
$\bar{u}_L$	$\bar{3}$	(0, ½)	0	-½	$-\frac{1}{3}$	$-\frac{2}{3}$
e^-_L	1	(½, 0)	-½	0	-1	-1

Table 1: Left-handed fermions of the first generation

An inspection of the table reveals a few features which cannot be explained within the standard model:

(i) The electric charges of the quarks and the leptons are quantized in a related way. Thus $Q(u) = -\frac{2}{3}Q(e^-)$ and the hydrogen atom is exactly neutral. This is not at all guarranteed if the SU(2) and U(1) gauge interactions are unrelated.

(ii) The sum of the electric charges of all left-handed fermions in (½,0) representations vanishes. This is the famous condition for the vanishing of the triangle anomaly in SU(2)xU(1) or $SU(2)_L xSU(2)_R xU(1)_{B-L}$. It is the only ingredient of the standard model that explicitly connects quarks and leptons and which tells us that a model with quarks and no leptons (or vice versa) is not renormalizable.

(iii) There are certain color-charge or color-(B-L) combinations which exist (and repeat themselves in higher generations). Other combinations do not exist. We have limitations such as $|Q|\leq 1$, $|B-L|\leq 1$ as well as surprising correlations. For instance, 3Q is identical to the color triality, although no relation between charge and color is implied by the model.

The above regularities cannot be accidental. They must be explained by some theoretical structure which goes beyond the standard model, either by embedding the three different gauge groups in a larger simple group or by constructing all quarks and leptons from more fundamental constituents.

9. The Generation Puzzle

A further question which cannot be settled within the standard model is the generation puzzle. We have three identical generations of quarks and leptons. The standard model does not contain any quantum number which distinguishes among the generations. Yet, we suspect that such a quantum number must exist. Three classes of solutions have been considered for a generation-labelling quantum number. In all cases we are looking for a symmetry which is already spontaneously broken at the stage of creating fermion masses. The existence of Cabibbo mixing tells us that any "generation number" cannot remain exactly conserved.

The three possibilities are:

(i) A discrete generation label. A discrete symmetry is introduced, such that each generation obtains a different eigenvalue under the symmetry operation. It is necessary that, say, e, μ and τ will have different eigenvalues. It is not necessary that e and u have the same eigenvalues, although it would be more elegant if they do. The scalar particles must have well-defined transformation properties under the discrete symmetry and the allowed Yukawa couplings are severely restricted by the symmetry. The mass matrix for analogous states in different generations contains matrix elements contributed by different scalar fields. If scalar fields with a non-vanishing generation number obtain vacuum expectation values, the discrete symmetry is broken and Cabibbo mixing is introduced.

As an example we may consider a discrete Z_n symmetry, under which e^o_L, μ^o_L and τ^o_L possess the quantum numbers X_e, X_μ, X_τ (where X is additively conserved mod(n)). Here e^o_L is a massless electron appearing in the standard model Lagrangian. If the discrete symmetry is vectorial, $X(e^o_L) = X(e^o_R)$ etc. If it is axial, $X(e^o_L) = -X(e^o_R)$ etc. In the first case a scalar field with X=0 can induce diagonal mass terms for e^o, μ^o and τ^o. The necessary X-values for scalar fields which contribute to mass-matrix elements are:

$$\begin{pmatrix} 0 & X_e - X_\mu & X_e - X_\tau \\ X_\mu - X_e & 0 & X_\mu - X_\tau \\ X_\tau - X_e & X_\tau - X_\mu & 0 \end{pmatrix}$$

On the other hand, if X is an axial quantum number, the three diagonal mass-matrix elements must be contributed by three different scalar fields. The necessary values are:

$$\begin{pmatrix} 2X_e & X_e+X_\mu & X_e+X_\tau \\ X_e+X_\mu & 2X_\mu & X_\mu+X_\tau \\ X_e+X_\tau & X_\mu+X_\tau & 2X_\tau \end{pmatrix}$$

In view of the different scales of the masses of different generations, we believe that the axial option is preferrable[5]. However, both possibilites should be considered.

The main disadvantage of the discrete symmetry idea is its artificial origin. Normally, one introduces the discrete symmetry arbitrarily for the sole purpose of "explaining" the generation structure.

The option of a discrete generation-label will become attractive only if the necessary symmetry is found naturally within some theoretical model which goes beyond the standard model.

(ii) A Continuous Global Symmetry. A variation on the same theme would be a continuous global symmetry under which each generation obtains a different eigenvalue. The entire discussion repeats itself, except for one new difficulty: If the continuous symmetry is spontaneously broken, an unwanted Goldstone boson appears. Here, again, the ad hoc nature of the symmetry is usually unattractive.

(iii) A Gauged Generation Label. A third possibility which avoids the dangerous Goldstone boson is to consider an extra "horizontal" gauge group under which different generations form a gauge multiplet. The simplest example is a U(1) gauge symmetry. The complications are: A severe anomaly constraint; the existence of a new gauge boson (or bosons); the danger of flavor changing neutral currents associated with "horizontal" gauge bosons. Here, again, the symmetry can be a vector symmetry or an axial symmetry, with the latter possibility preferred, as before[5].

In our opinion, the discrete and the gauge options have the best chances. The discrete symmetry is preferred, if it is discovered within the framework of a larger group or a new underlying theory. It is not attractive if it is concocted artificially.

In all cases, the generation puzzle can be solved only by physics outside the standard model.

10. Too Many Particles; Too Many Parameters

The standard model contains three arbitrary coupling constants of the three gauge groups and a large number of arbitrary parameters in the Higgs sector. We must produce six quark masses, three charged lepton masses, possibly three neutrino Dirac masses as well as Majorana masses, three Cabibbo angles and one phase for the quark sector, possibly a similar set for the leptons, a mass scale for W_L and possibly for W_R and an unknown number of additional parameters in the Higgs sector. The exact counting depends on detailed assumptions but it varies between 15 and 30 for most variations.

It is difficult to accept the notion that all of these parameters are arbitrary fundamental constants of nature, on equal footing with, say, the fine structure constant. A theory which goes beyond the standard model (in almost any direction) may enable us to correlate these parameters and reduce their degree of arbitrariness. An ideal situation would be to compute all parameters of the Higgs sector (and thus all fermion masses and mixing angles) from some underlying dynamical theory.

One should also add that, at the level of the standard model, we seem to have too many species of particles: six quarks and six leptons. A more fundamental theory may either reduce the number of building blocks or group them into fewer families. Both possibilities might be attractive. Both require physics beyond the standard model.

11. Elementary Scalar Particles?

Our final "complaint" against the standard model is perhaps the most important one. All previous problems (explaining the structure of a generation, a missing generation label, too many parameters) are either unanswered questions or matters of taste. The next problem is almost (not quite) a matter of self-consistency.

The problem is, of course, the existence of elementary scalar particles. It is, by now, widely known that the existence of such particles at low energies require extremely "fine tuning" of the parameters of the theory, or else they would acquire enormous masses. The same problem is sometimes referred to as the "hierarchy problem". *If one assumes a minimal* degree of "naturalness", such elementary scalars are unacceptable.

Two classes of solutions have been proposed, both going beyond the standard model. One is the direction of supersymmetry, where the scalar particles may be protected from acquiring a large mass by symmetry arguments. This is an attractive idea which is now being studied by many authors. So far, no realistic or even semi-realistic model has been proposed, but the approach is certainly promising.

The second solution is to suggest that the scalars in the standard model are fermion-antifermion condensates[6]. No elementary scalars exist. The normal fermions of the standard model cannot form the required condensates. Therefore, new fundamental fermions must be postulated, as the building blocks of the scalar condensates. This is the common basic idea of all technicolor schemes. The problem of scalars is solved, but new difficulties emerge. In particular, the number of particles and the number of arbitrary parameters grows very rapidly and the relative simplicity of the standard model is lost.

12. Developments Beyond the Standard Model

Having briefly introduced (in sections 8-11) several motivations for considering physics beyond the standard model, we now present a telegraphic review of some of the popular approaches to this problem:

(i) Left-Right Symmetric Model. This is a somewhat trivial extension of the standard model. It does not even address any of the questions of sections 8-11. It is a perfectly reasonable variation of the model. In the following sections, when we talk about "the standard model" we often refer to its left-right symmetric version. However, it really does not go very far beyond the model.

(ii) Grand Unified Theories. These provide a satisfactory answer to the questions of section 8. Both SU(5) and SO(10) fully explain the structure of one generation and account for all the points which we have raised in this context. The same models, of course, have the great advantage of reducing the three fundamental interactions into one, predicting the correct value of $\sin^2\theta_W$. However, they do not shed any light on the generation puzzle, offer no solution for the hierarchy problem and do not reduce the number of arbitrary parameters (although the number of particle species is greatly reduced). GUTs lead to definite unconfirmed predictions for proton decay, to an unpleasant "desert" of 13 orders of magnitude in energy and to a problem of magnetic monopoles.

(iii) Technicolor Schemes. These theories solve the problem of elementary scalars at the expense of introducing new particles and parameters. They, normally, have nothing to say about the structure within a generation or about the generation puzzle.

