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OLD TENSOR MESONS IN QCD SUM RULES

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Abstract

Tensor mesons f, A_2 and A_3 are analysed within the framework of QCD sum rules. We account for the effects of gluon and quark condensates phenomenologically, via matrix elements $\langle 0|G^2|0\rangle$ and $\langle 0|\bar{\psi}\psi|0\rangle$. The G^2 term in the sum rules is found with the help of a new computational technique developed recently. Our results for the masses of f, A_2, A_3 and their coupling constants agree with experiment.

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The applications of QCD sum rules [1] to resonance physics are rather numerous now [2]. Still, we decided to consider a new example, f, A_2 and A_3 mesons, in order to demonstrate the effectivity of a wonderful computational technique developed mainly in refs. [3-5].

Let us recall first the basic ingredients of the QCD sum rules. The two-point functions induced by currents with appropriate quantum numbers are calculated in euclidean domain. Apart from the simplest bare loop (fig.1) one includes also non-perturbative corrections due to the gluon and quark condensates (figs. 2 and 3, respectively). On the other hand, the result can be expressed, with the help of the general dispersion relation, in terms of the corresponding spectral density. In this way structures in the spectral densities turn out to be correlated with vacuum parameters. The so called borelization [1] helps to perform quantitative analysis. As an output we get rather accurate estimates of the masses and coupling constants of the lowest-lying states.

Our results for f, A_2 and A_3 mesons are as follows:

(i) The masses are reproduced within theoretical uncertainty of about 80 MeV. The sum rules explain, in a natural way, why f is nearly degenerate with A_2 while A_3 is much heavier.

(ii) The coupling of f to the corresponding quark current is determined. Unlike the ρ -meson case [1] this coupling constant is not directly measurable. However, one can express it, under an additional assumption, in terms of the $f\pi\pi$ amplitude and compare in this way the predicted value with the "experimental" one. Agreement is quite satisfactory.

It is worth noting that masses of f and A_2 mesons were found also in a recent paper [6] based on similar principles. There is an overlap between our work and that of Reinders et al.[6] We discuss, however, some essential aspects which go beyond the analysis of ref.[6]. All technical details are also different.

After these preliminaries let us proceed to a systematic description of the procedure. First of all we should fix currents which produce corresponding mesons from the vacuum state. The most convenient choice is

$$j^{(+)}_{\mu\nu} = \frac{i}{2}\left(\bar{\psi}\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu \psi + \bar{\psi}\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu \psi\right) \quad (J^P = 2^+), \qquad (1)$$

$$j^{(-)}_{\mu\nu} = \frac{i}{2}\left(\bar{\psi}\gamma_\mu\gamma_5 \overleftrightarrow{\mathcal{D}}_\nu \psi + \bar{\psi}\gamma_\nu\gamma_5 \overleftrightarrow{\mathcal{D}}_\mu \psi\right) \quad (J^P = 2^-), \qquad (2)$$

where $\bar{\psi}\psi = 2^{-1/2}(\bar{u}u + \bar{d}d)$ for f meson and $\bar{\psi}\psi = 2^{-1/2}(\bar{u}u - \bar{d}d)$ for A_2 and A_3. The definition of the covariant derivative is standard,

$$\overleftrightarrow{\mathcal{D}}_\mu = \frac{1}{2}\left(\overrightarrow{\mathcal{D}}_\mu - \overleftarrow{\mathcal{D}}_\mu\right),$$

$$\overrightarrow{\mathcal{D}}_\mu = \overrightarrow{\partial}_\mu - i\frac{g}{2}t^a A^a_\mu, \quad \overleftarrow{\mathcal{D}}_\mu = \overleftarrow{\partial}_\mu + i\frac{g}{2}t^a A^a_\mu.$$

Notice that these tensor currents have the lowest possible dimension (d=4). In the f -meson case one might add, in principle, a gluon pice of the type $G^a_{\mu\alpha}G^a_{\nu\alpha}$. We keep only the quark term for reasons discussed below.

In the non-relativistic limit currents (1),(2) reduce to P and D waves, respectively. This feature is desirable from the point of view of constituent quark models.

