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QCD CALCULATION OF $\pi\gamma\gamma$ VERTEX AT EQUAL EUCLIDEAN q^2 OF BOTH PHOTONS

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A b s t r a c t

The form factor of the $\pi^0\gamma\gamma$ vertex at equal space-like q^2 of the photons $\left(q_1^2 = q_2^2 = -Q^2 \right)$ and a small p^2 of the pion is calculated within the QCD operator product expansion. Explicit expressions for leading perturbative $O(\alpha_s)$ and non perturbative $C(Q^{-2})$ preasymptotic corrections are derived. To find the latter correction matrix elements of operators of dimension $d = 5$ between the pion and vaccum are calculated. The result for the form factor smoothly matches at $Q^2 \simeq 0.5$ GeV^2 the estimate based on the vector meson dominance model normalized to the famous current algebra (with ABJ anomally) prediction for the on-shell $\pi^0\gamma\gamma$ vertex.

1. Introduction

This paper invokes perturbative and non perturbative methods of QCD at short distances to calculate the form factor $F(p^2, q_1^2, q_2^2)$ of the $\pi^0\gamma\gamma$ vertex at equal space-like values of q^2 of both photons: $q_1^2 = q_2^2 = -Q^2$ $(Q^2 > 0)$ and at small p^2 of the pion: $p^2 \approx m_\pi^2 \simeq 0$. It turns out that this form factor can be calculated with a reasonable accuracy for values of Q^2 as low as $Q^2 \gtrsim 0.5$ GeV^2 without detailed knowledge of the quark wave function inside pion. Together with the famous current algebra result for $F(0,0,0)$ (see e.g. Ref. 1) this gives a description of the function $F(0, -Q^2, -Q^2)$ for all values of Q^2, but in the intermediate region $0 < Q^2 \lesssim 0.5$ GeV^2. We find however that an estimate of the form factor based on the vector meson dominance model (VDM) interpolates rather smoothly between the results at $Q^2 = 0$ and $Q^2 \gtrsim 0.5$ GeV^2 (and at higher values of Q^2 we find that VDM breaks down).

It seems conceivable that the form factor can be measured experimentally in the process $e^+e^- \to e^+e^- \pi^0$ (or $ee \to ee\pi^0$) at colliders having not necessarily a super high energy ($\sqrt{s} \gtrsim 2 - 3$ GeV would be quite enough) but with luminosity substantially higher than that of existing machines. At present the form factor we discuss is mainly of a theoretical interest since its description within QCD is rather simple in comparison with other applications of the short-distance QCD to processes at moderate Q^2. For example a calculation of electromagnetic form factors of charged mesons (π, K, ρ) [2] for reasonable values of Q^2 requires additional assumptions about quark wave functions of the mesons. Another example is the QCD description of the deep inelastic scattering where again the interesting region of moderate Q^2 is sensitive to contribution of higher twist operators whose matrix elements over nucleon can hardly be estimated [3]. On the contrary in the process to be discussed the $1/Q^2$ corrections to the leading term are related through the current algebra to known vacuum matrix elements (see eq. (7) below). In this respect the calculations in this paper are analogous to the QCD analysis of the e^+e^- annihilation into hadrons in resonance regions [4,5], however the former do not need dispersion relations for interpretation in terms of measurable quantities.

To formulate the result of this paper we define the normalization and the sign of the $\pi^0\gamma\gamma$ form factor F in such a way that to the arrangement of momenta and polarizations shown in fig. 1 corresponds the following expression for the vertex

$$F(p^2, q_1^2, q_2^2)\, \epsilon_{\mu\nu\lambda\sigma}\, q_{1\lambda}\, q_{2\sigma}\, a_{1\mu}\, a_{2\nu}\, \varphi_\pi \qquad (1)$$

where q_1, q_2 and a_1, a_2 are respectively the 4-momenta and the polarization amplitudes of the outgoing photons, and p and φ_π are the 4-momentum and the amplitude of the pion. On the pion mass-shell the form factor can also be defined by the following formal relation