(iv) Combined GUTs with Technicolor. Such models could potentially combine the unification offered by GUTs with the solution of the scalar problem offered by Technicolor. Specific models[7] even provide for several generations in one large multiplet of an overall group unifying color, technicolor, electromagnetic and weak interactions. This is, in principle, an attractive approach which should be further studied. All explicit models suffer from theoretical or phenomenological difficulties, but the general direction should not be abandoned.

(v) Supersymmetry Schemes. No realistic supersymmetry scheme have been, so far, proposed. The hierarchy problem can presumably be solved in a supersymmetric theory. No wisdom on any of the issues of sections 8-10 has been gained from such models. However, the supersymmetry approach is far from being exhausted. It is extremely promising.

(vi) Horizontal Symmetries. These are concocted in order to "explain" the generation puzzle. They usually have little to say on any other issue. The only exceptions are certain horizontal gauge symmetries that are automatically obtained within Grand Unified Technicolor schemes (see (iv) above).

None of the above approaches has led to an acceptable theory which goes beyond the standard model and answers all the questions raised in sections 8-11. Each of these approaches may lead, in the future, to such a theory.

An alternative to the above approaches is the possibility that quarks and leptons are composite. Needless to say, no satisfactory composite model exists. However, the idea of compositeness is as attractive as any of the other approaches listed above. We devote the rest of these notes to a detailed discussion of this idea.

13. Composite Quarks and Leptons: Motivation and Hopes

If quarks and leptons are bound systems of some new fundamental fermions, we may hope to reach the following situation:

(i) The new underlying theory includes few species of fundamental fermions, interacting with each other through few types of fundamental interactions. The total number of parameters is presumably extremely small: several coupling constants and possibly (but not necessarily) a few mass parameters. All masses of the composite quarks and leptons should, in principle, be calculable from the parameters of the fundamental theory, in the same way that all hadronic masses and coupling constants are, in principle, calculable from the QCD coupling and a few quark masses.

(ii) The pattern of quarks and leptons within one generation should be fully explained in terms of the features of the fundamental fermions. For instance, if both quarks and leptons are composites of the same set of fundamental fermions, their charge quantization must clearly be related. The peculiar relation between electric charge and color-triality may simply emerge from the color triality of one charged fundamental fermion. The mysterious vanishing of the sum of electric charges in one generation may look simple when translated into the charges of the constituents. The restrictions on $|Q|$ and $|B-L|$ may be related to the number of constituents within a composite quark or lepton, in the same way that the limitations on the Strangeness or Isospin of Hadrons follow from the number of valence quarks in a Hadron.

(iii) The different generations may be excitations of a composite system, similar to excited hadrons, nuclei or atoms. The type of excitation in each case must be different, however.

(iv) The scalar particles, as well as the quarks and leptons, are presumably composites of the new fundamental fermions. No fundamental scalar particles are necessary. The fundamental fermions may be massless or may have explicit mass terms, but need not gain masses through symmetry breaking. The problem of fine tuning may thus be avoided.

(v) Other features are left open. Color, Electromagnetism and the weak interactions may all exist in the underlying theory. Alternatively, one or more of these interactions may turn out to be a residual force. Additional color-like or other forces may be needed in order to bind the new fermions inside the quarks and the leptons. The underlying theory may be left-right symmetric, with Parity being spontaneously broken at the composite level. Alternatively, the fundamental theory may already include explicit parity violation.

It is not at all clear that a composite model with all the above desired features can be constructed, but it is certainly worth exploring. At present, there is no direct experimental evidence of any kind, for the compositeness of quarks and leptons. Any explicit model should also allow us to confront it with experiment, in the near or distant future.

14. Composite Quark and Leptons: The Scale Problem

All present experiments are consistent with a "pointlike" behavior of quarks and leptons. Different experiments lead to different limits with "effective cutoffs" between 10 and 200 GeV. Hence, the pointlike nature is guaranteed at least down to 10^{-15} cm, probably down to 10^{-16} cm. Any composite structure must be confined to these distances. It may, of course, occur on a much smaller distance scale or, equivalently, a higher energy scale. The masses of all observed quarks and leptons are much smaller than the present lower bound on the "compositeness scale" Λ, where $\Lambda \sim 1/r$ and r is an "effective radius" of a quark or a lepton. Thus, if quarks and leptons are composite systems, their Compton wave length is substantially larger than their effective radius. This is an unusual situation, unequalled by any previously studied composite system. In the case of Atoms, $Mr \sim M/\Lambda >> 1$. For hadrons $Mr \sim M/\Lambda \sim 1$. For composite quarks and leptons $Mr \sim M/\Lambda << 1$.

Several qualitative features immediately emerge from the mismatch of scales:

(i) If the scale of the fundamental theory is above 100 GeV (possibly many orders of magnitude above it), only a miracle or a symmetry principle can force the composite fermions to be almost massless on that scale. In contrast, hadronic masses are of the same order of magnitude as the scale of QCD. Atomic masses are determined by the masses of their constituents and are much larger than their inverse radii.

(ii) Radial and orbital excitations in Atoms, Nuclei and Hadrons are given by energy differences of order $\Lambda \sim 1/r$. Similarly, even if we have composite quarks and leptons with $M << 1/r$, their radial and orbital excitations are probably of mass $M \sim 1/r$. Such excitations are not very interesting. In particular, they cannot account for the higher generations which are also well below the compositeness scale. On the other hand, pair excitations are usually governed by the masses of the constituents and not by the physical radius of the system. Hence, in Atoms and Nuclei such excitations are usually irrelevant. In Hadrons they are comparable to (and sometimes indistinguishable from) orbital excitations. In the case of composite quarks and leptons, pair excitations are likely to be the lowest excitations if the fundamental fermions are massless or have small masses. In particular, higher generations may be related to pair excitations.

(iii) If the compositeness scale Λ is very large, not only the quarks and leptons but all particles appearing in the low energy world must be approximately massless in comparison with Λ. In each case, barring miracles, a symmetry principle should be involved. For composite fermions, it may be a chiral symmetry. For vector particles - a gauge symmetry. For scalar particles - there are several alternatives, each with its own problems. In all cases, these symmetry requirements are the most difficult and the most crucial aspect of any composite scheme.

(iv) If all particles are either at masses of order Λ or at masses which are negligible with respect to Λ, an effective low-energy Lagrangian must be very similar to the standard model Lagrangian. It should be approximately gauge invariant and approximately renormalizable (except for terms proportional to Λ^{-n} for positive n). The effective Lagrangian should, in principle, be derivable from the fundamental Lagrangian which describes the physics at energies of order Λ and which includes all the fundamental fields but no composite particles.

(v) If Λ is sufficiently large, experimental probes of the composite nature of quarks, leptons, scalars (and, possibly, some vectors) may be extremely difficult and indirect. We will discuss them separately in section 15.

(vi) All intuitive arguments are highly suspicious in the unusual situation of $M \ll 1/r$. Any mechanism which keeps the composites so light must lead to subtle cancellations between large quantities. This by itself is perfectly "natural" if it is justified by a convincing symmetry argument. However, the subtlety of the cancellation often prevents us from using simple-minded arguments about "similar" composite systems and their properties. We will see a few examples of this, as we proceed.

15. Composite Quarks and Leptons: Experimental Bounds

The following phenomena are not induced by the simplest Lagrangian of the standard model:

Proton decay, neutron oscillations, neutrino masses, neutrino oscillations,

$\mu\to e\gamma$, $\mu N\to eN$, $K\to e\mu$, strangeness changing neutral currents, V+A weak currents. unconventional contributions to $(g-2)_e$ and $(g-2)_\mu$. None of the above have definitely been observed. There are, of course, numerous additional processes and phenomena which, if observed, would signal physics beyond the standard model. However, the above list probably contains the most likely candidates for an experimental breakthrough into some new physics. The present experimental limits for each of these processes provide constraints for any given composite model. However, the constraint imposed by each process must depend on the details of the model. We do not attempt here a complete discussion of these constraints. A few examples will suffice:

(i) If quarks and leptons are composites of the same fundamental fermions, processes such as $u+u\to\bar{d}+e^+$ may proceed by a simple rearrangement of the constituents. Such a process is allowed by the conservation laws of $SU(3)_C\times SU(2)\times U(1)$. If no additional selection rule of the underlying theory prevents it, this reaction will be described by an effective four-fermion term in the low energy Lagrangian. Such a term is of dimension six and the corresponding amplitude will be of order Λ^{-2} where Λ is the high energy scale of the model. Thus, if no new selection rule intervenes, the proton lifetime in such a composite model will be proportional to Λ^4 leading to the bound $\Lambda\gtrsim 10^{15}$ GeV. However, it is conceivable that the internal structure of quarks and leptons forbids the effective four-fermion term, and higher terms are necessary in order to induce proton decay. In such a case Λ may be much smaller. Proton decay may even be forbidden in some composite models.

(ii) The anomalous magnetic moment of the electron and the muon is predicted with amazing accuracy by QED (including "conventional" weak and strong corrections). Any internal lepton structure must lead to deviations from the predicted values of g-2. If all new phenomena beyond the standard model begin at a scale Λ, the new corrections to g-2 are expected to be of order m/Λ or m^2/Λ^2 depending on whether the theory exhibits a chiral symmetry[8]. In the case of a chiral symmetry, the quadratic expression is valid, leading to bounds of the order of $\Lambda \gtrsim 500$ GeV. If the linear term is allowed, Λ must be much larger: $\Lambda \gtrsim 10^6$ GeV. The weaker chiral bound is relatively model independent. It is very hard to imagine a compositeness scale below 500 GeV without noticeable changes in g-2.