As usual the two-point function is introduced in the following way:

$$i\int e^{iqx} d^4x \langle 0|T\{j^{(\pm)}_{\mu\nu}(x), j^{(\pm)}_{\alpha\beta}(0)\}|0\rangle$$
$$= \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\Pi^{(\pm)}(q^2) + \dots \tag{3}$$

where on the right-hand side we single out the invariant structure referring to pure spin-2 states. Spin-0 and spin-1 states do not appear here while they contribute to other invariant structures which are not written out explicitly (and will not be considered in this paper). Notice that the contribution of a given tensor meson to $\Pi^{(\pm)}$ reduces to

$$\mathrm{Im}\, \Pi^{(\pm)}_{res}(s) = m^6 g^2 \pi \delta(s - m^2) \quad . \tag{4}$$

where m denotes the resonance mass and g is its coupling constant,

$$\langle 0|j_{\mu\nu}|\mathrm{resonance}\rangle = m^3_{res} g_{res} \varepsilon_{\mu\nu}$$

($\varepsilon_{\mu\nu}$ stands for the density matrix).

For massless quarks the best way to perform computations is to do them directly in configurational space. For instance, the bare loop of fig.1 is determined by the following expression ($J^P = 2^+$):

$$\Pi^{(0)}_{\mu\nu,\alpha\beta}(q) = -\frac{1}{4} i \int d^4x\, e^{iqx}\, \mathrm{Tr}\, \{ \overleftrightarrow{\partial}_\nu(x) \overleftrightarrow{\partial}_\beta(y) S^{(0)}(y,x)$$
$$\times \gamma_\mu S^{(0)}(x,y)\gamma_\alpha \}_{y\to 0} + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu, \alpha \leftrightarrow \beta) \tag{5}$$

where $S^{(0)}(x,y)$ is the bare quark propagator,

$$S^{(0)}(x,y) = \frac{1}{2\pi^2} \frac{\hat{x}-\hat{y}}{[(x-y)^2]^2} \tag{6}$$

and the derivative $\overleftrightarrow{\partial}$ acts in the following way

$$\overleftrightarrow{\partial}_\nu(x) S(y,x) S(x,y) = \frac{1}{2} S(y,x) \frac{\partial}{\partial x_\nu} S(x,y) - \frac{1}{2}\left(\frac{\partial}{\partial x_\nu} S(y,x)\right) S(x,y).$$

After simple arithmetics one finds $\Pi^{(0)}_{\mu\nu,\alpha\beta}(x)$. It is not difficult to perform the Fourier transformation. The basic formula is

$$\int \frac{d^4x}{(x^2)^n} e^{iqx} = \frac{i(-1)^n 2^{4-2n} \pi^2}{\Gamma(n-1)\Gamma(n)} (q^2)^{n-2} \ln(-q^2) . \tag{7}$$

On the right-hand side we omitted a polynomial in q^2 with divergent coefficients. This polynomial is inessential - gives no contribution to the sum rules - since the Borel procedure[1] eliminates it. The final result for the diagram of fig.1 is

$$\Pi^{(\pm)}(\text{fig.}1) = -\frac{3}{5} \frac{1}{32\pi^2} (q^2)^2 \ln Q^2 , \tag{8}$$
$$Q^2 \equiv -q^2 .$$

The computation of the $\langle G^2 \rangle$ contribution, fig.2, is a bit more lengthy. Consider the vacuum gluon field as an external one and impose the gauge condition

$$x_\mu A^a_\mu(x) = 0 , \tag{9}$$

where $A^a_\mu(x)$ is the four-potential. This gauge was invented by Schwinger long ago[7] and the rediscovered in QCD in refs.[8,9,3].

It has many virtues. In particular, all quantities in this gauge can be expressed in terms of the field strength tensor $G^a_{\mu\nu}$, say

$$A_\mu(x) = \frac{1}{2} x_\rho G_{\rho\mu}(0) + \frac{1}{3} x_{\alpha_1} x_\rho \mathcal{D}_{\alpha_1} G_{\rho\mu}(0) + \frac{1}{4\cdot 2!} x_{\alpha_1} x_{\alpha_2} x_\rho \mathcal{D}_{\alpha_1} \mathcal{D}_{\alpha_2} G_{\rho\mu}(\ ) + \ldots \quad (10)$$