$$F(m_\pi^2, q_1^2, q_2^2)\, \epsilon_{\mu\nu\lambda\sigma}\, q_{1\lambda}\, q_{2\sigma}\, \varphi_\pi =$$

$$= i\int e^{-\frac{i}{2}(q_1-q_2)x} \langle 0|\, T\{ j_\mu(x), j_\nu(0)\}\, |\pi^0(p)\rangle\, d^4x \qquad (2)$$

where j_μ is the electromagnetic current. In what follows only its part associated with up and down quarks will be essential:

$$j_\mu = e\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d\right). \qquad (3)$$

We remind that the current algebra with the ABJ anomally gives [1] the value of $F(0,0,0)$:

$$F(0,0,0) = -\frac{\sqrt{2}\,\alpha}{\pi f_\pi} \qquad (4)$$

where $f_\pi \simeq$ 130 MeV is the $\pi \to \mu\nu$ decay constant

$$\langle 0 |\, \bar{d}\,\gamma_\mu \gamma_5 u \,| \pi^+(p) \rangle = i f_\pi p_\mu \varphi_\pi . \qquad (5)$$

(Note that the sign of γ_5 used corresponds to the formula $Tr(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma\gamma_5) = 4i\,\epsilon_{\mu\nu\lambda\sigma}$ and $\epsilon_{0123} = 1$. It should be also noted that in what follows we suppress the amplitude φ_π in expressions for matrix elements). The $\pi^0 \to \gamma\gamma$ decay rate is given by

$$\Gamma(\pi^0 \to \gamma\gamma) = |F(m_\pi^2, 0, 0)|^2 \frac{m_\pi^3}{64\pi} . \qquad (6)$$

Using the current algebra result (4) in this formula one reproduces the experimental number for the π^0 decay rate thus confirming the natural suggestion that the form factor F varies only slightly when one of its arguments is shifted by m_π^2. Throughout this paper we shall employ this suggestion usual for applications of the current algebra to pions.

Our final result for the so defined form factor has the form

$$F(0,-Q^2,-Q^2) =$$

$$= -\frac{4\sqrt{2}\,\pi\alpha f_\pi}{3Q^2}\left(1-\frac{5}{6}\frac{\alpha_s(Q^2)}{\pi}-\frac{1}{9}\frac{m_0^2}{Q^2}+\dots\right), \tag{7}$$

where $\alpha_s(Q^2)$ is the QCD running coupling constant:

$$\alpha_s(Q^2) = \frac{4\pi}{b\ln(Q^2/\Lambda^2)}, \tag{8}$$

and m_0^2 is the following ratio of vacuum matrix elements

$$m_0^2 = \frac{\langle 0|\bar\psi\,\sigma_{\alpha\beta}F_{\alpha\beta}\,\psi|0\rangle}{\langle 0|\bar\psi\psi|0\rangle}. \tag{9}$$

Here ψ stands for u- or d- quark field,

$$\sigma_{\alpha\beta} = \frac{i}{2}(\gamma_\alpha\gamma_\beta-\gamma_\beta\gamma_\alpha), \quad F_{\alpha\beta} = \frac{g}{2}G^a_{\alpha\beta}\lambda^a \tag{10}$$

$g^2 = 4\pi\alpha_s$, λ^a are the Gell-Mann matrices and $G^a_{\alpha\beta}$ is the gluon field tensor. The dots in eq. (7) stand for terms of higher order in α_s/π and for higher nonperturbative terms ($O(1/Q^6)$) which we will only briefly mention in this paper (see sect. 4).

