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BOND-GRAPH MODELING OF NUCLEAR-REACTOR DYNAMICS

J. Louis Tylee
EG&G Idaho, Inc.
Idaho Falls, Idaho 83415

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Abstract

A tenth-order linear model of a pressurized water reactor (PWR) is developed using bond graph techniques. The model describes the nuclear heat generation process and the transfer of this heat to the reactor coolant. Comparisons between the calculated model response and test data from a small-scale PWR show the model to be an adequate representation of the actual plant dynamics. Possible application of the model in an advanced plant diagnostic system is discussed.

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Introduction

Recently, there has been a renewed interest in the protection and control of nuclear power plants. Specifically, much attention is being paid to the problem of supplying the plant operator with the information needed to properly ascertain plant status. The current approach is to use digital computer-generated CRT color graphics to display plant measurements. A suggested improvement to this approach is to implement on-line small dynamic models of different plant subsystems. These models would be used to check the consistency of the plant measurements being taken and alert the operator to any anomalies noted. The models could also be used to predict the plant response to proposed operator control actions or could be used in a Kalman filter to provide optimal estimates of critical plant variables. In this paper, such a model of the reactor in a pressurized water reactor (PWR) power plant is developed and evaluated.

In deriving a model for use in a real-time diagnostics system, we must take into account two conflicting considerations: accuracy and speed. That is, a highly detailed model would probably not be able to operate in a

real-time environment, yet a fast, over-simplified model would be of little value. The development procedure then is to consider several candidate model structures and find the model which represents an optimum balance between accuracy and speed. It is thus desirable to be able to modify the candidate models with a minimum amount of computer reprogramming--the bond graph modeling approach [1] provides such a capability. In fact, in developing linear models, such as the reactor model considered here, no programming at all is required. The program ENPORT [2], once given a description of the graph structure, automatically generates the system differential equations and simulates its response to specified inputs. These advantages of model adaptability and automatic equation derivation dictated the selection of bond graph modeling in this project.

The model described in this paper is a linearized, discrete representation of the nuclear heat generation process and the transfer of the generated heat through the fuel rods into the reactor coolant. Reactivity feedback effects due to changes in fuel temperature, coolant temperature, and control rod motion are modeled, as is the direct deposition of heat into the coolant. A linear model is selected as opposed to a nonlinear model because of its simpler formulation, faster time propagation, and the ability to directly apply optimal estimation and control theories. The accuracy of the model has been validated by comparing model predictions to actual transient data obtained from a small test reactor.

Bond Graph Model

For the bond graph novice, references [1], which describes the modeling of electrical, mechanical, hydraulic, thermal, and other dynamic systems

using bond graphs, and [3], which defines the bond graph language, should provide ample background. We first develop the bond graph for the reactor kinetics and then the nuclear fuel rod.

Reactor Kinetics Model

In most transient reactor studies, the generation of power within the nuclear fuel is calculated using the time-dependent point kinetics equations [4]. These equations are derived by performing population balances on the various neutrons generated in the fission process. Unfortunately, the variables in these equations cannot be classified using the standard effort-flow definitions of bond graphs, thus we must rely on what is called a pseudo-bond graph. A pseudo-bond graph is simply a mathematical convenience used to define differential equations for systems in which the variables (efforts, flow, powers) do not necessarily correspond to the accepted bond graph definitions. Figure 1 shows such a pseudo-bond graph of the reactor power linear differential equations.

From Figure 1, we can write the power generation equation as:

$$\frac{\ell}{\beta}\,\dot{\delta P} = P_o\delta\rho - \delta P + \sum_{i=1}^{N} \lambda_i \delta\Psi_i \qquad (1)$$

where: ℓ - prompt neutron generation time (s)

β - effective fraction of delayed neutrons

δP - deviation of reactor power from nominal (MW)

$\delta\rho$ - net reactivity ($)

P_o - nominal (initial) reactor power (MW)

λ_i - decay constant of ith delayed neutron group (s^{-1})

$\delta\Psi_i$ - ith delayed neutron group energy deviation from nominal (MW-s)

and N is the number of delayed neutron groups modeled (usually six).