and

$$S(x,y)\Big|_{y\to 0} = S^{(0)}(x,y) + \frac{i(\hat{x}-\hat{y})((x-y)_\alpha y_\beta G_{\beta\alpha}(0))}{4\pi^2[(x-y)^2]^2} - \frac{1}{8\pi^2} \frac{(x-y)_\alpha}{(x-y)^2} \tilde{G}_{\alpha\varphi}(0)\gamma_\varphi\gamma_5 + \quad (11)$$

gluonic operators of higher dimension,

$$\left(\tilde{G}_{\alpha\varphi} = \frac{1}{2}\epsilon_{\alpha\varphi\mu\nu} G_{\mu\nu} \;;\; G_{\mu\nu} = \frac{g}{2} t^a G^a_{\mu\nu}\right).$$

A few comments are in order here. Eq.(10) was derived first in [8,10]. The expression for $S(x,0)$ was found in [3-5] where only currents without derivatives were considered. In our case they do contain a derivative (see (1),(2)), so it is impossible to put y=0 from the very beginning. One should keep terms linear in y and tend y to zero only after differentiation. Once we are interested in G^2 effects the expansion in (11) should be continued, in principle, up to terms of this order. Operators proportional to DG are, clearly, irrelevant. Operators containing two G's might contribute, but, fortunately, this does not happen. There arise two distinct structures bilinear in G. One of them is proportional [4,5] to

$$G^a_{\mu\alpha} G^a_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} G^a_{\alpha\beta} G^a_{\alpha\beta} \quad .$$

The vacuum expectation value of this combination vanishes. Another one enters with coefficient proportional to y^2 (and higher powers of y).

After these remarks the calculation of diagrams 2 is trivial. We write expression analogous to (5) with $\overleftrightarrow{D}$ and S instead of $\overleftrightarrow{\partial}$ and $S^{(o)}$, and use eqs.(10),(11). It takes an hour or so to perform all computations, the final result is

$$\Pi^{(\pm)}(fig.2) = \frac{1}{18} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle \ln Q^2 . \tag{12}$$

It is one and the same for both currents (1) and (2) and coincides with that obtained by Reinders et al.[6]

Diagrams of fig.3 are so simple that no comments are needed. Fig.3b gives no contribution to the invariant amplitude (3) which we consider here. We get

$$\Pi^{(\pm)}(fig.3) = \frac{\pi}{4} \alpha_s \frac{1}{q^2} \times \begin{cases} \frac{1}{2} \langle (\bar{u}\gamma_\alpha t^a u \pm \bar{d}\gamma_\alpha t^a d)^2 \rangle \text{ for } j^+_{\mu\nu} \\ \frac{1}{2} \langle (\bar{u}\gamma_\alpha\gamma_5 t^a u - \bar{d}\gamma_\alpha\gamma_5 t^a d)^2 \rangle \\ \qquad \text{for } j^-_{\mu\nu} , \end{cases} \tag{13}$$

or, saturating by the vacuum intermediate state [1]

$$\Pi^{(\pm)}(fig.3) = \pm \frac{4\pi}{9} \frac{1}{Q^2} \langle \alpha_s^{1/2} \bar{q}q \rangle^2 . \tag{14}$$

Assembling three pieces together we find $\Pi^{(\pm)}$ in euclidean domain. To obtain the announced sum rules we make the Borel transformation of Π .

$$\hat{B}\Pi = \lim_{\substack{Q^2\to\infty,\ n\to\infty \\ Q^2/n \equiv M^2 = const}} \frac{1}{(n-1)!} (-Q^2)^n \left(d/dQ^2\right)^n \Pi .$$

Notice that

$$\hat{B}(Q^2)^k \ln Q^2 = (-1)^{k+1}\Gamma(k+1)(M^2)^k . \tag{15}$$

The sum rules in the $2^{\pm}$ channels look as follows

$$\frac{1}{\pi M^2}\int \operatorname{Im}\Pi^{(\pm)} e^{-s/M^2} ds = \frac{3}{80\pi^2} M^4 \left\{1 - 0.48\left(\frac{0.6\,\mathrm{GeV}^2}{M^2}\right)^2 \pm 0.29\left(\frac{0.6\,\mathrm{GeV}^2}{M^2}\right)^3\right\} \tag{16}$$

where we used the standard numerical estimates of $\langle \alpha_s G^2 \rangle$ and $\langle \alpha_s^{1/2} \bar{q} q \rangle$ [1].

