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<u>On the Quantum Mechanics of DIC</u>

<u>A Tropical Rhapsody on Northern Themes</u>[*]

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During the last decade an important and, to a large extent, unantecipated new facet became incorporated to the explored aspects of the dynamics of nuclear systems. This came about through experimental investigation, and latter through theoretical studies of the so called Deep Inelastic Collisions (DIC) between heavy ions[R1]. The capturing point was the identification of aspects of such collisions that are strongly suggestive of diffusion or relaxation-like processes (correlations between Q-value and preferred scattering angle, interpretations in terms of a "collision time" related to preferred scattering angle and behavior of distributions of final state dynamical variables as a function of such a "collision time", for instance). This led to a branch of theoretical investigations centered on the use of transport-like (irreversible) dynamical equations and of methods akin to those of non-equilibrium statistical mechanics. These equations typically describe the diffusion-like time evolution of probability distributions associated with relevant observable quanti

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(*) Rhapsody: 1-A short epic poem, or a portion of a longer epic, recited by a rhapsodist at one time.
2-A confused or disconnected series of sentences or statements, composed under excitement, and without dependence or natural connection; a confused or rambling composition.
3-In music, a composition of irregular form and in the style of an improvisation.
(Webester's New Twentieth Century Dictionary)

ties, a circumstance that readily poses the problem of their connection with an ultimate, underlying quantum mechanical evolution with different general properties. The introduction of coarse-grained descriptions can, eventually, bridge the gap[N1], but it should be borne in mind that this will only lead generally to properly irreversible behavior provided the underlying, reversible dynamics has adequate mixing properties [N2,R2]. This point appears not to have been sufficiently emphasized, maybe due to the circumstance that reasonable albeit mixing-nonspecific randomness assumptions have readily given rise to acceptable transport-type equations through coarse-graining procedures.

Besides this "statistical" branch, however, a different line of approach to the DIC developed from more standard approaches to the physics of nuclear structure, in which the "irreversibility" aspects of these phenomena are considerably de-emphasized. It contains both the Time-Dependent Hartree-Fock (TDHF) description of the collision of heavy ions [R3] and the so called Coherent Surface Excitation Model (CSM) of heavy ion reactions[R4]. The TDHF uses a single determinant to describe the state of the colliding systems at all times. As this constraint is incompatible with the characteristic linear structure of quantum mechanical phase-spaces ("superposition principle"), but still allows for enough structure for a classical phase-space (symplectic structure on the complex Grassman manifold), one gets a hybrid theory with a strong classical flavor and plagued with interpretation problems, some of which may yet turn out to be quite revealing when properly understood (e.g. interfragment correlations in final states). The CSM, on the other hand, makes deliberate use of classical dynamical equations both for the relative motion of the colliding fragments and for the considered "in trinsic" degrees of freedom, which are taken to be nuclear fields (possibly damped) known, from nuclear structure studies, to be endowed with strong coherence properties. The model has been also provided with a statistical anex to describe mass (and the associated momentum) transfer through the use of proximity ideas. A crucial point deserving special emphasis, however, is that correlated quantum fluctuations of intrinsic and scattering degrees of freedom in the final state have been found to be an essential ingredient for the proper interpretation of the (classical) dynamical results[N3].

* * *

Let us pause here for a slightly more technical recollection of this point. If one treats the intrinsic collective fields quantum-mechanically, still keeping the classical treatment of the relative motion, one finds that, provided the coupling of relative motion to the harmonic fields is linear in the latter, and provided they are initially in their ground state, they are driven by the time-dependent interaction due to the relative motion (any relative motion) into coherent states of the form

$$|I\rangle = e^{-\frac{|I|^2}{2} + I a^{+}} |0\rangle$$

where the c-number complex amplitude I(t) is given in terms of a time integral of the time-dependent interaction f(t) :

$$I(t)^{*} = \frac{1}{\sqrt{w}} \int^{t} dt' \, f(t') e^{-iwt'}$$

w being the oscilator frequency of the field. The same result has been obtained in a different context via the more abstruse technique of looking for saddles[R5] of path integral repre -

sentations of the inclusive effective propagator for relative motion, when the later is duly stuffed with the appropriate fully quantal influence functionals for the linearly coupled harmonic fields; and it contains an inconsistency which ap - pears to be typical of such mixtures of classical and quantum mechanics in a single game: the mean square fluctuation of the total energy is not conserved in such partly-classical dynamical schemes, so that energy conservation (appart from damping effects !) holds only "on the average" [(N4)]. This actually comes about in a very natural way, which can be described as follows. Being described in classical terms, relative motion takes place along a trajectory and with a prescribed time-dependence; relative kinetic energy is therefore a well defined quantity at any time (forget mass transfer, for simplicity !). This classical motion, however, generates a time-dependent "external" force acting on the intrinsic (quantum !) system that is thereby unitarily driven away from its initial state, say, one of its "free" eigenstates. This external driving will then, in general, change the (quantum) dispersion of intrinsic energy which can become quite large and even - tually freezes once the collision process is completed.