The ratio of the vacuum mean values (9) has been first considered in Ref. 6 in connection with analysis of leptonic decays of charmed D and F mesons. It has been concluded that the ratio (9) is positive and its value is $m_0^2 \approx$ 0.5 - 1 GeV^2. The same ratio is involved in the recent QCD calculation of baryon masses [7] which favours higher values of m_0^2 in this interval: $m_0^2 \simeq 1\ \mathrm{GeV}^2$, and this is the numerical value we use below. (It should be mentioned that in a more accurate determination of m_0^2 one should take into account the anomalous dimensions of the operators and specify the normalization point. We do not go here into these details since the term with m_0^2 in eq. (7) is essential at values of Q^2 which are close to those considered in Refs. 6,7, therefore we neglect a small renormalization of the ratio (9) at our level of accuracy).

The derivation of eq. (7) will be given in the next two sections. In section 2 we consider the operator expansion for the T-product of currents involved in eq. (2), and then in section 3 we calculate non trivial matrix elements of $d = 5$ operators between the pion and vacuum. Section 4 contains numerical estimates including a comparison with the vector dominance model and a brief discussion of higher non perturbative corrections.

2. The Operator Expansion

For large space-like values of the 4-vector $q = (q_1-q_2)/2$ the integral in eq. (2) is dominated by small intervals x^2 between the points in which the currents are acting. Therefore it is appropriate to use operator expansion for the T-product of the currents involved in eq. (2).

In performing the operator expansion it is convenient to keep in mind further sandwitching of the operators between the pion and vacuum. Thus as seen from eq. (2) one can retain only terms antisymmetric in the Lorentz indices μ and ν.

Note also that for $q_1^2 = q_2^2 = -Q^2$ and fixed p^2 what counts is the dimension of operators in the expansion rather than their twist. This follows from the relation

$$pq = \frac{1}{2}\left(q_1^2 - q_2^2\right) = 0$$

which shows that additional Lorentz indices of operators do not result in factors of order Q^2. (An opposite situation arises when one considers the same form factor F for essentially asymmetric q_1 and q_2, i.e. when $|q_1^2 - q_2^2| \sim |q_1^2 + q_2^2|$. In this case contribution of comparable magnitude is received from operators of different dimension but with the same twist. An example of such a situation in a somewhat analogous problem was considered in Ref. [8] where form factors for decays of heavy quarkonia were analysed at a fixed q_1^2 and large q_2^2).

Next remark is that because of the relation

$$\epsilon_{\mu\nu\lambda\sigma}\, q_{1\lambda} q_{2\sigma} = \epsilon_{\mu\nu\lambda\sigma}\, q_{1\lambda} p_\sigma \tag{11}$$

the amplitude (1) has a kinematical zero at $p = 0$ (i.e. when $q_2 = -q_1$). However, this zero appears only after calculation of the pion-to-vacuum matrix element, while the coefficient functions in the operator expansion are non-zero at $q_2 = -q_1$. Therefore when calculating the coefficient functions it is legitimate to put

$$q_1 = -q_2 = q \tag{12}$$

and to account for the pion momentum to the first order when calculating matrix elements.

We proceed now to calculation of the coefficient functions. In the Born approximation the operator expansion can be performed by using the following formula

$$T_{\mu\nu}(\bar\Psi,\psi)=i\int e^{-iqx}\,T\left(\bar\Psi(x)\gamma_\mu\psi(x),\bar\Psi(0)\gamma_\nu\psi(0)\right)d^4x=$$
$$=-\bar\psi\left\{\gamma_\mu\frac{1}{\gamma\mathcal{P}+\gamma q-m_\psi}\gamma_\nu+\gamma_\nu\frac{1}{\gamma\mathcal{P}-\gamma q-m_\psi}\gamma_\mu\right\}\psi\,, \qquad (13)$$

where $\gamma a=\gamma_\sigma a_\sigma$. $\mathcal{P}$ is related to the covariant derivative

$$\mathcal{P}_\sigma=iD_\sigma=i\partial_\sigma+gA_\sigma^a\frac{\lambda^a}{2}\,, \qquad (14)$$

and ψ stands for u or d. Expansion of the expression (13) in local operators amounts to expansion in series in ($\gamma\mathcal{P}-m$). The first term of this series is obviously given by

$$T_{\mu\nu}^{(3)}(\bar\Psi,\psi)=-\bar\psi(0)\left\{\frac{\gamma_\mu(\gamma q)\gamma_\nu}{q^2}-\frac{\gamma_\nu(\gamma q)\gamma_\mu}{q^2}\right\}\psi(0)$$
$$=\frac{2}{Q^2}\,i\epsilon_{\mu\nu\lambda\sigma}\,q_\lambda\,\bar\psi(0)\gamma_\sigma\gamma_5\psi(0)\,, \qquad (15)$$