(iii) The K_S-K_L mass difference is an extremely small and well-measured quantity. It is approximately accounted for by the "classical" box diagram describing $\bar{d}s \to \bar{s}d$ with W^+ and W^- exchange and an internal u and c quark contributions[9]. It is reasonable to assume that no other diagram is allowed to contribute, to this process, a contribution which is larger than that of the "classical" diagram. This requirement leads to bounds[10] on the Cabibbo angles involving the t-quark, on possible right-handed currents involving c-quarks, on the mass of W_R,[11] on possible masses and couplings of horizontal gauge bosons[12] and on any mechanism which yields strangeness changing neutral transitions. The limits imposed by this mass difference are model dependent, but they have already ruled out a large number of models. Whenever we get a bound from this process, it is usually a nontrivial bound because of the smallness of the measured mass difference.

16. Massless Composite Fermions

We have already explained why composite quarks and leptons must be approximately massless with respect to their compositeness scale Λ. Such masslessness must emerge from a symmetry principle. The simplest symmetry which may prevent a fermion from acquiring a mass is a chiral symmetry. Thus we must look for a composite model with a chiral symmetry.

The chiral symmetry is essentially automatic if the fundamental fermions appearing in the underlying Lagrangian are massless. However, the existence of a chiral symmetry in the underlying level does not necessarily guarantee its preservation at the composite level. The chiral symmetry may be broken spontaneously, leaving no reason for massless fermions at the composite level.

Thus the necessary logical sequence of assumptions is as follows:

(i) The *fundamental* Lagrangian contains massless fermions and therefore possesses a chiral symmetry.

(ii) The full chiral symmetry or, at least, a chiral subgroup remains unbroken at the composite level.

(iii) The chiral symmetry of the effective Lagrangian containing the composite fermions, prevents the latter from gaining a mass. We have composite massless fermions.

Three questions immediately arise:

(a) If the new fundamental fermions are massless, why don't we observe them?

(b) What is the interaction which binds the fundamental fermions inside the composite quarks and leptons?

(c) If both fundamental and composite fermions are massless, what provides us with the necessary "compositeness scale" Λ?

All three questions can be immediately answered by one postulate, if we assume a new color-like force (called "hypercolor") with a scale parameter Λ_H. All fundamental fermions carry hypercolor. They are confined by hypercolor forces of characteristic scale Λ_H into hypercolor-singlet composite fermions with an effective radius $r \sim \Lambda_H^{-1}$. The confined fundamental fermions cannot be experimentally observed. The binding and the scale are provided by the hypercolor gauge force.

The above scenario is an attractive framework for the construction of a composite model[13]. However, it is crucial that the chiral symmetry or at least a chiral subgroup must remain unbroken at the composite level. This is not apriori impossible but it differs from the observed pattern of chiral symmetry breaking in QCD. There (see section 3), the chiral SU(2)xSU(2) symmetry is spontaneously broken into a pure vector SU(2) symmetry, and no composite massless fermions emerge. The hypercolor situation must, for some reason, be different!

1". Anomaly Contraints

Let us assume that we have constructed a composite model of quarks and leptons based on an $SU(N)_H$ hypercolor gauge group and containing K fundamental massless fermions, all assigned to the N-dimensional representation of $SU(N)_H$. The underlying Lagrangian automatically possesses a global $SU(K)_L xSU(K)_R xU(1)$ symmetry. The U(1) factor is a vector charge counting the number of fermions. An additional axial U(1) factor is broken by instanton terms.

The model contains "flavor" triangle anomalies corresponding to products of three SU(K) currents or to products of two SU(K) currents and the U(1) current. Such anomalies are perfectly legitimate, since the SU(K)xSU(K)xU(1) symmetry is not gauged. However, in the zero momentum limit, a given anomalous term can be exactly calculated both from the underlying theory and from the low-energy effective theory containing the composite particles. The results must be the same, thus imposing a severe constraint on the spectrum of composite particles.

If, in the underlying level, the anomaly does not vanish, the theory must produce massless composite particles[13,14]. We may consider three logical possibilities:

(i) The chiral symmetry is not broken at all. There are composite massless fermions. Their contribution to each anomaly must be exactly equal to that of the fundamental massless fermions. Thus a severe constraint is imposed, connecting the fundamental fermions to the composite fermions. This is the famous 't Hooft condition[13].

(ii) The chiral symmetry is completely broken. No chiral subsymmetry remains. The only massless composite particles are Goldstone bosons. Their contribution to the anomaly is equal to that of the fundamental fermions. However, since the Goldstone bosons have unknown couplings, the anomaly constraint can only be used in order to compute these couplings, leading to equations similar to the Goldberger-Trieman relations.

(iii) The chiral symmetry is broken, but a chiral subgroup remains conserved. The subgroup may be continuous or discrete. In this case, massless Goldstone bosons must exist but massless fermions may also exist. The combined contributions of the massless composite bosons and fermions must balance the anomaly of the underlying theory.

The anomlay constraint is particularly powerful in case (i). It is not very useful in case (ii), but we are interested in massless composite fermions, and they do not occur in that case. Case (iii) allows composite massless fermions, and the anomaly constraint is somewhat less powerful.

18. Model Building: Theoretical Framework and Requirements

We are now in a position to establish a theoretical framework for a composite model and to present a general list of requirements which such a model must satisfy. We hasten to add that in defining this framework we have already restricted ourselves in many ways. A variety of other models might be constructed and it may turn out that we are on the wrong track. We briefly discuss some of the alternatives which we discard, in the next section.

The framework we choose is the following: We assume a fundamental Lagrangian including only massless fermions and gauge bosons. There are no fundamental scalars. The gauge group is a hypercolor $SU(N)_H$. Additional gauge groups for color, electromagnetic, weak and other interactions may also be present. All fundamental massless fermions carry nontrivial hypercolor. The Lagrangian automatically possesses a chiral symmetry.

At energies below the hypercolor scale Λ_H, only hypercolor singlets are allowed as "free" particles. An effective low-energy Lagrangian describes physics well below Λ_H. It includes composite hypercolor-singlet particles as well as some of the original gauge bosons (except the hypergluons). Some of the composite hypercolor-singlet fermions are massless because of a leftover chiral symmetry which is not spontaneously broken. This may be the original chiral symmetry of the underlying Lagrangian or a chiral subgroup. The effective low-energy Lagrangian must be very similar to the standard model Lagrangian. It should be approximately gauge invariant under $SU(3)_C$x $SU(2)$x$U(1)$ or $SU(3)_C$x$SU(2)_L$x$SU(2)_R$x$U(1)_{B-L}$. Only terms proportional to Λ_H^{-n} (n positive) may violate these symmetries. If Λ_H is sufficiently large and all particles in the effective Lagrangian are sufficiently light, the latter should also be renormalizable[15] (except for Λ_H^{-n} terms). This is the famous Veltman "theorem".

The following requirements should be imposed on the model:

(i) It should be simple and economic. After all, one of our main motivations was the proliferation of particle species and of parameters. We do not want to reintroduce them at the underlying level (although we may end up doing so).

(ii) The model should be realistic. It should reproduce, with a few reasonable assumptions, the observed spectrum of quarks and leptons. The most beautiful and self-consistent model is worthless if it predicts fractional-charge leptons and only color-sextet quarks. Unfortunately, the literature is too full with such schemes which obey one constraint or another but bear no resemblance to the real world.

(iii) The model should be self-consistent in all respects, but in particular - it should obey the anomaly constraints discussed in section 17, in one of the three possible ways mentioned there. This is a nontrivial requirement for a realistic model.

(iv) The model should survive experimental tests concerning the processes discussed in section 15. In particular, predictions for proton decay and $\sin^2\theta_W$ should be studied, and (g-2) should remain unperturbed within the present accuracy. New clear experimental tests would, of course, be welcome, but here we may be asking too much.

(v) Since the pattern of chiral symmetry-breaking is so crucial, a complete dynamical description and understanding of it is necessary. Here we are asking a great deal, because the same problem in QCD is not yet solved. However, if a composite model for quarks and leptons is to survive, the chiral problem must be understood.

(vi) A satisfactory generation-labelling scheme must be naturally found within the quantum numbers of the model, providing for identical generations which are distinguished by a spontaneously broken new quantum number.

In order to avoid confusion let us clearly state at this stage that no existing model comes close to fulfilling all of these requirements within the above framework. However, that should not stop us from continuing our investigation and from studying various schemes.

19. Model Building: Alternatives Which We Do Not Pursue

Now that we have chosen a specific theoretical framework (section 18) we should mention some interesting alternatives which we will not discuss any further in these notes:

(i) The fundamental fermions may actually be extremely heavy and not massless. It is hard to see how they can "naturally" be bound into massless composite fermions, but we cannot completely rule out such a possibility.

(ii) The basic binding force may not be colorlike. Various authors suggested magnetic forces, gravity and other ideas. It is entirely possible that something completely new and wonderful will do the job. The hypercolor assumption, is of course, the most conservative and least original assumption we could have made.