One may also, perhaps instructively, consider the classical relative motion as a "classical limit" of some wave-packet motion. In doing so, however, one realizes that the partly-classical treatment corresponds to a factorized ansatz for the complete amplitude in a relative motion factor and an intrinsic factor. From this point of view one may consider this type of treatment as envolving a "mean field approxi - mation"[(R6)] of sorts, namely one which will specifically rule out correlations between intrinsic and collective degrees of freedom[(N5)].

Now, it is through correlations of precisely this sort that such a general requirement as detailed energy conservation can be properly installed in the theory[N6]. The CSM copes with this problem within the bounds of a fully classical treatment, by introducing a dispersion in the initial conditions of the (classical) intrinsic modes to stand for the quantum fluctuations inherent to their ultimately quantum constitution, an approach which is strongly reminiscent of semi-classical treatments of inelastic collision processes which are popular among chemists [N7,R7].

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We may, however, take a different turn at this point and consider what are the devices through which a fully quantum treatment of the collision copes with these matters. A reasonable enough description of the initial state (for a time-dependent description) can be written as

$$\langle\vec{r}\,|t=0\rangle = |I_o\rangle \; \langle\vec{r}\,|U\left[\vec{k}_o,\vec{r}_o\right]\rangle \; .$$

This is a factorized pure state displaying a definite intrinsic factor $|I_o\rangle$ and a pre-collision couple of wave-packets centered a distance $\vec{r}_o$ appart and wit relative mean momentum $\vec{k}_o$. An explicit representation for the intrinsic factor will not be needed here. While the description of the pre-collision set-up by a pure state is strictly speaking unrealistic due to incoherent inhomogeneities of the beam, statistical fluctuations of the <u>initial</u> state take place within energy intervals which are negligible in view of the order of magnitude of collision times. This is therefore a negligible ingredient from the point of view of the final product distributions.

Now let this state evolve <u>unitarily</u> through the hamiltonian $H = H_I + H_r + V_{rI}$. The action of the interaction term V_{rI} will "desseparate" the state of the system, i.e., will destroy the factorized form through the establishment of correlations involving r and intrinsic variables. A convenient way of writing the state is then (Schrödinger decomposition)[R8]

$$\langle\vec{r}|t\rangle = \langle\vec{r}|e^{-iHt}|t=0\rangle = \sum_n A_n(t)|I_n(t)\rangle\langle\vec{r}|U_n(t)\rangle$$

where $\langle I_n(t)|I_{n'}(t)\rangle=\delta_{nn'}$, $\langle U_n(t)|U_{n'}(t)\rangle=\delta_{nn'}$, and phases can be arranged so that the $A_n(t)$ are always real. In this representation, the interaction of $\vec{r}$ and intrinsic variables appears as a dependence of the $A_n(t)$ on time, as shown by the evolutic.. law

$$\dot{A}_m(t) = \mathrm{Im} \sum_n \langle I_m(t)U_m(t)|H|I_n(t)U_n(t)\rangle A_n(t)$$

which gives $\dot{A}_m=0$ when $V_{\vec{r}I}=0$ [R9]. While being rather peculiar from many points of view, this representation has the nice features of explicitely displaying both the minimal <u>form</u> of correlations between intrinsic and relative degrees of freedom <u>and</u> the maximal coherence of each subsystem when considered separately. This information is contained in the decomposition into doubly orthogonal terms n and in each of the normalized states $|I_n(t)\rangle$, $|U_n(t)\rangle$ respectively. Having in mind specifically a scattering situation in which, for simplicity, only inelastic channels are taken into account (i.e., all channels are defined by the vanishing of the interaction term $V_{\vec{r}I}$ of H) [N8], it is easy to verify that, for large enough times, the intrinsic energy fluctuation in each of the $I_n(t\to\infty)\rangle$ can eventually be made to vanish, i.e., each term of $\langle\vec{r}|t\to\infty\rangle$ can eventually become associated with a definite channel. This comes about through the vanishing of the overlaps between relative motion wave-packets in the standard channel decomposition of $\langle\vec{r}|t\rangle$ through the spatial separation of packets having different inelasticities. This, in fact, makes the channel decomposition identical with the Schrödinger decomposition. The complete overall quantum coherence of $|t\rangle$ is of course guaranteed, together with detailed energy conservation requirements, which result obviously in the intrinsic-relative motion correlations.