Although the delayed neutrons, those which appear long after the fission event, account for less than one percent of the fission neutrons, they play an important role in the reactor kinetics. Figure 2 shows a pseudo-bond graph for the equation describing the energy properties of the ith delayed neutron group. From this graph, we can write:

$$\frac{1}{\theta_i}\,\dot{\delta\Psi}_i = \delta P - \frac{\lambda_i}{\theta_i}\,\delta\Psi_i \qquad (2)$$

where θ_i is the fraction of the delayed neutrons in group i. Forming N of these bond graphs (one for each delayed neutron group) and connecting them at a constant flow junction, i.e., a 1 junction, will yield the summation term required in equation (1).

The remaining variable to address in discussing the reactor kinetics equations is $\delta\rho$, the net reactivity seen in (1). The net reactivity is simply a measure of the critical state of the reactor; a positive $\delta\rho$ indicating a supercritical reactor, a negative $\delta\rho$ a subcritical reactor, while a zero $\delta\rho$ indicates the reactor is critical. In a PWR, there are four major sources of reactivity. Moving the control rods into a reactor core tends to decrease the reactor power while withdrawing the rods tends to cause a power increase. Chemical shim, that is modifying the concentration of a neutron absorbing chemical, such as boric acid, in the reactor coolant, is also used to control power level. Finally, changes in fuel temperature and core coolant temperature affect the net reactivity due to the nuclear Doppler effect [4] and the change in moderator density, respectively. In the linear model developed here, we consider each of these reactivity sources except chemical shim, so $\delta\rho$ is modeled as:

$$\delta\rho = \delta\rho_{rod} + \alpha_f \delta T_f + \alpha_m \delta T_m \qquad (3)$$

where: $\delta\rho_{rod}$ - net reactivity due to control rod motion ($)

α_f - Doppler reactivity coefficient ($/K)

δT_f - deviation of fuel temperature from nominal (K)

α_m - moderator (coolant) reactivity coefficient ($/K)

δT_m - deviation of moderator temperature from nominal (K)

Both α_f and α_m are usually negative which assures reactor stability during power excursions. The fuel and coolant temperatures are computed in the fuel rod model which is discussed next.

Fuel Rod Model

A typical commercial PWR will have thousands of fuel rods in the core. Each rod is made up of small UO_2 pellets stacked in Zircaloy tubes or cladding which are usually sealed with helium gas. The heat generated within the fuel rod is transferred to the flowing coolant which is eventually used to create steam which in turn drives turbines to generate power. In this model, we consider only a single average fuel rod with one temperature node in the nuclear fuel and one node in the surrounding clad. A node representing the average core coolant temperature is also included in the model whose bond graph is shown in Figure 3. (This fuel rod model is also a pseudo-bond graph since, even though temperature and heat flow are standard effort and flow variables for thermal systems, the product of temperature and heat flow is not a power as is required in a true bond graph.)

A first thing to note in Figure 3 is that not all of the power generated within the fuel is transferred through the fuel rod, i.e. the power input

to the fuel node is $\gamma\delta P$, where $\gamma < 1$. This is typical of PWR plants--some of the power, usually about two percent, is directly deposited into the core coolant. Note that this direct heat deposition is included in the bond graph with the additional flow source at the core coolant node. From Figure 3, we obtain three differential equations:

$$M_f C_{pf} \dot{\delta T}_f = \gamma\delta P - U_{fc}(\delta T_f - \delta T_c) \tag{4}$$

$$M_c C_{pc} \dot{\delta T}_c = U_{fc}(\delta T_f - \delta T_c) - U_{cm}(\delta T_c - \delta T_m) \tag{5}$$

$$M_m C_{pm} \dot{\delta T}_m = (1-\gamma)\delta P + U_{cm}(\delta T_c - \delta T_m) - 2W_c C_{pm}(\delta T_m - \delta T_{in}) \tag{6}$$

where: M_f - mass of fuel in core (kg)