(iii) We have ruled out fundamental scalars. However, this cannot be an absolute ban. There could be models with scalars appearing at a high energy scale without (or only with a minor) "fine tuning" problem.

(iv) Supersymmetry offers another possible reason for massless composite fermions (besides chiral symmetry). They could be the Goldstone fermions of a broken supersymmetry. Attempts in this direction[16] have not been very successful, but we should keep trying.

(v) Finally, one should always remember the possibility that at very short distances the basic rules are changed. We may face the breakdown of various principles of quantum field theory in a totally unexpected way. The trouble with such a suggestion is that, having stated it, there is little else we can do with it.

Our line of analysis is to ignore the above interesting possibilities and to proceed within the framework of section 18.

20. The Chiral Symmetry Dilemma

We have assumed that the composite quarks and leptons are kept massless (or approximately massless) by a chiral symmetry. Hence, at least some part of the chiral symmetry of the underlying Lagrangian must remain unbroken at the composite level. We face a dilemma which stems from the following statements:

(i) We believe that in two-flavor massless QCD, the chiral symmetry is completely broken. No chiral subgroup remains intact. Three-flavor massless QCD probably behaves similarly.

(ii) If we neglect all interactions except hypercolor (all other interactions are probably much weaker at the Λ_H-scale), a hypercolor model with K fundamental massless fermions is isomorphic to K-flavor massless QCD.

(iii) If the theoretical framework of section 18 is valid, some chiral symmetry should remain unbroken in the hypercolor case.

(iv) In no case do we have a full dynamical understanding of chiral symmetry and its breaking.

It is hard to reconcile statements (i),(ii),(iii), but no negative proof can be given. In that respect, statement (iv) is crucial!

What are the logical possibilities?

(a) A reasonable attitude, advocated by many prominent theorists, is simply to declare that (i),(ii) and (iii) are inconsistent, hence - our theoretical framework is unacceptable and one should not continue to pursue our discussion beyond this point. Perhaps this is true. Perhaps not.

(b) A clever way out is to consider a composite model in which left- and right-handed fermions have different transformation properties under the gauge group. Such a model is not isomorphic to QCD and statement (ii) does not apply to it. In such a model an $f\bar{f}$ condensate cannot break the chiral symmetry without breaking the original gauge symmetry. Two options are open: Either there is no condensation or the gauge symmetry breaks itself into a smaller subgroup. The first possibility has been studied by various authors and no realistic model was found[17]. The second possibility is the interesting "tumbling" approach[18]. Here, again, no realistic model was found. However, the left-right asymmetric classification may still be the correct solution. We will not study it any further in these notes, but it is an important direction of research.

(c) It is possible that the pattern of chiral symmetry-breaking depends on the number of flavors K. This could happen at least in two ways. There may be a phase transition at some K-value, $K > 3$, leading to a different pattern for QCD and for a hypercolor theory with $K > 3$ flavors of fundamental fermions. It is also possible that the general $SU(K)_L x SU(K)_R$ chiral symmetry always breaks, leaving a small conserved chiral subgroup which is trivial for $K = 2$ but is nontrivial for large K. An example could be a discrete Z_K chiral group. A chiral Z_2 cannot protect any fermion from acquiring a mass. A chiral Z_4 or Z_6 can do it. There is no dynamical reason to expect any of these speculations to be true, but there are no complete arguments against them.

(d) Another possible speculation is that the presence of the color or electroweak interactions somehow influences the pattern of chiral symmetry breaking in a hypercolor scheme. This is the most obvious difference between the hypercolor case and QCD. The simplest attitude would be to treat color and electroweak interactions as minor perturbations which cannot change anything. However, subtle effects may occur. For instance, imagine a situation in which the chiral symmetry can break via $f\bar{f}$ or $ff\bar{f}\bar{f}$ condensates, the potential having two similar minima. A small perturbation could conceivably change the balance between the two minima, making the $ff\bar{f}\bar{f}$ condensate the likely one. At this point we may also add that the usual $N_C \to \infty$ argument[1] for the breaking of chiral symmetry in QCD does not necessarily remain valid if N_C/N_f is held fixed. In some composite models, such a fixed ratio may be a necessary requirement.

The above discussion can be summarized very simply: One can speculate about scenarios which provide the required pattern of chiral symmetry breaking- for a composite model. All such scenarios are not supported by any decent dynamical arguments, but they cannot be ruled out. The alternative is to abandon ship.

We continue by simply assuming that somehow the hypercolor dynamics will leave some chiral symmetry unbroken.

21. Composite Gauge Bosons?

Within the framework described in section 18 we must face at least four types of gauge bosons: Hypergluons, gluons, photon, weak bosons. Which of these bosons must be elementary? Can some of them be composite?

There is a certain confusion in discussing the possibility of composite gauge bosons. There are theories in which a certain fermion-antifermion pair of fields may appear to have some or all the properties of a gauge boson[(19)]. In some sense this is a composite gauge boson, but it appears in the same fundamental Lagrangian with the fermion fields and all other fields of the theory. Such a possibility is very interesting but it is not related to our discussion here.

A different concept of a composite gauge boson is this: It does not appear at all in the fundamental Lagrangian of the theory (in the same way that other composite objects do not appear there). It does appear in an effective Lagrangian together with all other composite particles. Here we would like to study whether some of our gauge bosons may appear as composites in this sense.

The hypergluon must clearly appear as a fundamental massless gauge field in the underlying Lagrangian. In fact, it will not appear at all in the low-energy Lagrangian. What about the gluon, photon and weak bosons? Consider first the massless gauge bosons (gluon and photon). If an exactly massless gauge boson appears in the low-energy Lagrangian, the Lagrangian must be exactly gauge invariant under the corresponding gauge group. This gauge invariance cannot be broken by higher dimension terms which are proportional to Λ_H^{-n} (n positive). If no small corrections of any kind are allowed to break the exact gauge invariance of the effective Lagrangian, it is essentially unavoidable that the original underlying Lagrangian also possesses the same local gauge symmetry. But in that case, it must contain the corresponding massless gauge bosons as fundamental fields. We therefore claim that the gluon and the photon are not composite. They have the same status as the hypergluon in the underlying Lagrangian which must now be gauge invariant at least under $SU(N)_H x SU(3)_C x U(1)_{EM}$.

The above argument does not necessarily apply to the massive W and Z weak bosons. The weak gauge symmetry of the effective Lagrangian could be an approximate symmetry, broken by higher dimension terms which vanish as $\Lambda_H \to \infty$. It is conceivable that this approximate gauge symmetry is not fully present at the underlying level. In fact, the longitudinal components of W and Z are "born" from the scalar fields which are now formed as condensates of the fundamental fermions. In some sense, at least the longitudinal W and Z must be composite in such a scheme.

The possibility of composite W and Z which do not appear in the underlying Lagrangian is extremely interesting. It leads to exciting consequences but also to serious difficulties. First, the difficulty:

If $\Lambda_H >> M_W, M_Z$, the composite W and Z are approximately massless on the compositeness scale. No chiral symmetry will help us here. There must be another symmetry reason for such a situation. It is true that a renormalizable gauge theory will guarantee massless gauge bosons (except for spontaneous breaking) and vice versa but this is a circular argument. Starting from a hypercolor scheme with fundamental fermions we have no good symmetry-argument or dynamical argument for a light composite W and Z.

On the other hand, if W and Z are composite, the weak interactions become short range residual hypercolor forces and are not a fundamental interaction of nature (see section 7). All fundamental gauge symmetries remain exactly conserved. All other symmetries are spontaneously broken. There is a complete one-to-one correspondence between the gauge symmetries of the fundamental Lagrangian and the exact symmetries of the universe. Such a possibility is extremely attractive, in our opinion.

Finally, if the weak gauge symmetry is not present at the underlying level, the composite model may be much more economic, both in terms of its gauge group and in terms of its fundamental fermions. However, the symmetries of the basic Lagrangian must somehow allow the full electroweak symmetry to emerge in the composite level, a nontrivial demand!

We summarize: all massless gauge bosons are declared elementary. We are willing to consider the possibility of composite W and Z, while being aware of the difficulty in justifying their small mass.

22. A Search for a Minimal Scheme

We are now ready to look for the most economic scheme which obeys the framework of section 18 and is capable of producing at least the spectrum of one generation. In the spirit of the previous section we do not insist on having the full electroweak gauge group in the fundamental Lagrangian.

The minimal gauge group is, of course, $SU(N)_H \times SU(3)_C \times U(1)_{EM}$. The simplest assignment for the fundamental fermions is to the N-dimensional representation of $SU(N)_H$ (both left-handed and right-handed). Hence, a composite hypercolor fermion can only be constructed from N fundamental fermions (N odd). The most economic odd N is, of course, N=3. However, a brief inspection of the table in section 8 discloses a much stronger reason for choosing N=3. It is clear from the table that the smallest needed values of Q and B-L are 1/3 and, at the same time, $|Q|\leq 1$, $|B-L|\leq 1$. If all fundamental fermions have $|Q|=1/3,0$ and $|B-L|=1/3$ all necessary values of Q and B-L (and only these values!) are easily obtained from three constituents. We therefore select N=3 and the full gauge symmetry[20] is $SU(3)_H xSU(3)_C xU(1)_{EM}$.