If one now wishes to concentrate on observables pertaining to relative motion alone (such as $d^2\sigma/dE\,d\Omega$, for instance), the relevant information is contained in

$$tr_I\left[\langle\vec{r}|t\rangle\ \langle t|\vec{r}\,'\rangle\right] = \sum_n A_n^2(t)\ \langle\vec{r}|U_n(t)\rangle\ \langle U_n(t)|\vec{r}\,'\rangle$$

which represents a statistical mixture of orthonormal states with probabilities A_n^2. Similarly, the state of the intrinsic subsystem is given by the mixture

$$tr_{\vec{r}}\left[\langle\vec{r}|t\rangle\ \langle t|\vec{r}\,'\rangle\right] = \sum_n A_n^2(t)\,|I_n(t)\rangle\ \langle I_n(t)|$$

where the same weights A_n^2 appear. The time-dependence of these weights is caused by the interaction term $V_{\vec{r}I}$ of the hamiltonian H, the asymptotic decoupling of H_I and $H_{\vec{r}}$ in a scattering situation leading eventually for their stabilization, as well as of the states $|U_n\rangle$ and $|I_n\rangle$.

* * *

We may now use these diverse elements in order to set up a conceptual scheme from which to approach the physics underlying the DIC, and the apparent irreversibility taking place in these processes in particular. As a first point, we see that as far as observations done on a given subsystem are concerned (such as $d^2\sigma/dEd\Omega$), the system behaves as a statistical mixture governed by weights (such as $A_n^2(t)$) but can still show coherence properties governed by the factor states (such as $\langle \vec{r}|U_n(t)\rangle$). Thus the properties of this statistical mixture depend not only on the dynamical properties as given by the hamiltonian H but also on the particular subsystem which is brought into focus in the observation. This means that we can produce, in principle, a large, potential multimultiformity of mixtures out of a given completely coherent state, corresponding to different subsystem decompositions of the system as a whole. The ideal "a priori" symmetry of different decompositions is distorted, however, when one finds oneself restricted to observations that are not just ideal but actually carried out in practice. That this is not just a very prosaic and incidental type of constraint is illustrated by the fact that similar restrictions are also involved as a basic ingredient in dynamical analyses of quantum measurements (R10), where they ultimately dispose with certain correlations given by the unitary dynamical law of quantum mechanics. A certain parallelism, therefore, suggests itself between the observed entropy generation in DIC and that which accompanies quantum measurements in general.

In the case of DIC one should expect any observability constraints to be related a) to the nature of the degrees of freedom in terms of which the asymptotic decoupling that results from the unbound character of the system undergoing the collision is naturally expressed (i.e., giving asymptotically stationary probabilities A_n^2 in the associated Schrödinger decomposition of the state vector) and b) to the occurence of semiclassical regimes in association with the degrees of free dom distinguished in a given Schrödinger decomposition. To the extent that one considers just asymptotic measurements (corss sections), the Schrödinger decomposition will eventually coincide with the channel decomposition of the state vector, and statement a) expresses the difficulty of implementing observables detecting correlation terms (interference effects) involving different physical channels. Excluding such observables, the relevant information will be given essentially by the asymptotic values of the $A_n^2(t)$ and by the relative motion states $\langle \vec{r} | U_n \rangle$, and contained in the reduced density matrix $tr_I \left[\langle \vec{r} | t \rangle \langle t | \vec{r}' \rangle \right]$ having these objects as ingredients. To the extent that a transport theory (such as that of Weidenmüller (R1)) gives a good description of this quantity, it implies that the final reduced density matrix can be linked to initial conditions by means of an "effective", irreversible dynamical law. The validity of the results given by such a dynamical law at <u>intermediate</u> times, of course, remains untested. Such validity would be associated with a transport type (master) equation for the time evolution of $tr_I \left[\langle \vec{r} | t \rangle \langle t | \vec{r}' \rangle \right]$ at all times. From the point of view of observations done on the corresponding subsystem, then, the process could be described as an "approach to equilibrium", and therefore endowed with mixing properties, as mentioned earlier. These mixing properties, however, must be again associated not only to the dynamical law for the entire system (i.e., to the hamiltonian H) but <u>also</u> to the particular decomposition of the system that one considers.