Eq.(16) is actually not quite accurate since it does not account for anomalous dimensions of $j^{\pm}_{\mu\nu}$. These currents are logarithmically renormalized in perturbation theory which results in factors like

$$\left[\frac{\alpha_s(s)}{\alpha_s(\mu^2)} \right]^{\gamma/6} . \tag{17}$$

Moreover, in the f-meson case the mixing of quark and gluon operators is possible and it actually takes place in the leading log approximation.

The logarithmic effects almost do not affect the predicted value of the resonance mass. Besides that they can be minimized by an appropriate choice of the normalization point μ . We will analyse the sum rules in the vicinity of $M^2 \sim 0.5$ GeV^2, and the most natural choice is $\mu = 1$ GeV. Integrals (16) are saturated then by the domain $s = 1$-2 GeV^2, and factors (17) are close to unity. As for the mixing it is small and becomes noticeable only at $\ln s/\mu^2 \gtrsim 4$. Such values of s lie deep in the continuum domain and are irrelevant to the analysis we undertake here. Thus it seems reasonable to neglect all logarithmic renormalizations.

Their minor role is confirmed also by phenomenology. Indeed, the logarithmic renormalizations are essentially different in the f and A_2 channels, and, nevertheless, f and A_2 are nearly degenerate in mass.

Let us proceed now to the analysis of sum rules (16). To determine the mass of the lowest-lying resonance it is convenient to consider the quantity $\rho(M^2)$,

$$\rho^{(\pm)} = -\frac{d}{d(1/M^2)} \ln \int Im\, \Pi^{(\pm)} e^{-s/M^2} ds \,. \qquad (18)$$

At small M^2 all higher-state contributions die away, and $\rho(M^2) \to m^2_{res}$.

From eq.(16) we get

$$\rho^{(\pm)} = 3M^2 \left\{ 1 + 0.32 \left(\frac{0.6}{M^2}\right)^2 \mp 0.29 \left(\frac{0.6}{M^2}\right)^3 + \ldots \right\} \,. \qquad (19)$$

Unfortunately, the sum rule technique in its present form does allow one to tend M^2 to zero in the mathematical sense. Therefore we should introduce the correction factor f_{cont}, as in[1], and try to find a domain of M^2 in which a compromise between two opposite requirements is achieved. On one hand, to ensure relative smallness of the continuum contribution it is necessary to consider $M^2 \lesssim 1\ GeV^2$. On the other hand, to keep control over the power series we should choose $M^2 \gtrsim 0.8\ GeV^2$. By inspecting the domain

$$0.8\ GeV^2 < M^2 < 1\ GeV^2$$

we get (see fig. 4,5)

$$m_f \approx m_{A_2} \approx 1.25\ \text{GeV}, \qquad m_{A_3} \approx 1.63\ \text{GeV} \tag{20}$$

in good agreement with experiment*) (1.27, 1.32 and 1.66 GeV, respectively). Notice that the splitting between the opposite parity states is entirely due to the quark condensate which tends to make A_3 heavier and A_2 and f lighter. There is no splitting between f and A_2 in the approximation considered, and degeneracy is eliminated only after accounting for non-leading terms. This, clearly brings in a suppression and explains the fact that $m_f \approx m_{A_2}$.

Returning now to sum rule (16) and assuming that

$$\operatorname{Im} \Pi^{(+)} \simeq m_f^6 g_f^2 \pi \delta(s - m_f^2) + \frac{3}{160\pi} s^2 \theta(s - s_0)$$

(where $m_f^2 = 1.62\ \text{GeV}^2$, $s_0 = 2.5\ \text{GeV}^2$)we fix also the f-meson coupling to the current (1), normalized at the point $\mu \simeq 1$ GeV (see above):

$$g_f \approx 0.040\ . \tag{21}$$

Of course, this residue is not known experimentally .Still, an indirect comparison with data is possible. To this end we proceed as follows.