(the upper index of T indicates the dimension of the operator involved).

The next term of the expansion which gives contribution to the amplitude (2) arises in the second order in ($\gamma\mathcal{P}-m$). When calculating its explicit form it is convenient to suppress manifestly symmetric in μ and ν terms and also to keep in mind that the amplitude under consideration is antisymmetric in permutation of the Lorentz index of q with either of the indices μ and ν. Using also the commutation relation

$$[\mathcal{P}_\alpha, \mathcal{P}_\beta] = i F_{\alpha\beta}, \tag{16}$$

and the equation of motion

$$(\gamma \mathcal{P} - m_\psi)\psi = 0, \tag{17}$$

and neglecting terms of order m_ψ^2 one can arrive at the following expression for terms in the operator expansion due to d = 5 operators which potentially give contribution to the $\pi^0 \gamma\gamma$ vertex:

$$\begin{aligned} T^{(5)}_{\mu\nu}(\bar\psi, \psi) = \frac{4q_\lambda}{Q^4} \bar\psi(0) \{ &\epsilon_{\mu\alpha\nu\sigma} F_{\alpha\lambda} \gamma_\sigma \gamma_5 \\ &+ \epsilon_{\mu\lambda\alpha\sigma} F_{\alpha\nu} \gamma_\sigma \gamma_5 + 2i F_{\lambda\nu} \gamma_\mu - i F_{\mu\nu} \gamma_\lambda \\ &- \epsilon_{\mu\nu\lambda\sigma} i \gamma_\sigma \gamma_5 (\mathcal{P}_\alpha \mathcal{P}_\beta + \mathcal{P}_\beta \mathcal{P}_\alpha) \frac{q_\alpha q_\beta}{q^2} \} \psi(0). \end{aligned} \tag{18}$$

After sandwiching between pion and vacuum terms of the type (15) give the leading at large Q^2 asymptotic behaviour of the form factor, while those of the type (18) describe the non perturbative correction suppressed by one additional power of Q^{-2}.

As to the leading perturbative correction proportional to $\alpha_s(Q^2)$, it arises from the corresponding radiative correction to the coefficient function of the d = 3 operator. Clearly this correction does not contain logarithmic terms like $\alpha_s \ln Q^2$ since the only d = 3 operator relevant is the axial current (see eq. (15)) which has vanishing anomalous dimension. The finite nonlogarithmic terms can be found by any convenient procedure, e.g. by using dimensional regularization. (No difficulty with dimensional regularization arises due to γ_5 since the relevant structure is

$$2i\,\epsilon_{\mu\nu\lambda\sigma}\,\gamma_\sigma\gamma_5 = \gamma_\nu\gamma_\lambda\gamma_\mu - \gamma_\mu\gamma_\lambda\gamma_\nu\,,$$

and the right hand side can be generalized to a space-time dimension $n \neq 4$ in the standard way). The result of this perturbative calculation amounts to multiplication of the expression (15) by the following factor

$$\left(1 - \frac{5}{6}\,\frac{\alpha_s(Q^2)}{\pi} + O\left(\left(\frac{\alpha_s}{\pi}\right)^2\right)\right). \tag{19}$$

As it is mentioned above we suppress here perturbative corrections to coefficient functions of $d = 5$ operators.