To the extent that the Schrödinger decomposition approaches the channel decomposition, the residual coherence present, say, in the relative states $\langle \vec{r}\ U_n \rangle$ is trivial, in the sense that it is given just by the form of the initial wave-packet (assumed to be as narrow as needed in momentum space to eliminate distortion effects coming from the energy dependence of scattering amplitudes) and by energy conservation (apart from the scattering amplitude itself). At finite times, however, correlations would be nontrivial and related in particular to collective effects engaged by the interaction $V_{\vec{r}I}$. One can also conceive of other Schrödinger type decompositions (i.e., one that singles out some degree of freedom other than $\vec{r}$, such as the orientation of the principal axes of the quadrupole tensor of the system) which could be relevant at finite times and fall under an observability criterion such as b) above. Phenomenological regularities sketched at the beginning appear to indicate that the corresponding "effective" dynamics of such varied decompositions must be intimately interlocked. These coherent effects, conspicuous at finite times, are alien to the statistical transport theories. Their works, however, preside, albeit in ghostly form, the collision dynamics in approaches like TDHF or CSM.

R's and N's

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R3 - S.E.Koonin, in Progress in Particle and Nuclear Physics 4 (1980)283.

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N1 - This is a comprehensive reference to coarse-grained descriptions, intended to cover not only the approach developed by Nöremberg (see R1 and references cited therein) but also that based on the use of ensembles of random h amiltonians, as developed by Weidenmüller and collaborators. Such ensembles are used in statistical theories of spectra to describe average properties of stationary states of a complex system. In transport theories of nuclear collisions they are likewise related to average properties of the intrinsic states. Their use in transport theories implies thus some sort of "idealized" coarse graining, the idealization consisiting in the replacement of an actual "coarse sampling" by a judiciously defined "dynamical fuzying". (See also N2).

N2 - This refers to coarse-grained <u>observation</u> of a system that is allowed to time-evolve according to its full microscopic dynamical law. The "increasing mixing character" of coarse-grained probability <u>distributions</u> for DIC under time evolution has been discussed in R2 (MCN). This is related there to the bistochastic character of the matrix that represents the change in the distribution. It should be noted, however, that bistochasticity does not imply dynamical approach to coarse-grained equilibirum without other assumptions. If $\rho(t)$ is the coarse-grained distribution, then $\rho(t_1) = \mathcal{B}(t_1-t_o)\rho(t_o)$ and $\rho(t_2) = \mathcal{B}(t_2-t_o)\rho(t_o)$, both $\mathcal{B}(t_1-t_o)$ and $\mathcal{B}(t_2-t_o)$ being bistochastic matrices defined by the microscopic, reversible dynamical law. For $t_2>t_1>t_o$, however, the relation between $\rho(t_2)$ and $\rho(t_1)$ is more complicated, and in particular $\rho(t_2)$ <u>can</u> be <u>less</u> mixed than $\rho(t_1)$ (while it is guaranteed that <u>both</u> are <u>more</u>

mixed than $\rho(t_o)$. This possibility is explicitely ruled out by restricting the microscopic dynamical law in the appropriate way ("mixing" dynamical law).

N3 - Part of these correlations manifest themselves in TDHF calculations, which make no assumption as to the factorizability of the final state into parts corresponding to each of the final fragments. TDHF calculations are however constrained by the overall determinantal ansatz, and the detailed effects of this constraint on the possible final correlations appears not to have been sufficiently explored.

N4 - Cf. the brief communication made by A.Einstein to the Brazilian Academy of Sciences, on the 7ht of May of 1925, on experiments done by Geiger and Bothe on the idea of "statistical energy conservation" of Bohr, Kramers and Slater. A transcription of this communication appears in Suplemento Cultural de "O Estado de São Paulo", 81(1978)p.3, in an article by R.V.Caffarelli on Einstein's visit to Rio de Janeiro.

N5 - The mean field of TDHF is based on an entirely different sort of factorization (factorization of the many-body density into one-body density factors) which does not single out collective and intrinsic parts. Correlations between these parts are however allowed, provided they are tailored to fit the determinantal constraint (see also N3).

N6 - The want of detailed energy conservation affects also transport descriptions, whenever they make use of a classical treatment of the degrees of freedom associated with the relative motion of fragments.

N7 - These treatments aim mainly at the calculation of scattering amplitudes to definite channels, unlike in the CSM treatment of DIC, which has an inclusive character. In the single channel problem, the question of energy conservation is taken care of automatically as shown in detail e.g. in Pechukas, Ref. R7. Inclusive treatments as proposed in R5 and R6 (which follow closely the path marked by Pechukas) involve precisely the mixture of classical and quantum mechanics that leads to problems regarding detailed energy conservation.

N8 - This can be extended to more complicated cases (e.g., involving rearangement) at the expense of enough technical complications (e.g. : to treat the transference of a group of particles one can analyse the wave function in terms of three components associated with the transfered group and with initial and final "cores", respectively).