M_c - mass of cladding in core (kg)

M_m - mass of moderator (coolant) in core (kg)

C_{pf} - specific heat of fuel (MW-s/kg-K)

C_{pc} - specific heat of clad (MW-s/kg-K)

C_{pm} - specific heat of coolant (MW-s/kg-K)

δT_c - deviation of clad temperature from nominal (K)

γ - fraction of power into fuel rod

U_{fc} - heat transfer coefficient between fuel and clad (MW/K)

U_{cm} - heat transfer coefficient between clad and coolant (MW/K)

W_c - core coolant flow rate (kg/s)

δT_{in} - deviation of core inlet temperature from nominal (K)

Combining this fuel rod model with the power model in Figure 1, six of the delayed neutron group bond graphs in Figure 2, and the net reactivity equation in (3), yields the complete bond graph model of the reactor dynamics shown in Figure 4. Note that the three subsystems, i.e. net reactivity,

reactor kinetics, and fuel rod are connected using active bonds (only an effort or flow variable is transmitted). The kinetics bond graph is connected to the fuel rod model via transformers between bonds 24 and 25 and bonds 38 and 39, with respective transformer moduli of γ and $(1-\gamma)$. Recall from equation (3) that $\delta\rho$ requires knowledge of δT_f and δT_m, hence the transformers linking the fuel rod and reactivity submodels. The modulus of the transformer between bonds 26 and 27 is the inverse of α_f, the Doppler coefficient, while the transformer modulus between bonds 36 and 37 is the inverse of α_m, the moderator density reactivity coefficient. Finally, the effort source at bond 44 is $\delta\rho_{rod}$, the control rod reactivity.

In the next section, we will use the bond graph in Figure 4, which yields ten first-order differential equations, to simulate an actual PWR transient.

Model Validation

If the linear bond graph reactor model just developed is to provide accurate operator information in a plant diagnostics system, it is important that it be a valid representation of the dynamics of an actual PWR. Such a validation was performed using test data from the LOFT (Loss of Fluid Test) reactor located at the Idaho National Engineering Laboratory near Idaho Falls, Idaho. The LOFT plant can generate 50 MWt and is used to analyze both operational transients and loss of coolant accidents in pressurized water reactor plants.

To obtain the transient response of the reactor model, the computer program ENPORT [2] was employed. ENPORT is a program written especially to analyze linear bond graphs. The bond graph is described to the program by

numbering each bond as in Figure 4 and parameter values are supplied. Using this information, ENPORT generates the system state equations in continuous and discrete time and plots the response of the system variables to specified inputs. This automatic simulation capability relieves the analyst of many tedious programming details.

As described in the fuel rod model derivation, in a PWR, the heated primary coolant from the reactor flows through steam generators to create the steam needed to drive the plant turbines. The primary coolant then returns to the reactor for reheating. The test transient used here to validate the physical accuracy of the reactor model is a decrease in the level setpoint of the steam generator water level controller. This transient was initiated with the LOFT reactor at 50% power and model parameters for this test condition are given in the appendix. Decreasing the level setpoint causes an immediate closure of the feedwater valve by the level controller which results in less heat being extracted from the primary coolant by the steam generator. This increase in primary coolant temperature is shown in Figure 5, where the reactor inlet temperature response during the transient is plotted. Recall that it is this temperature change that is the effort source at bond 43 of the reactor bond graph. By describing the inlet temperature variation to ENPORT using the polynomial curve fit shown in Figure 5, the response of other system variables can be computed. Figures 6 and 7 show two such responses.

Figure 6 compares the ENPORT calculation of average core coolant temperature (the effort variable at bond 40) with the LOFT test data. The increasing inlet temperature is seen to cause a corresponding increase in coolant temperature and the model is seen to predict the test data accurately, especially in the first 40 seconds. The increasing core coolant

temperature causes a corresponding decrease in reactor power (the flow variable at bond 4) via moderator density reactivity feedback as seen in Figure 7. Again, the ENPORT model provides a faithful prediction of the actual response. The power begins to increase after 40 seconds due to the corresponding drop in coolant temperature. The linear model seems to provide an excellent representation of the LOFT reactor dynamics.