Consider the matrix element $\langle \pi | \theta^q_{\mu\nu}(\mu \sim 1\ \text{GeV}) | \pi \rangle$ where $\theta^q_{\mu\nu} = 2^{1/2} j^+_{\mu\nu}$ is the u,d-quark piece of the energy-momentum tensor (fig.6a). On one hand, phenomenologically $\langle \pi | \theta^q_{\mu\nu}(\mu) | \pi \rangle$ reduces at small q^2 to $2 p_\mu p_\nu \rho^q(\mu)$. (Here $\rho^q(\mu)$ is the fraction of the pion momentum carried by u and d quarks as it would be measured in the deep inelastic scattering $e + \pi \to e$ +

*) Close values of m_{f,A_2} and $s_0(2^+)$ are found in [6].

+ anything with momentum transfer of order μ . This fraction seems to be the same for pion and nucleon, and at $\mu \simeq$ 1 GeV it is presumably close to 0.6).

On the other hand, one can saturate this matrix element by the f- meson and higher-state contributions (fig.6b). One can argue [11] that the latter is integrally small, of order $O(\alpha_s)$. The hypothesis of the dominance of the lowest-lying state is well known to be valid in the ρ -meson case - it leads to a surprisingly successful relation between f_ρ and $g_{\rho\pi\pi}$ (fig.7).

Accepting this hypothesis for the tensor current we get

$$\sqrt{2}\, g_f\, g_{f\pi\pi}\, p_\mu p_\nu = 2\rho^q(\mu)\, p_\mu p_\nu \tag{22}$$

where $g_{f\pi\pi}$ stands for the $f \to \pi^+\pi^-$ decay constant,

$$A(f \to \pi^+\pi^-) = \frac{g_{f\pi\pi}}{m_f} \chi_{\mu\nu}\, p_\mu^{(1)} p_\nu^{(2)} \;, \quad \Gamma(f \to \pi^+\pi^-, \pi^0\pi^0) = \frac{g_{f\pi\pi}^2 m_f}{1280\pi}\left(1 - \frac{4m_\pi^2}{m_f^2}\right)^{5/2}, \tag{23}$$

$$(g_{f\pi\pi})_{exp} = 23 \pm 1 \, .$$

This implies, in turn, that

$$(g_f)_{\text{"}exp\text{"}} = \frac{\sqrt{2}\,\rho^q(\mu)}{g_{f\pi\pi}} = 0.037 \pm 0.002 \tag{24}$$

where we used eq.(23) and $\rho^q(\mu \simeq 1\ GeV) = 0.6$. The agreement with our theoretical estimate, eq.(21), is even better than one could expect beforehand.

Summarizing, we have shown that the sum rule approach of ref[1] accounting for the leading effects of quark and gluon condensates, can be easily extended to incorporate the old tensor mesons: f , A_2 and A_3 . The pattern of their masses and coupling constants is well reproduced. We have also checked the effectivity of the new computational method proposed earlier in ref. [3-5].

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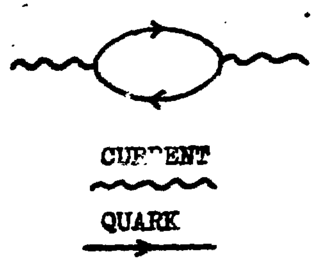


Fig.1. Bare quark loop determining $\Pi^{(\pm)}_{\mu\nu,\alpha\beta}$ in the limit of asymptotic freedom.