3. Matrix elements

To calculate the form factor F as given by eq. (2) one should use in this equation the terms of the form (15) and (18) for the product of electromagnetic currents of the up and down quarks and also explicitely account for the quark electric charges and the electromagnetic coupling e which enters the definition (3) of the current. Doing so we get

$$\begin{aligned}
&F(0,-Q^2,-Q^2)\,\epsilon_{\mu\nu\lambda\sigma}\,q_\lambda P_\sigma = \\
&4\pi\alpha\,\langle 0|\,\frac{4}{9}\,T_{\mu\nu}(\bar u,u) + \frac{1}{9}\,T_{\mu\nu}(\bar d,d)\,|\pi^0\rangle = \\
&\frac{4\pi\alpha}{3\sqrt{2}}\,\langle 0|\left\{T^{(3)}_{\mu\nu}(\bar d,u) + T^{(5)}_{\mu\nu}(\bar d,u) + \ldots\right\}|\pi^+\rangle\,,
\end{aligned} \tag{20}$$

where in the last equation we have performed isotopic rotation since discussion of matrix elements of the charged operators $T^{(n)}_{\mu\nu}(\bar{d},u)$ somewhat shortens notations.

The matrix element of the leading at large Q^2 term $T^{(3)}_{\mu\nu}(\bar{d},u)$ (see eq. (15)) is directly given by eq. (5). Therefore including also the perturbative correction factor (19) we get the following expression for the asymptotic behaviour of the form factor

$$F(0,-Q^2,-Q^2) = -\frac{4\sqrt{2}\,\pi\alpha f_\pi}{3Q^2}\left(1-\frac{5}{6}\frac{\alpha_s(Q^2)}{\pi}+\dots\right). \tag{21}$$

Let us consider now the matrix elements of the d = 5 operators involved in $T^{(5)}_{\mu\nu}(\bar{d},u)$ (see eq. (18) in which one should put $\bar{\psi} = \bar{d}$ and $\psi = u$). The on-shell matrix elements of the operators corresponding to the first two terms in figure brackets in eq. (18) have the following generic form

$$\langle 0|\bar{d}\,F_{\alpha\beta}\,\gamma_\sigma\gamma_5\,u|\pi^+\rangle = \frac{1}{3}\,C\,(g_{\beta\sigma}P_\alpha - g_{\alpha\sigma}P_\beta)\,, \tag{22}$$

where C is a constant to be determined. To find this constant we derive from eq. (22)

$$\langle 0|\partial_\alpha(\bar{d}\,F_{\alpha\beta}\,\gamma_\beta\gamma_5\,u)|\pi^+\rangle = -i\,C\,m_\pi^2\,. \tag{23}$$

On the other hand using the current algebra representation for the pion field

$$\pi(x) = \frac{1}{f_\pi m_\pi^2} \partial_\mu (\bar{u}(x)\gamma_\mu \gamma_5 d(x)) = \frac{i(m_u + m_d)}{f_\pi m_\pi^2} \bar{u}(x)\gamma_5 d(x),$$

and employing the standard reduction technique we get

$$\langle 0 | \partial_\alpha (\bar{d} F_{\alpha\beta} \gamma_\beta \gamma_5 u) | \pi^+ \rangle =$$

$$\frac{(m_u + m_d)}{f_\pi} \langle 0 | \int d^4x \, \delta(x_0) [\bar{d}(0) F_{0\beta}(0) \gamma_\beta \gamma_5 u(0), \bar{u}(x)\gamma_5 d(x)] | 0 \rangle$$

$$= -i \frac{(m_u + m_d)}{f_\pi} \langle 0 | (\bar{u}(0) \sigma_{0\beta} F_{0\beta}(0) u(0) + \bar{d}(0) \sigma_{0\beta} F_{0\beta}(0) d(0)) | 0 \rangle$$

$$= -i \frac{(m_u + m_d)}{4 f_\pi} \langle 0 | (\bar{u} \sigma_{\alpha\beta} F_{\alpha\beta} u + \bar{d} \sigma_{\alpha\beta} F_{\alpha\beta} d) | 0 \rangle$$