We next turn to the electric charge. It is clear that fermions with Q=1/3 and Q=0 are necessary. Hence, the most economic model will include two species: a charged fermion T and a neutral fermion V. We call them rishons[21] (=primary, in Hebrew). T and V stand for "Tohu Vavohu" or "formless and void", which were the building blocks of the early universe according to the book of Genesis (Chapter 1, verse 2).

Having determined the hypercolor and charge assignments, all we have to do is to decide on the color. The T and V rishons cannot have the same color. If they do, all hypercolor-singlets would have the same color-triality and we will not be able to form hypercolor-singlet quarks and leptons. In section 8 we have already mentioned the mysterious correlation between the color and charge of all quarks and leptons: 3Q equals the color triality. Hence, a "step" of Q=1/3 must change the color triality by one unit. If we now assign T and V to any pair of different color trialities, the correct color-charge relation is guaranteed, as long as three rishons make a quark or a lepton. The simplest representations corresponding to the three color trialities are $1,3,\bar{3}$. The color assignments of T and V can therefore be, respectively: (3 and $\bar{3}$) or (1 and 3) or ($\bar{3}$ and 1).

All three possibilities should be studied. In all cases the Q=+1 positron is given by TTT and the Q=0 neutrino is VVV. If we wish the two "wave-functions" to have the same pattern and symmetry we must choose T in 3 and V in $\bar{3}$.

Otherwise, TTT will be color antisymmetric and VVV will be color symmetric, leading to completely different space-time structure because of Fermi statistics.

The above argument (which is not absolutely compelling) leads us to our final choice of rishon assignments[20]:

$$T(3,3)_{1/3}$$

$$V(3,\bar{3})_{0}$$

where the two figures in parenthesis stand for hypercolor and color, respectively, and the subscript gives the electric charge.

The fundamental Lagrangian can now be written. The only exact symmetries of the universe are the three gauge symmetries. Having determined the gauge group and the fermion classification, everything should, in principle, follow.

Anyone who tries to construct all of matter from two species of fundamental building blocks (even if each of them appears in nine color-hypercolor combinations) should not be surprised to find insurmountable difficulties. However, it is surprising how much one can achieve with such a simple scheme, before the difficulties take over.

23. The Rishon Model Lagrangian and its Symmetries

Before we write our Lagrangian explicitly, we must make a list of minimal demands concerning the necessary required symmetries of a realistic model, within our agreed theoretical framework. We must have the following symmetry operations:

(i) Hypercolor, color and electromagnetism. These are exact gauge symmetries and they are guaranteed in our model by construction.

(ii) Additional additive quantum numbers: Since our model is left-right symmetric, it is likely to require an $SU(2)_L xSU(2)_R xU(1)$ symmetry and not only $SU(2)xSU(1)$. Inside $SU(2)_L xSU(2)_R xU(1)_{B-L}$ there are three additive quantum numbers. They can be chosen as I_{3L}, I_{3R} and B-L. Alternatively, we can choose two vector charges $[(I_{3L}+I_{3R})$ and $(B-L)]$ and one axial charge $(I_{3L}-I_{3R})$. The electric charge is a linear combination of the two vector charges: $Q = (I_{3L}+I_{3R})+\frac{1}{2}(B-L)$. Hence, in addition to electric charge we need two additive quantum numbers - one vector and one axial.

(iii) The above additive quantum numbers are only the diagonal operators of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In addition, we must find, only in the composite level, the full electroweak group, with all quarks and leptons forming $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$ doublets.

(iv) We also need a generation label which should already be present somewhere in the model. The most economic possibility is a continuous U(1) symmetry or a discrete Z_n group. In both cases, an axial group is preferred (see section 9).

We summarize: In addition to the original gauge group we need in the fundamental Lagrangian a vector charge, an axial charge and a (hopefully axial) generation number. The full $SU(2)_L \times SU(2)_R$ should emerge in the composite level.

We now write the full Lagrangian and study its additional ungauged symmetries, in order to see whether they agree with the above requirements.

The underlying Lagrangian of the rishon model is:[22]

$$
\begin{aligned}
L = \bar{T}_{jj'}(i\delta^j_k\delta^{j'}_{k'} \cdot \not\partial + g_H\delta^j_k(\lambda^a)^{j'}_{k'}\not{A}^a_H + g_C\delta^{j'}_{k'}(\lambda^a)^j_k\not{A}^a_C + \\
+ \tfrac{1}{3}e\delta^j_k\delta^{j'}_{k'}\not{A})T^{kk'} + \bar{V}^j_{j'}(i\delta^k_j\delta^{j'}_{k'}\not\partial + g_H\delta^k_j(\lambda^a)^{j'}_{k'}\not{A}^a_H + \\
+ g_C\delta^{j'}_{k'}(\lambda^a)^k_j\not{A}^a_C)V^{k'}_k - \tfrac{1}{4}(F_{EM})_{\mu\nu}(F_{EM})^{\mu\nu} - \tfrac{1}{4}(F^a_C)_{\mu\nu}(F^a_C)^{\mu\nu} - \\
- \tfrac{1}{4}(F^a_H)_{\mu\nu}(F^a_H)^{\mu\nu}
\end{aligned}
$$

Here, T and V are Dirac four-spinors, each representing a right-handed and a left-handed massless fermion; a=1,...,8 is an index specifying an SU(3) generator; λ^a is the corresponding 3×3 matrix; $j,k=1,2,3$ are color indices; $j',k'=1,2,3$ are hypercolor indices; $A^a_{H\mu}$, $A^a_{C\mu}$, A_μ are the hypergluon, gluon and photon fields, respectively: $(F^a_H)_{\mu\nu}=\partial_\mu A^a_{H\nu}-\partial_\nu A^a_{H\mu}+g_H f_{abd}A^b_{H\mu}A^d_{H\nu}$; $(F_{EM})_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; upper and lower color indices correspond to the 3 and $\bar{3}$ representations, respectively.

The basic Lagrangian may include, in addition, terms of the form $\theta F\tilde{F}$, potentially yielding strong and hyperstrong CP violation. We do not discuss them any further in these notes.

The Lagrangian contains no mass parameters. The only free parameters are g_H, g_C and e (or $\Lambda_H,\Lambda_C,\alpha$). Note also that for Λ_H values between 10^3 and 10^8 GeV, g_C/g_H at Λ_H is around 0.15-0.25 while $e\sim0.3(\frac{e^2}{4\pi}=\alpha)$.

Our Lagrangian automatically exhibits additional discrete and global symmetries. It clearly obeys parity and charge conjugation invariance. Superficially, it appears to conserve separately the number of left-handed and right-handed T and V rishons, yielding four conserved charges: Two vector and two axial. The two vector-charges can be chosen as n_T and n_V, the net number of T and V rishons, respectively. Alternatively, we may choose to use the sum (n_T+n_V) and the difference (n_T-n_V) as our vector charges.
Consider a vector charge defined by $\frac{1}{3}(n_T-n_V)$. The T and V rishons correspond to eigenvalues of $+\frac{1}{3}$ and $-\frac{1}{3}$ respectively. Hence, all combinations of three rishons or antirishons can produce only the following eigenvalues: $+1$, $+\frac{1}{3}$, $-\frac{1}{3}$, -1. These are precisely the required values of B-L (section 8). Hence, we identify:

$$(B-L) = \frac{1}{3}(n_T-n_V).$$

The other vector charge is the total rishon number:

$$R = \frac{1}{3}(n_T+n_V)$$

Since $Q=\frac{1}{3}n_T$, we find: $Q = \frac{1}{2}R+\frac{1}{2}(B-L)$. Consequently, we now discover that:

$$\tfrac{1}{2}R = I_{3L}+I_{3R}.$$

The two vector charges of the Lagrangian exactly correspond to the two diagonal vector charges of $SU(2)_L xSU(2)_R xU(1)_{B-L}$.

There are also two axial U(1) symmetries in the Lagrangian. They correspond to the axial currents $\bar{T}\gamma_\mu\gamma_5 T$ and $\bar{V}\gamma_\mu\gamma_5 V$. Such axial U(1) symmetries are usually broken by instanton effects. Their divergences are proportional to combinations of $(F\tilde{F})_C$ and $(F\tilde{F})_H$. However, because of the equal color multiplicities of T and V, the divergence of the current $Y_\mu = \bar{T}\gamma_\mu\gamma_5 T - \bar{V}\gamma_\mu\gamma_5 V$ is not affected by the instantons and the Lagrangian obeys an axial $U(1)_Y$ symmetry. This axial symmetry would not have been there if T and V were in an $SU(3)_C$ triplet and singlet, respectively.

The axial Y-charge is a good candidate for the required I_{3L}-I_{3R} quantum number. We will later see that, indeed:

$$\tfrac{1}{2}Y = (I_{3L}-I_{3R})$$

It is remarkable that we have now found that the full continuous symmetry of the Lagrangian is $SU(3)_H xSU(3)_C xU(1)_R xU(1)_{B-L} xU(1)_Y$. The three U(1) charges (two vectors and one axial) exactly correspond to the three diagonal charges of $SU(2)_L xSU(2)_R xU(1)_{B-L}$.