It should be noted here that the water level setpoint decrease transient displayed in Figures 5 through 7 represents a relatively small perturbation on the LOFT plant and hence the linear model was able to predict it quite well. In a more severe transient, where the plant would be perturbed far from the nominal operating point upon which the model is based, a fixed linear model may not be adequate. To overcome this inadequacy, it would be necessary to have the on-line capability to generate a new linear model when it has been determined that the plant has moved sufficiently away from nominal. How to make such a determination and ways to perform the relinearization are current areas of study.

Conclusions

Mentioned earlier as a positive benefit of the bond graph modeling approach and the ENPORT program is the ease at which a model can be modified and resimulated. We could now exploit this adaptability in attempts to improve the reactor model. Studies have shown that for relatively slow changes in the net reactivity, the differential equation for core power, equation (1), can be replaced by the algebraic approximation:

$$\delta P = P_0 \delta \rho + \sum_{i=1}^{N} \lambda_i \delta \Psi_i \qquad (7)$$

This approximation is possible because the power equation has a time constant much smaller than all other system time constants. To implement this approximation in the bond graph of Figure 4, we simply eliminate bond 4 and reverse the causality of bond 3. The main advantage to using the algebraic power equation would be faster running times since the solution time step could be increased with the elimination of the fast dynamics of the differential power equation.

It may also be possible to use fewer delayed neutron groups in the power calculation. Previous work [5] has shown that adequate accuracy can be retained with as few as one delayed neutron group in the kinetics equations. It would be quite easy to eliminate delayed neutron equations from the bond graph of Figure 4 to examine the effect of fewer groups on the model performance. Again, the prime benefit from the elimination of delayed neutron groups would be decreased running time since each deleted group results in one less state equation and hence a lower order model.

Just as easily as we can examine the effects of decreased model complexity on the model's physical accuracy, we could also augment the reactor bond graph with additional model structures. Examples of such augmentations would be finer nodalization of the fuel rod with more nodes in the fuel and cladding, or perhaps additional core coolant nodes to simulate the axial variation of power within the reactor core. We could also modify the bond graph to model more than one fuel rod, perhaps modeling a hot fuel rod (i.e., high power density) which would be useful in calculating the reactor DNBR (departure from nucleate boiling ratio) during transients. All of these additions would of course enhance the model accuracy at the expense of increased model running time.

In this paper, we have seen that a linear reactor model can provide

excellent representations of actual PWR plant dynamics. The problems associated with being restricted to linear models can probably be eliminated by having the capability of relinearizing the model as it deviates from its operating point. Such linear models when implemented on-line could provide a fast and accurate plant diagnostic tool.

Acknowledgements

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Appendix

LOFT Reactor Model Parameters at 50% Power Conditions:

Reactor kinetics:

P_o = 50 MW	λ_1 = 3.87 x^{-1}	θ_1 = 0.026
ℓ = 19.52 x 10^{-6} s	λ_2 = 1.40 s^{-1}	θ_2 = 0.128
β = 0.007259	λ_3 = 0.311 s^{-1}	θ_3 = 0.407
ν = 0.9812	λ_4 = 0.115 s^{-1}	θ_4 = 0.188
	λ_5 = 0.0307 s^{-1}	θ_5 = 0.213
	λ_6 = 0.0127 s^{-1}	θ_6 = 0.038

Fuel:

α_f = -5.400 x 10^{-3} S/K	C_{pf} = 3.038 x 10^{-4} MW-s/kg-K
M_f = 1477.4 kg	U_{fc} = 0.1010 MW/K

Clad:

M_c = 303.1 kg	U_{cm} = 1.318 MW/K
C_{pc} = 3.186 x 10^{-4} MW-s/kg-K	

Coolant (moderator):