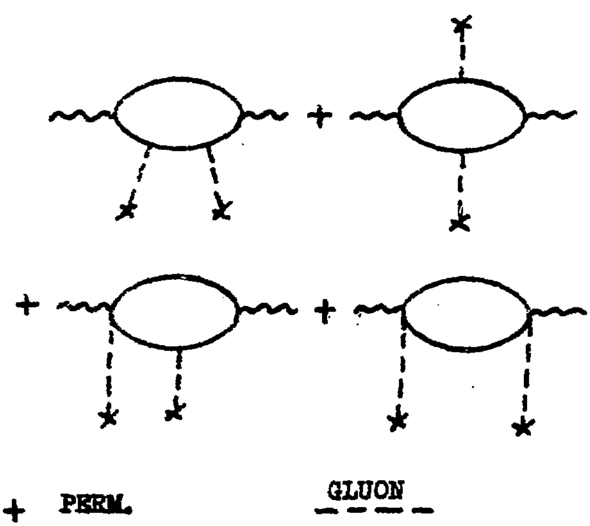
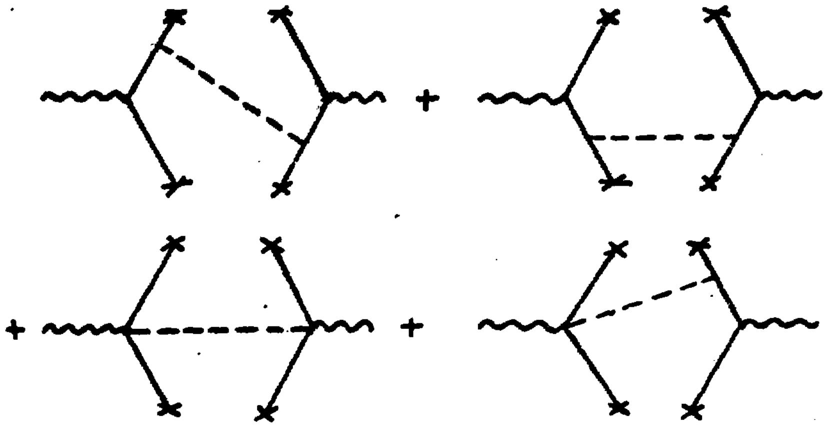
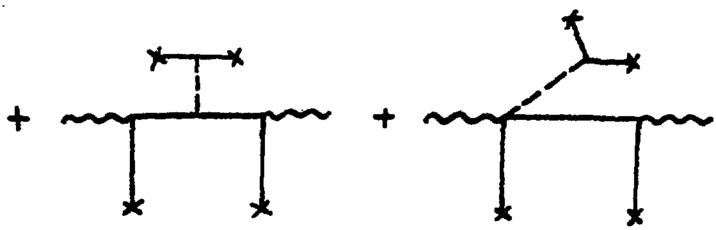


Fig.2. Graphic representation for the $\langle G^2\rangle$ term in $\Pi^{(\pm)}_{\mu\nu,\alpha\beta}$.



(3a)



(3b)

Fig.3. The leading contribution due to the quark condensate. The vacuum matrix elements $\langle \bar{\psi}\psi\bar{\psi}\psi \rangle$ are reduced to $\langle \bar{\psi}\psi \rangle^2$ with the help of the vacuum saturation hypothesis.

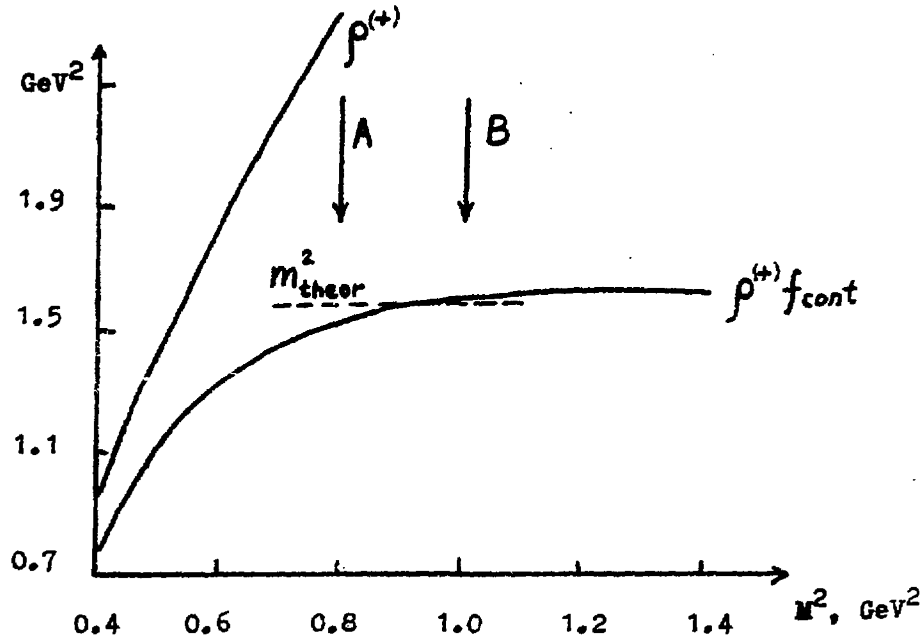


Fig.4. Theoretical prediction for m^2_{f,A_2} . In the interval $0.8 < M^2 < 1.0\ GeV^2$ uncertainties due to higher power terms and unknown details of the continuum contribution are optimal. The continuum threshold s_o is chosen in the following way: $s_o = 2.5\ GeV^2$.