The fourth U(1) factor corresponds to the current $X_\mu = \bar{T}\gamma_\mu\gamma_5 T + \bar{V}\gamma_\mu\gamma_5 V$. The divergence $\partial_\mu X_\mu$ is nonvanishing. However, both color and hypercolor instantons break the axial X charge by 12 units (this is, again, true only if both rishons are in color 3 or $\bar{3}$). Hence, a discrete axial Z_{12} symmetry remains unbroken[23]. We thus found a discrete axial symmetry within the model, a suitable candidate for a generation label (see section 9). We will return to the generation puzzle in section 26.

Note that we have found all the necessary non-gauged quantum numbers within the fundamental Lagrangian. All of them will be spontaneously broken, leaving only the original gauge symmetries as exact symmetries.

24. Composite Quarks and Leptons in the Rishon Model

Let us temporarily ignore all dynamic considerations, and construct the simplest composite fermions. We are interested only in hypercolor singlets, and we need to consider only three-rishon and three-antirishon states. For three rishons we have TTT, TTV, TVV and VVV. For three antirishons: $\bar{V}\bar{V}\bar{V}$, $\bar{V}\bar{V}\bar{T}$, $\bar{V}\bar{T}\bar{T}$, $\bar{T}\bar{T}\bar{T}$.

Let us now arbitrarily assume that for each of these combinations, the only light state is the lowest-color state. This single assumption completely determines all the quantum numbers of a well-defined set of light composite fermions. The states are listed in Table 2. They correspond exactly to the observed spectrum of quarks and leptons in one generation. We see that the electric charges, B-L values and colors are precisely the required ones.[20]

	Hypercolor	1	1
Minimal allowed color	B-L \ $\frac{1}{2}R$	$\frac{1}{2}$	$-\frac{1}{2}$
1	1	TTT (e^+)	$\bar{V}\bar{V}\bar{V}$ ($\bar{\nu}_e$)
3	$\frac{1}{3}$	TTV (u)	$\bar{V}\bar{V}\bar{T}$ (d)
$\bar{3}$	$-\frac{1}{3}$	TTV ($\bar{d}$)	$\bar{V}\bar{T}\bar{T}$ ($\bar{u}$)
1	-1	VVV (ν_e)	$\bar{T}\bar{T}\bar{T}$ (e^-)

Table 2: All hypercolor-singlet three-constituent composite fermions, assuming the minimal color for each configuration.

We also note that for each value of color and B-L, we find two different composite fermions, differing only by their R value. These are the required doublets of $SU(2)_L \times SU(2)_R$. In fact, for an arbitrary hypercolor scheme, we can always construct composite fermions from N fundamental fermions or from N antifermions. There are always only two ways, independent of the value of N_H. With our assignments of rishons, each pair of composites (from rrr and

$\bar{r}\bar{r}\bar{r}$, respectively) exactly produce the required SU(2) doublet.

The rishon wave-function within each lepton can be easily studied. The three rishons are totally antisymmetric under both hypercolor and color. Fermi statistics require a completely antisymmetric wave function. Thus:

$$e^+_L \equiv (T_R T_R) T_L \quad ; \quad \nu_{eL} \equiv (V_R V_R) V_L \quad ; \text{ etc.}$$

where the parenthesis denotes a Lorentz scalar di-rishon in totally anti-symmetric hypercolor and color representations. Here, again, we used the color assignments of the T and V rishons. Had we assigned one of them to an $SU(3)_C$ singlet, one of the above leptons would require an extremely complicated wavefunction with derivative couplings. For the quark wavefunctions we have several alternatives. Our favourite[22] is:

$$u_L \equiv (T_L T_L) V_L \quad ; \quad \bar{d}_L \equiv (V_L V_L) T_L \quad ; \text{ etc.}$$

Since $Y(T_L) = -Y(T_R) = -Y(V_L) = Y(V_R) = 1$, we can now determine the Y-values of all quarks and leptons. It is now easy to verify that , remarkably, for

$$\tfrac{1}{2}Y = I_{3L} - I_{3R}$$
$$\tfrac{1}{2}R = I_{3L} + I_{3R}$$

we obtain precisely the correct I_{3L} and I_{3R} values of all right-handed and left-handed quarks and leptons, including the non-trivial assignment of left-handed fermions to $(\tfrac{1}{2},0)$ and left-handed antifermions to $(0,\tfrac{1}{2})$ representations of $SU(2)_L \times SU(2)_R$.

The overall conclusion of this section is that, with one simple assumption about the lowest allowed color, the model reproduces the correct spectrum of one generation, including all quantum numbers. What we have not done is to prove that these states are light; that no other states are light; that $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is an approximate gauge symmetry and that the anomaly constraints (see section 17) are satisfied. We will discuss all of these problems in the next few sections, but we cannot claim to solve all of them.

Before we leave this topic we must return to section 8, where we presented three features of the structure of one generation: (i) Related charge quantization of quarks and leptons; (ii) The vanishing sum of charges of all quarks and leptons in a generation; (iii) The existence of some color-charge combinations and the absence of others. All three features are neatly explained in our simple scheme.

All charges come from T-rishons. Hence, all charges are quantized in the same way. The electron is $\bar{T}\bar{T}\bar{T}$. The proton is $2u+d = 4T+\bar{T}+2V+2\bar{V}$. A hydrogen atom contains $4(T+\bar{T})+2(V+\bar{V})$ and is neutral (even if we do not specify the T and V charges).

(ii) One generation contains e^-, ν_e, three u-quarks and three d-quarks. Their total rishon content is $6(T+\bar{T}+V+\bar{V})$. Clearly, any charge must vanish when summed over all fermions of one generation (even if we do not specify the T and V charges).

(iii) Finally, the color-charge correlations are trivially explained by the fact that at the rishon level, a change of $Q=\frac{1}{3}$ is accompanied by a change of color triality.

25. The Weak Interactions in the Rishon Model

The fundamental Lagrangian (section 23) does not contain the weak gauge bosons or the associated group $SU(2)_L xSU(2)_R xU(1)_{B-L}$. It only contains the $U(1)_{EM}$ subgroup as a gauge symmetry and the two other diagonal charges as global symmetries. Charged $(Q = \pm 1)$ weak currents cannot operate between single rishon states, which always have $Q = \pm\frac{1}{3}$ or 0.

At the composite level we notice pairs of hypercolor singlet fermions with identical $SU(3)_C xU(1)_{B-L}$ properties. When we write the known pieces of the low-energy effective Lagrangian (corresponding to parts (i), (ii) and (iii) of section 2), we find an explicit global SU(2) symmetry, under which all composite fermions are in doublets. This SU(2) symmetry did not exist in the fundamental Lagrangian. If the quarks and leptons of section 24 are massless composite fermions, we actually have a chiral $SU(2)_L xSU(2)_R$ global symmetry at the composite level. The full continuous symmetry is then $SU(3)_C xSU(2)_L x SU(2)_R xU(1)_{B-L}$, exactly the group of the left-right symmetric extension of the standard model.[20,22]

However, we do not yet have any weak bosons and we do not have an $SU(2)_L xSU(2)_R xU(1)_{B-L}$ gauge symmetry. What is now missing is part (iv) of the Lagrangian in section 2.

Let us understand the nature of the weak interactions in such a scheme. Consider the forces between two hypercolor-singlet composite objects such as two leptons. At large distances there will be no net hypercolor forces between two $SU(3)_H$-singlets. However, at short distances, we expect complicated short-range residual forces, reflecting the color and hypercolor interactions between the rishons inside the two composite objects. These forces are analogous to the residual color forces operating between two colorless hadrons (see section 7). The residual color forces between hadrons are identified with the "strong" or hadronic or nuclear force. We now conjecture that the residual forces among hypercolor singlet composites of rishons be identified with the conventional weak interactions. The "tail" of the hadronic forces is determined by the masses of the lightest composite colorless mesons which can be exchanged (i.e. pions, ρ-mesons, etc.). The "tail" of the weak force would now be determined by the masses of the lightest composite hypercolor-singlet bosons which can be exchanged. We would hope that these are the W and Z bosons and that they are responsible for the observed weak interactions. However, there is an important difference between the QCD situation and ours. There, the lightest vector particles (ρ,ω,ϕ) have masses of order Λ_C. In our case we want W and Z to have masses which are well below Λ_H, and we have already remarked that for $M_W, M_Z << \Lambda_H$ a new symmetry principle is required (see section 21). We do not know of any such principle and we therefore must consider our conjecture as extremely speculative.

Let us, however, assume that six relatively light composte W and Z bosons are formed, corresponding to the six non-electromagnetic generators of $SU(2)_L xSU(2)_R xU(1)_{B-L}$. Their rishon content is easy to determine: $W_L^+ = (T_L T_L)T_R(V_R V_R)V_L$; $W_R^+ = (T_R T_R)T_L(V_L V_L)V_R$: etc. It is not difficult to construct their wave functions in such a way that their couplings to quarks and leptons will be universal (up to terms of order Λ_H^{-n}). In fact, if the effective low-energy Lagrangian is to be approximately renormalizable, all W and Z couplings must follow the pattern of the standard model.

We must emphasize: <u>If</u> the effective Lagrangian is renormalizable (except for Λ_H^{-n} terms), it <u>must</u> be approximately gauge invariant under $SU(2)_L xSU(2)_R x$ $U(1)_{B-L}$, with higher-dimension terms breaking the gauge symmetry. However, we cannot directly prove that the effective Lagrangian is renormalizable.