(the commutator involved in this chain of relations is a canonical one and we have also used in the last transition the relation

$$\langle 0 | \bar{\psi} \sigma_{\lambda\beta} F_{\sigma\beta} \psi | 0 \rangle = \frac{1}{4} g_{\lambda\sigma} \langle 0 | \bar{\psi} \sigma_{\alpha\beta} F_{\alpha\beta} \psi | 0 \rangle$$

which follows from Lorentz invariance of the vacuum).
The quark masses ($m_u + m_d$) can be eliminated with the help of the well known equation (see e.g. in Ref. 5):

$$f_\pi^2 m_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle .$$

As a result we get for the constant C in eqs. (22), (23) the following expression

$$C = -\frac{f_\pi}{4}\langle 0|\bar{\psi}\,\sigma_{\alpha\beta}F_{\alpha\beta}\,\psi|0\rangle / \langle 0|\bar{\psi}\psi|0\rangle$$
$$= -f_\pi m_0^2/4 \tag{24}$$

(the parameter m_0^2 is defined by eq. (9)).

The matrix elements corresponding to the third and the fourth terms in the figure brackets in eq. (18) are calculated by the same trick. Use parity conservation in QCD to write in general

$$\langle 0|\bar{d}\,i F_{\alpha\beta}\gamma_\rho u|\pi^+\rangle = \frac{1}{3}C'\epsilon_{\alpha\beta\rho\sigma}p_\sigma \tag{25}$$

with unknown coefficient C' . Apply then $\epsilon_{\alpha\beta\rho\lambda}\partial_\lambda$ to both sides of this equation, which gives

$$-iC'm_\pi^2 = \frac{1}{2}\langle 0|\partial_\lambda(\bar{d}\epsilon_{\alpha\beta\lambda\rho}\,i F_{\alpha\beta}\gamma_\rho u)|\pi^+\rangle,$$

and the latter matrix element can be calculated just in the same way as that in eq. (23). The result is C' = C where C is given by eq. (24).

The matrix element corresponding to the last term in eq. (18) can be calculated in linear in the pion momentum approximation. In this approximation it is determined by two constants A and B:

$$\langle 0|\bar{d}\,i\gamma_\sigma\gamma_5(\mathcal{P}_\alpha\mathcal{P}_\beta + \mathcal{P}_\beta\mathcal{P}_\alpha)u|\pi^+\rangle =$$
$$A g_{\alpha\beta}p_\sigma + B(g_{\alpha\sigma}p_\beta + g_{\beta\sigma}p_\alpha). \tag{26}$$

Multiplying both sides by $g_{\alpha\beta}$ and by $g_{\beta\sigma}$ and using eqs. (16) and (17) we get

$$(2A+B)\,p_\sigma = \langle 0|\bar{d}\, i\gamma_\sigma\gamma_5 \mathcal{P}^2 u|\pi^+\rangle\,,$$

$$(A+5B)\,p_\alpha = \langle 0|\bar{d}\, F_{\alpha\sigma}\gamma_\sigma\gamma_5 u|\pi^+\rangle - i(m_u+m_d)\langle 0|\bar{d}\,\mathcal{P}_\alpha\gamma_5 u|\pi^+\rangle \qquad (27)$$

(note that after the isotopic rotation in eq. (20) each quark should be considered as having mass $(m_u+m_d)/2$). The last term in the second equation is of order $f_\pi m_\pi^2 p_\alpha$ and we neglect it in comparison with the first one in the right hand side which is given by eqs. (22) and (24) and is equal to $C p_\alpha = -f_\pi m_0^2 p_\alpha/4$. As to the matrix element in the first of the equations (27) the operator $\mathcal{P}^2 = (\gamma\mathcal{P})(\gamma\mathcal{P}) - -\frac{1}{2}\sigma_{\alpha\beta}F_{\alpha\beta}$ due to eq. (17) can be substituted up to quadratic in quark mass terms by $-\frac{1}{2}\sigma_{\alpha\beta}F_{\alpha\beta}$. By means of the γ-matrix algebra the matrix element is then reduced to a combination of the amplitudes (22) and (25) which is equal to zero due to the relation $C' = C$. Therefore up to terms quadratic in the pion mass we find

$$2A + B = 0\,,$$

$$A + 5B = C$$

which gives $A = -C/9$, $B = 2C/9$.