α_m = -5.105 x 10^{-2} S/K	C_{pm} = 5.242 x 10^{-3} MW-s/kg-K
M_m = 220.9 kg	W_c = 476.3 kg/s

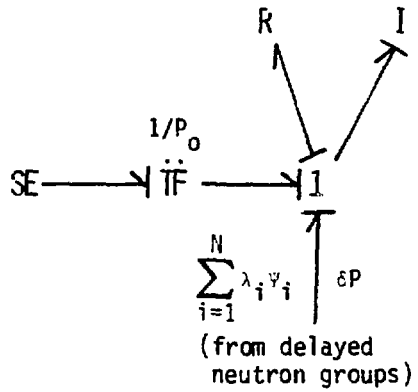


Figure 1. Pseudo-bond graph of nuclear power generation equation

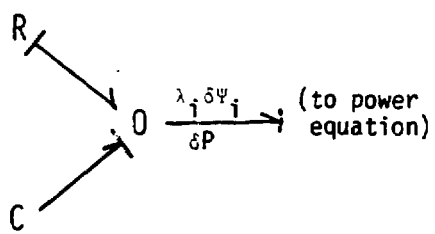
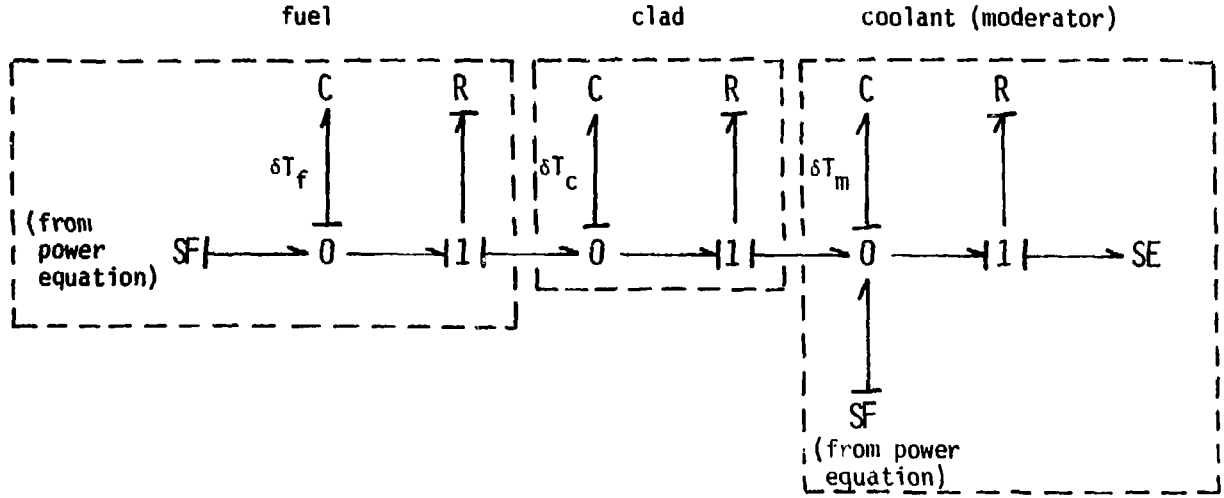