A particularly interesting consequence relates to the angle θ_W. The angle is defined by the direction chosen by the electromagnetic current within $SU(2)_L xSU(2)_R xU(1)_{B-L}$ (or, in fact, within the U(1)xU(1)xU(1) group of the three diagonal charges). In the rishon-model, the three U(1) groups are global symmetries at the underlying level. As global symmetries, each linear combination of the three charges is a conserved charge. However, if we insist on (an approximate) gauging of the three U(1) groups, the $U(1)_{EM}$ gauge group can be obtained if and only if the U(1) factors which couple to $\bar{T}\gamma_\mu T$ and $\bar{V}\gamma_\mu V$, have the same couplings. This, in turn, determines[21] $\sin^2\theta_W=0.25$, at the level of the low-energy effective Lagrangian. Whether this prediction is good or bad depends on small unknown corrections which depend on $M(W_R)$ as well as on other factors.

Our discussion in this section was somewhat confused, for a good reason. The central assumption is the existence of light composite W and Z bosons and we do not know how to justify it. The conjecture that the weak interactions are residual forces and the hypothesis of an approximate gauge invariance of the low-energy Lagrangian, both depend on the existence of composite W and Z and are therefore highly speculative.

26. An Axial Z_n Symmetry as a Generation Label

Corresponding quarks or leptons in different generations must have the same $SU(3)_C xSU(2)_L xSU(2)_R xU(1)_{B-L}$ quantum numbers. They differ by some "generation number" (see section 9). Higher generations are also approximately massless in comparison with Λ_H. Hence, they cannot be obtained by radial or orbital excitations of the first-generation "ground state". A possible excitation of a massless fermion which may lead to a different massless fermion is an excitation by one or more fermion pairs. We should therefore investigate the possibility of constructing a system of fermions and antifermions forming a scalar under the Lorentz group as well as under $SU(3)_C xSU(2)_L xSU(2)_R x\ U(1)_{B-L}$, but possessing a nonvanishing value of some "generation number". Such a system could be the difference between corresponding fermions in different generations.

In section 9 we have explained that a discrete axial quantum number would be a desirable candidate for a "generation number", provided it is naturally found in the model. In section 23 we have shown that a discrete axial Z_{12} subgroup of $U(1)_X$ is a good symmetry of the underlying Lagrangian of the rishon model, and may serve as an adequate candidate for a generation number[23]. In fact, any composite model within the theoretical framework of section 18, possesses such a Z_{2K} symmetry (K=number of fundamental fermion-flavors).

Can we construct a Z_{12}-nonsinglet with all other quantum numbers vanishing? The answer is yes. It is the combination $T_L\bar{T}_L V_L \bar{V}_L$ (or $T_R\bar{T}_R V_R \bar{V}_R$). Note that no single $r\bar{r}$ pair can be a Lorentz scalar and an $SU(2)_L xSU(2)_R$ singlet at the same time. We need at least two pairs. We may therefore conjecture that higher-generation quarks and leptons are obtained[23] from their first generation counterparts by the addition of the "bunches" ($T_L\bar{T}_L V_L \bar{V}_L$) or ($T_R\bar{T}_R V_R \bar{V}_R$). Such "bunches" carry $X = \pm 4$, where X is conserved mod(12). Consequently, if we insist that each generation should have a different X-value, we can only have three generations with $X = X_o$, X_o+4, X_o+8. The number three, here, can be traced to the number of colors in the model.

The possibility of utilizing the Z_{2K} symmetry as a generation number is more general than the specific case of the rishon model. However, an explicit test can come only when the pattern of symmetry breaking is understood and the masses within the different generations can be computed. This is, of course, far in the future.

27. Chiral Symmetry Breaking in the Rishon Model

The only continuous chiral symmetry in the fundamental Lagrangian of the rishon model is $U(1)_Y$. It must be spontaneously broken, for two reasons: (i) In the effective Lagrangian this global U(1) symmetry becomes part of $SU(2)_L xSU(2)_R xU(1)_{B-L}$ and it must be broken in order to allow the quarks and leptons to acquire their (small) masses. (ii) The triple product of the three currents associated with the Y, R and (B-L) quantum numbers, possesses an anomaly in the underlying level. In the composite level, the same product is obviously anomaly free. This can be reconciled only if $U(1)_Y$ is broken, and a Goldstone boson exists.

If $U(1)_Y$ is broken, a chiral subsymmetry must remain intact, in order to protect the composite quarks and leptons from acquiring a mass of order Λ_H. Such a subgroup can only be a discrete Z_N subgroup of $U(1)_Y$. The latter should be further broken at the level of the effective Lagrangian in order to allow for the small quark and lepton masses. A possible solution is, therefore, as follows:

(i) A quadrilinear or hexalinear fermion condensate obtains a vacuum expectation at the Λ_H scale, breaking $U(1)_Y$ and leaving a Z_M (M=4 or 6) chiral subsymmetry. The axial charge Y is now conserved modulo M. A massless Goldstone boson exists. It couples to the current Y_μ and guarantees that the anomaly constraint is obeyed. All $Y = \pm 1$ composite fermions are protected from acquiring a mass by the axial Z_M symmetry. All quarks and leptons have, of course, $Y = \pm 1$. The massless Goldstone particle cannot couple to $Y = \pm 1$ fermions (it carries $|Y| = M$)and is therefore experimentally unobserved. It does not appear in the effective low energy Lagrangian.

(ii) A bilinear $r\bar{r}$ condensate obtains a small vacuum expectation value at the Λ_C scale, breaking the remaining axial Z_M symmetry and providing small masses to the quarks and leptons. This condensate is the ϕ-field discussed in section 6. The massless Goldstoneboson is now allowed to couple to quarks and leptons but its coupling is of order m_f/Λ_H and is negligible. It can only be detected[22] by its effect on the energy losses of the sun or of red giants, leading to lower bounds on Λ_H.

With this scenario everything is perfectly consistent except for one major puzzle: Why should the symmetry breaking at the hypercolor scale proceed via complicated multifermion operators rather than the simple normal $r\bar{r}$ condensate? We have no good dynamical answer to this question. We were simply led to this situation by demanding the self-consistency of the model. If the dynamics does not justify our description, the model is not valid.

Another related important topic is the limit $g_C=e=0$ of the underlying Lagrangian. In that limit, the three colors of the T rishon and the three colors of the V rishon act as six identical "flavors". The Lagrangian then obeys an $SU(6)_L xSU(6)_R xU(1)$ symmetry. It is isomorphic to a six-flavor massless QCD theory. According to our discussion in section 20, the simplest guess for the pattern of chiral symmetry breaking will leave us only with a vectorial SU(6) symmetry. This is totally unacceptable to us as it would lead to composite fermions of mass Λ_H, falling into unwanted representations of SU(6). We already commented in section 20 on the various open options. Here we only add that the best situation from the rishon model point of view is a breaking of $SU(6)_L xSU(6)_R$ directly into $SU(3)_C xZ_M$ where Z_M is the discrete subgroup of $U(1)_Y$ discussed above. We can write complicated multifermion operators which can perform this but, as above, we cannot explain why they, and no others, form the required condensates.

Again and again we see that the most crucial difficulty of the rishon model, as well as any other composite model of quarks and leptons, is the pattern of chiral symmetry breaking.

28. Parity Violation, B-L Violation and Neutrino Masses in the Rishon Model

The underlying Lagrangian of the rishon model conserves (among other things) parity, charge conjugation and B-L. The only electrically neutral fermion is the V-rishon, but it has nontrivial color and hypercolor. Consequently, the V-rishon cannot obtain a Majorana mass. In other words - V and $\bar{V}$ differ by some conserved quantum numbers. They cannot mix.

At the composite level a (VVV) and a ($\bar{V}\bar{V}\bar{V}$) composite fermion are formed. Both are neutral, color-singlets and hypercolor-singlets. No conserved quantum number distinguishes between them. The VVV composite (ν_e) can now obtain a

Majorana mass. One way of achieving this is through a 6V condensate. Six V-rishons are the simplest Lorentz scalar combination with net R-number which conserves hypercolor, color and electric charge. Hence, it is the simplest condensate which could break R-number, B-L, parity and charge conjugation at the same time.

In section 6 we have discussed the necessary scalars in an $SU(2)_L x SU(2)_R x U(1)_{B-L}$ model. We found that they are ϕ-fields in $(\frac{1}{2},\frac{1}{2})_0$ (corresponding to $r\bar{r}$ in the rishon model) and Δ_R-fields in $(0,1)_2$. The Δ_R^0-field is precisely the 6V-condensate described here. We therefore see that, starting from the Lagrangian of section 23, the following features are all obtained by the simplest R≠0 condensate:

(i) Parity and charge conjugation are spontaneously broken, as weak interaction effects. Note that the exact hypercolor, color and electromagnetic gauge symmetries do not allow their respective interactions to violate C and P, but the residual weak interactions break C and P through the 6V-condensate.