This completes our discussion of matrix elements of the $d = 5$ operators involved in eq. (18). Collecting all terms together we get

$$\langle 0 | T^{(5)}_{\mu\nu}(\bar{d}, u) | \pi^+ \rangle = -\frac{8}{9}\frac{C}{Q^4}\epsilon_{\mu\nu\lambda\sigma} q_\lambda p_\sigma =$$

$$\frac{2}{9}\frac{f_\pi m_0^2}{Q^4}\epsilon_{\mu\nu\lambda\sigma} q_\lambda p_\sigma .$$

When used in eq. (20) this expression together with eq. (21) results in our final result given by eq. (7).

4. Comparison with the vector meson dominance model and discussion of results

It is interesting to insert numbers in eq. (7). We use $m_0^2 = 1$ GeV^2 and also take $\Lambda = 0.1$ GeV for determinatio of $\alpha_s(Q^2)$. Fig. 2 shows the plot of the form factor normalized to the current algebra value for $F(0,0,0)$:

$$\Phi(Q^2) \equiv F(0,-Q^2,-Q^2)/F(0,0,0) = \frac{4\pi^2 f_\pi^2}{3Q^2}\left(1 - \frac{5}{6}\frac{\alpha_s(Q^2)}{\pi} - \frac{1}{9}\frac{m_0^2}{Q^2}\right). \tag{28}$$

It should be kept in mind that this equation is an asymptotic one and is valid for large Q^2. However already for $Q^2 \simeq 0.5$ GeV^2 the preasymptotic corrections are quite moderate. The perturbative correction at $Q^2 = 0.5$ GeV^2 is around 10% and the non perturbative one is 22%. Therefore if one assumes that further corrections are of order of the first ones squared one can hope that the formula (28) has accuracy of order 10% at $Q^2 = 0.5$ GeV^2. For lower values of Q^2 this accuracy rapidly worsens. However as it is seen from fig. 2 just at the boundary of the applicability domain eq. (28) smoothly matches the estimate of $\Phi(Q^2)$ from VDM:

$$\Phi(Q^2)_{VDM} = \left(1 + \frac{Q^2}{m_\rho^2}\right)^{-2}. \tag{29}$$

Thus in the case considered we see a nice harmony between the current algebra with the ABJ anomally the vector meson dominance and the short-distance QCD which fit to each other in description of the form factor $F(0,-Q^2,-Q^2)$. This agreement is by no means trivial. It would have been completely destroyed if e.g. the non perturbative correction had opposite sign. Note also that in the case considered the situation is quite different from that in the QCD calculation of the charged pion electromagnetic form factor [2] where the asymptotic prediction is too low to match the vector dominance.

It is also plausible that the assumption about relative magnitude of further corrections can be verified. It is certainly true for the next perturbative term in the coefficient function of the $d = 3$ operator which can be evaluated by the standard technique of two-loop calculations in QCD. As to the next non perturbative term it appears due to $d = 7$ operators. It looks like that matrix elements of these can also be reduced by current algebra to rather simple vacuum mean values just in the same way as it has been done in this paper for $d = 5$ operators. An order of magnitude estimate of these terms can be done under assumption of factorization of vacuum matrix elements [5] and it gives the relative to the leading term value of these correction of order η/Q^4 where

$$\eta = \langle 0 | \frac{\pi \alpha_s}{18} G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) | 0 \rangle$$

(here in the numerical coefficient the dimensionality of the space-time and of the color group are accounted for). Analysis of charmonium and bottonium [4,5,9] results in the value $\eta \simeq 0.01 \ GeV^4$ and thus this rough estimate literally corresponds to the assumed behaviour of this correction as $m_o^2/(9Q^2)$ squared. However a detailed analysis of the terms associated with $d = 7$ operators is desirable for a more definite conclusion.