Figure 2. Pseudo-bond graph of delayed neutron group equation



$C_f = M_f C_{pf}$ $\quad C_c = M_c C_{pc}$ $\quad C_m = M_m C_{pm}$

$R_f = 1/U_{fc}$ $\quad R_c = 1/U_{cm}$ $\quad R_m = 1/2W_c C_{pm}$

$SF = \gamma \delta P$ $\quad SF = (1-\gamma)\delta P$

$SE = \delta T_{in}$

Figure 3. Fuel rod model bond graph

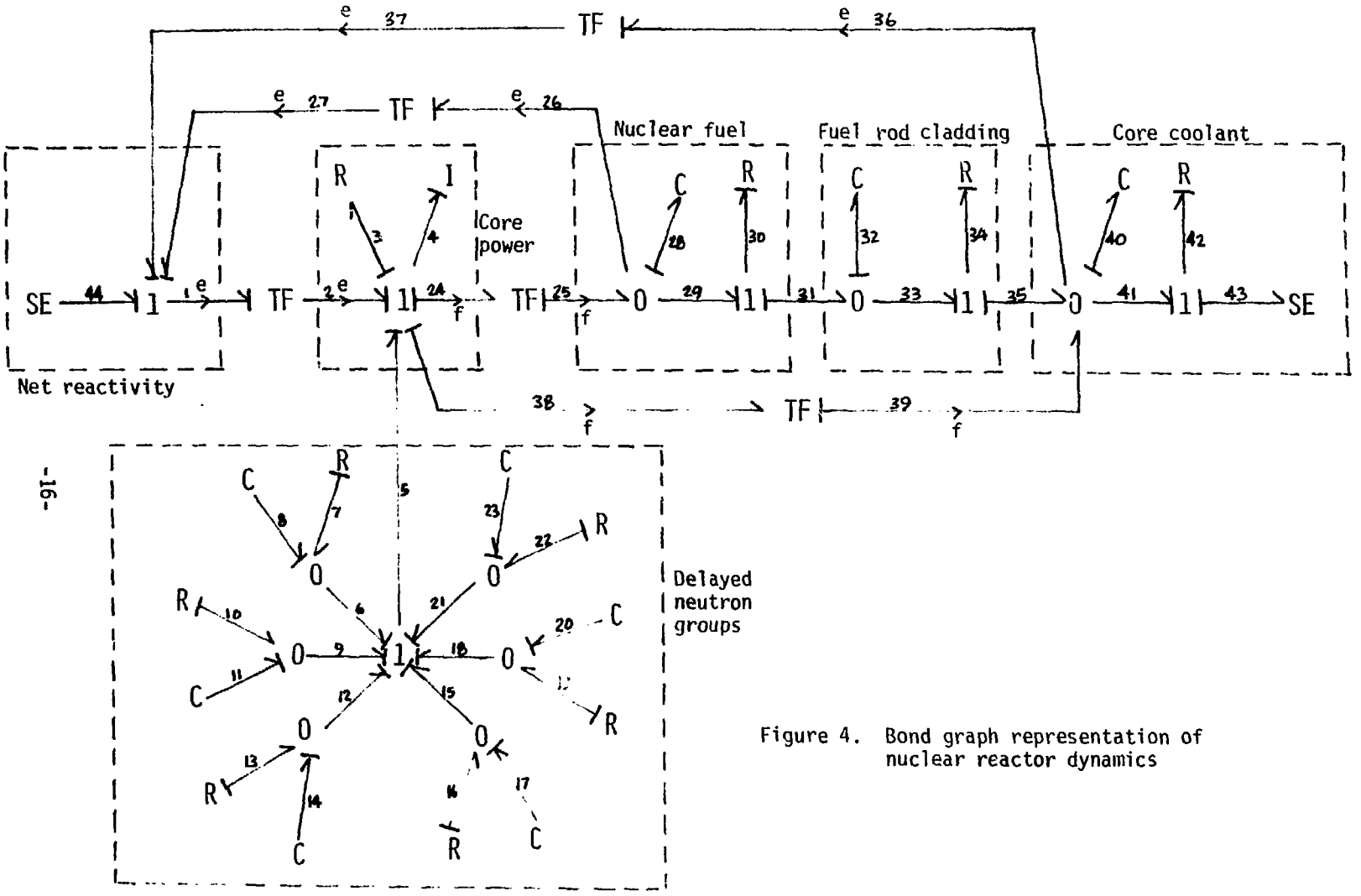