(ii) B-L symmetry is spontaneously broken. At the energies above Λ_H (or at the early stages of the universe) the numbers of T, $\bar{T}$, V and $\bar{V}$ rishons must be equal. There is complete matter-antimatter symmetry in the rishon terminology (remember that a hydrogen atom has equal amounts of rishon matter and antimatter; see section 24). At the scale in which the 6V-condensates form, B-L is violated and the numbers of V and $\bar{V}$ rishons in the universe become unequal (for instance - through neutrino masses or (B-L)-violating proton decays). The actual present total (B-L)-value of the universe need not vanish, as a result.

(iii) The W_R-boson and the right-handed neutrinos obtain masses which are determined by the vacuum expectation value <6V>. The left-handed neutrinos obtain tiny masses of order $m_\ell^2/m(\nu_R)$ where ℓ is the corresponding lepton.

It is quite satisfactory that all of these symmetry-breaking effects can be obtained in terms of one simple condensate. Again, we have no dynamical "handle" on the actual magnitude of <6V>.

29. Proton Decay in the Rishon Model

A (B-L) conserving proton decay can proceed by the reaction $u+u \to \bar{d}+e^+$, or in rishon language:

$$(TTV) + (TTV) \to (TVV) + (TTT)$$

This is a simple rearrangement process. It might proceed, for instance, by the exchange of a $T\bar{V}$ composite meson. The expected mass of any composite particle is Λ_H, unless it has a good reason to be massless. If we assume that the above reaction necessitates a propagator corresponding to a mass of order Λ_H, we obtain

$$\tau_p \sim \frac{\Lambda_H^4}{m_p^5}$$

leading to $\Lambda_H \gtrsim 10^{15}$ GeV. A similar conclusion can be obtained on more general grounds by considering the dimension of the simplest baryon-number violating four-fermion terms in the low-energy effective Lagrangian.

However, with the quark and lepton wave functions of section 24, the above process becomes forbidden. This can be seen[24] by considering the X-values of the quarks and the leptons. The simplest diagram (or multifermion term in the effective Lagrangian) require $\tau_p \propto \Lambda_H^8$, allowing for Λ_H-values as small as 10^7-10^8 GeV. A detailed investigation of the model actually shows that the dominant proton decay processes are (B-L)-violating[24], e.g. $p \to e^- \pi^+ \pi^+$ etc.

Proton decay is a crucial test for every composite model. When it is not completely forbidden, it usually provides the strongest lower bound on the compositeness scale. As long as there is no independent information which can pinpoint the value of the compositeness scale, there is no apriori reason for the proton lifetime to be in the observable range of 10^{30}-10^{33} years. In fact, this range corresponds to a very small "window" of Λ_H-values. Consequently, on purely statistical grounds, it is unlikely that proton decay will be observed within the next decade or two, even if it is allowed. The situation in a SU(5) GUT is, of course, very different. There we have inepependent information which fixes the unification scale exactly in the range which corresponds to a detectable proton lifetime.

30. The Rishon Model: Successes, Difficulties and Open Problems

We now summarize our detailed discussion of the rishon model (sections 22-29). We start with the difficulties. There are two major difficulties, both more general than the specific model:

(i) The pattern of chiral symmetry breaking. We will not repeat here the discussion of sections 3, 20 and 27. We only make a general statement: If quarks and leptons are composite and if their small masses are related to a chiral symmetry, the overall dynamical picture must be different from that of QCD. How different and why, may depend on the model but we always encounter a somewhat unusual pattern which cannot be explained, at present.

(ii) The notion of light composite vector bosons corresponding to an approximate gauge symmetry is far from understood. The related idea of a residual-force nature for the weak interaction is equally obscure. We have no good reason to keep a composite W and Z at masses well below Λ_H. Here, again, the problem goes beyond our specific model, and it will be encountered by any model with composite weak bosons.

Either one of these difficulties will suffice to kill the rishon model, if no satsifactory explanation is found. In the absence of a clear successful front-running composite model, we feel that these topics should be further studied, especially in view of their general implications.

The successes of the rishon model are mainly related to its simplicity and economy, its impressive reproduction of the spectrum of one generation and the interesting pattern in which the various quantum numbers are found (and later broken) within the model. The model has only three parameters with all other quantities determined from them, in principle. The only exact symmetries are gauge symmetries and all the original gauge symmetries remain exact. There is a natural candidate for a generation-number and a reasonable value for $\sin^2\theta_W$ is obtained. An attractive pattern of parity and B-L violation is built-in. These and the other features are too numerous to be accidental. The model may be wrong, but at least some of its novel ingredients are likely to survive, in our opinion.

Among the open problems of the model we should mention a complete understanding of the generation structure, a complete discussion of proton decay, a better understanding of $g_C \to 0, e \to 0$ limit and a study of possible variations and extensions.

All in all - a mixed bag of successes, difficulties and open problems.

31. A Distilled List of Novel Ideas

In our discussion of the rishon model, as well as in the preceding general discussion, a number of specific new ideas were presented, which are independent of any concrete model. It is important to realize that such ideas stand or fall on their own and their validity does not necessarily relate to the success of a given model. In this section we briefly reiterate three of these ideas and draw attention to the need of investigating each one of them separately.

(i) Weak Interactions as Residual Forces (see sections 7,25). Quite independent of any specific model, we may consider the short range weak forces between, say, two neutrinos. If the neutrino is composite and if its constituents possess some quantum number which vanishes for the composite system, it is likely that a short range residual ν-ν interaction will result. The idea of identifying it with the ordinary weak interaction is quite attractive. It almost necessarily follows that the weak bosons of the standard model are composite. However, the difficult problem of almost-massless composite bosons is present only if M_W is much smaller than the compositeness scale. Perhaps this is not so. The compositeness scale could, in general, be as low as 500 GeV. In any case - the possibility that the weak interaction is not a fundamental interaction should be kept in mind.

(ii) A Discrete Axial Generation Number (see sections 9,26). In almost any composite model with massless fundamental fermions, an axial U(1) symmetry is broken by instanton effects into a Z_N subgroup. The remaining discrete symmetry may be an attractive candidate for a generation labelling quantum number. It has several advantages: (a) It is axial (i.e. $X(f_L) = -X(f_R)$); (b) It is discrete and its spontaneous symmetry breaking does not require Goldstone particles; (c) It already exists as a necessary quantum number in the model. The details depend on the specific model, but the general suggestion may have a wider validity.

(iii) Unbroken Discrete Chiral Subgroup (see sections 20,27). No-one has, so far, succeeded in finding a realistic solution for the anomaly constraints in the case of an unbroken global chiral symmetry. An interesting solution may be to break the continuous chiral symmetry, leaving a discrete chiral sub-symmetry[25]. In such a case, the anomaly constraints pose no problem; the discrete symmetry can keep some composite fermions massless; the Goldstone bosons obtained from the breaking of the continuous chiral symmetry decouple from the composite massless fermions. The pattern is quite attractive and it may be interesting to look for a dynamical theory which actually leads to such a symmetry-breaking sequence leaving a conserved discrete chiral subgroup.

32. Summary and Outlook

We have pursued here a line of reasoning which followed several steps: (i) Assume the Standard Model; (ii) Show a need to go beyond it; (iii) Consider different classes of ideas; (iv) Concentrate on the Notion of Compositeness; (v) Study its general difficulties, mainly the Scale Problem; (vi) Assume a connection between composite massless fermions and an unbroken chiral symmetry; (vii) Establish a general framework based on hypercolor and a chiral symmetry; (viii) Establish the general requirements for a candidate model; (ix) Find the minimal scheme; (x) Study it and discover its successes and failures.

In the process we have also discussed many additional topics. We have often made specific choices on the basis of simplicity, economy or other matters of taste. In travelling such a long road, one is almost certain to make a few wrong turns. It is always important to go back and try other directions and options.

No-one has, at present, a satisfactory composite model of quarks and leptons. No-one knows with any certainty whether they are indeed composite. We strongly believe that this possibility should be intensively studied. Theoretically, one should not apriori abandon any reasonable direction. The consistency requirements as well as the phenomenological constraints are so severe that it is very difficult to construct a satisfactory model even without new experimental tests.

Experimentally, in addition to further crucial verifications of the standard model itself, we are eagerly waiting for the first experimental indication of physics beyond the standard model (see section 15). The relevant experiments are somewhat thankless, as long as one only continues to improve upon null results. However, any explicit positive signal would provide us with an extremely crucial guide for the new physics which we keep searching for. A right-handed weak current, a neutrino mass, a proton decay or a neutral flavor-changing transition would each open up a new domain of physics. Whether it indeed points in the direction of further compositeness only time will tell.

Finally - in probing into smaller and smaller distances we may always face a completely new set of rules. These notes were entirely written within the conventional language of quantum field theory with no exotic deviations from it (except for a brief remark in section 19). In order to justify radical new theoretical ideas we usually need a major crisis: either a clear conflict between the present theoretical rules and some experimental results or a manifest inconsistency within our theoretical understanding. Neither has happened, so far. As long as we do not have such a crisis, we should keep trying within the conventional set of rules. We are far from exhausting all the fruitful avenues of investigation.

ACKNOWLEDGEMENTS

Most of the topics described in these notes were studied in collaboration with Nathan Seiberg. I thank him for numerous discussions and many new ideas. The Banff school provided a very pleasant and stimulating atmosphere. I thank Professors A.N. Kamal and A. Capri for their warm hospitality and friendship. I also thank many of the speakers and students in the School for questions, comments, discussions and, above all, criticism.

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