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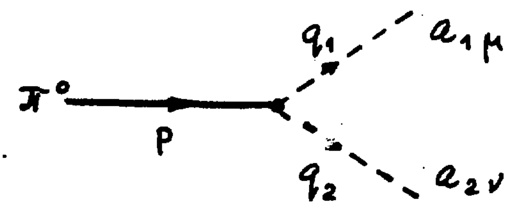

Fig. 1. Definition of momenta and polarizations in the $\pi^0\gamma\gamma$ vertex (eqs. (1) and (2)).

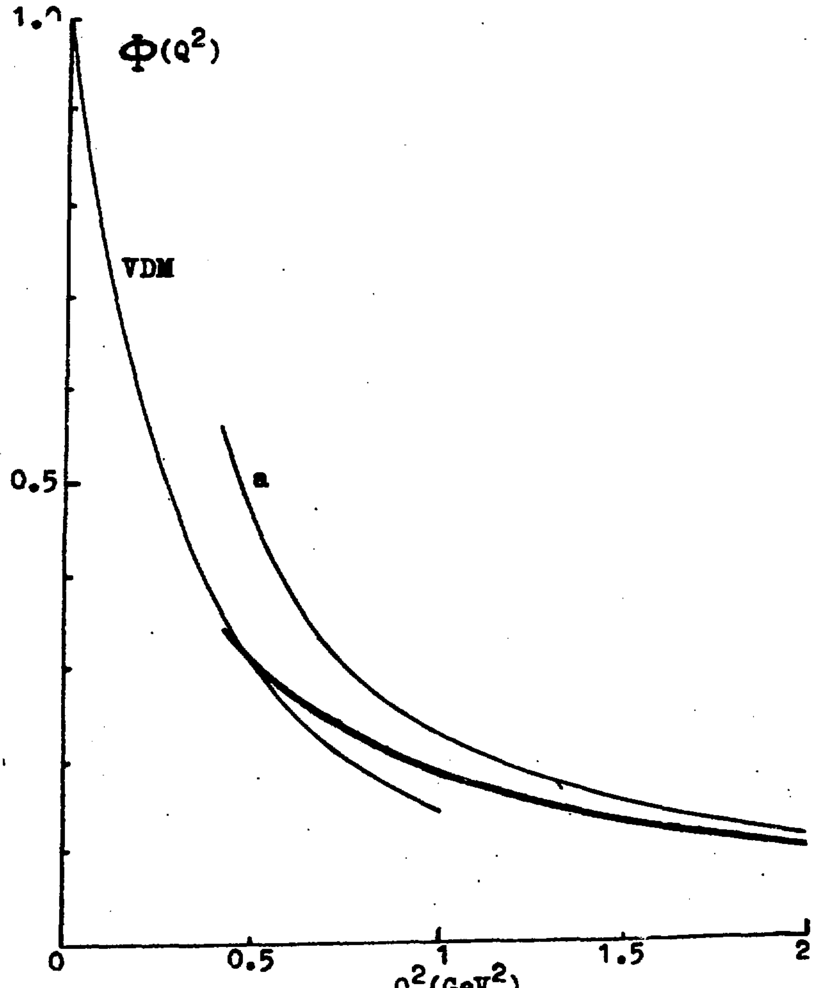

Fig.2. The plot of the ratio $\Phi(Q^2) = F(0,-Q^2,-Q^2)/F(0,0,0)$ as given by eq.(28) (heavy line). The curve labeled VDM is the vector meson dominance estimate. The curve labeled a is the plot of Φ (Q^2) without preasymptotic corrections ($\Phi(Q^2) = 4\pi^2 f_\pi^2/(3Q^2)$).

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