Figure 4. Bond graph representation of nuclear reactor dynamics

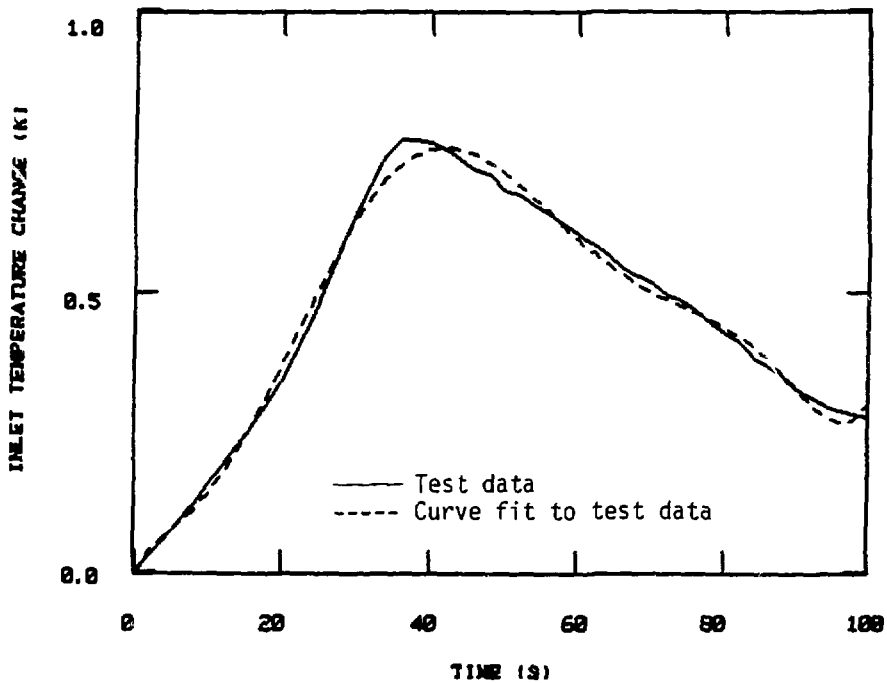


Figure 5. Steam generator water level controller setpoint decrease; reactor inlet temperature response

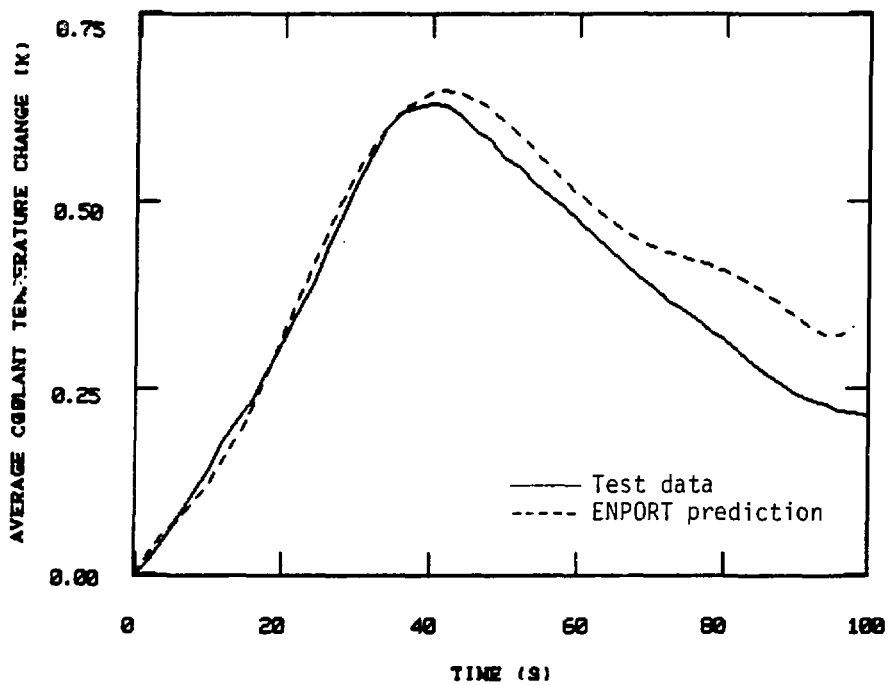


Figure 6. Steam generator water level controller setpoint decrease; core coolant average temperature response