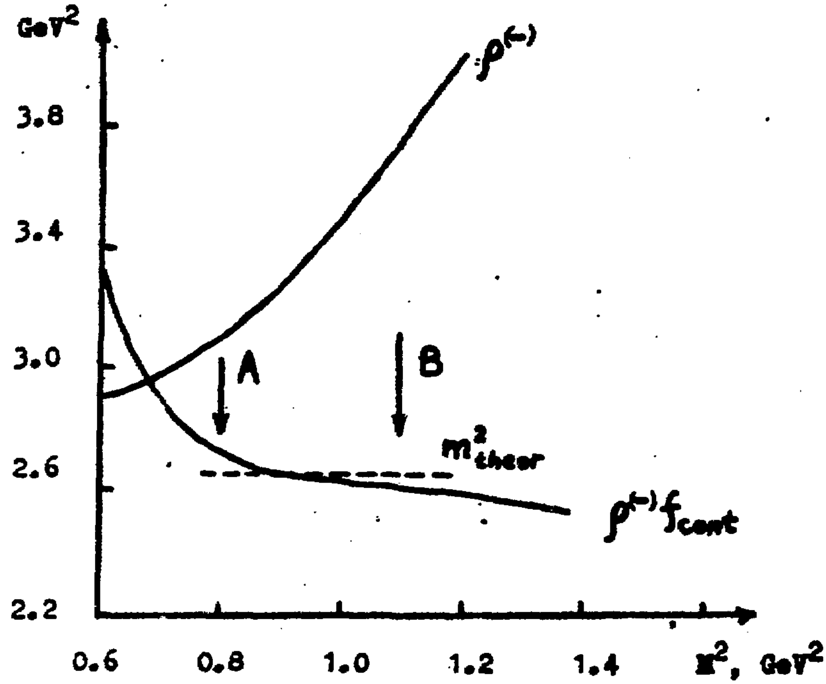


Fig.5. The same for $m^2_{A_3}$. (s_0 = 3.5 GeV^2).

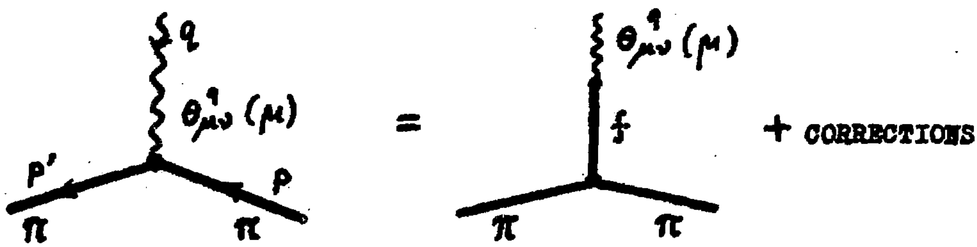


Fig.6. Graphic representation for the matrix element $\langle\pi|\theta^{q}_{\mu\nu}|\pi\rangle$. Each invariant amplitude in the π-π -current vertex *(a)* is reduced with the help of the general dispersion relation to a sum over various intermediate states (b). Singled out is the (dominant) f- meson contribution.

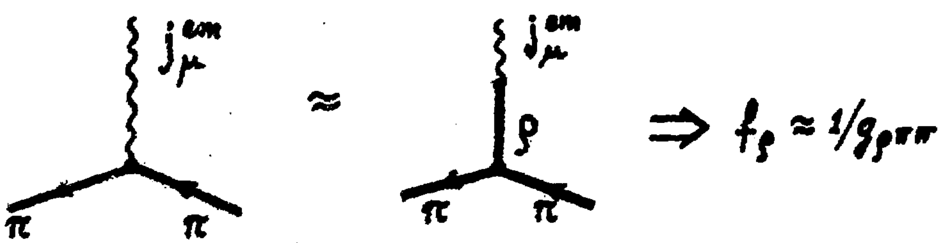


Fig.7. The ρ -meson dominance in the matrix element $\langle\pi|j^{em}_{\mu}|\pi\rangle$. Assuming the complete saturation we have $f_{\rho} = 1/g_{\rho\pi\pi} \approx 0.16$ while the true value is $(f_{\rho})_{exp} = 0.18 \pm 0.01$.

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