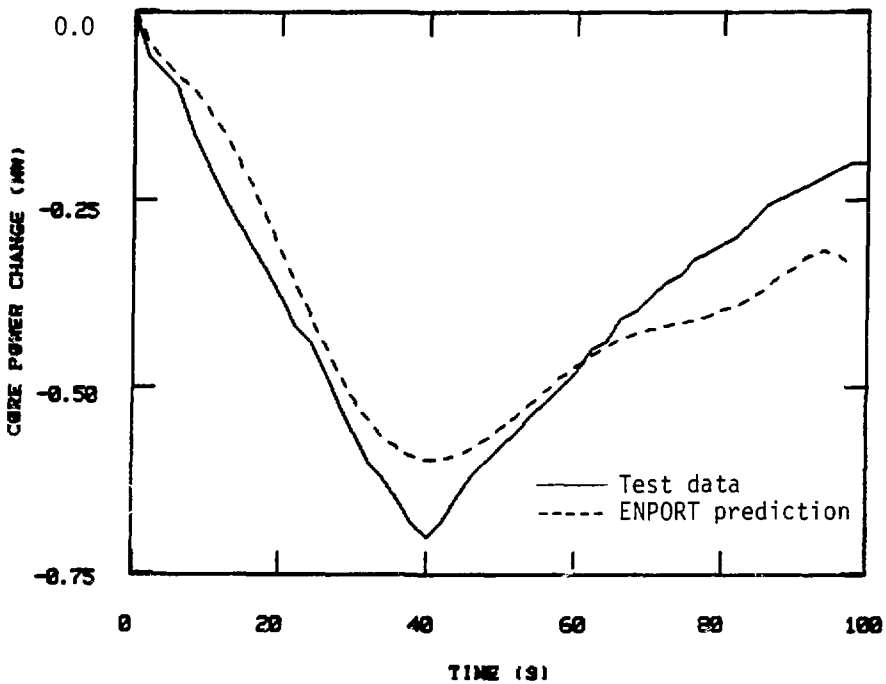


Figure 7. Steam generator water level controller setpoint decrease; core power response

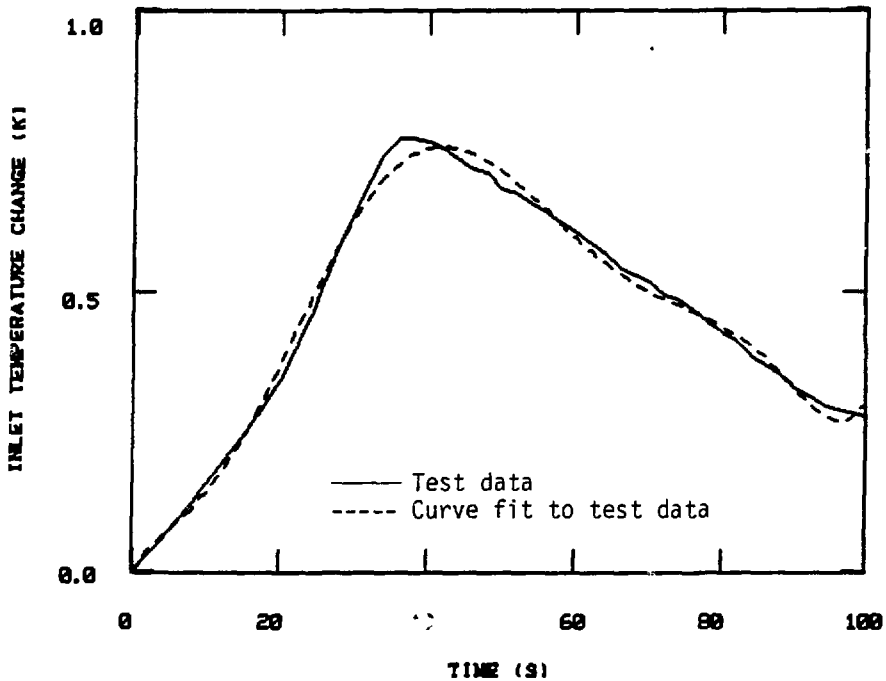


Figure 5. Steam generator water level controller setpoint decrease; reactor inlet temperature response