BR8206307

L. C.

Rec. 09.02.82

CBPF

Notas de Física

CBPF-NF-036/81

ANISOTROPIC SQUARE LATTICE POTTS FERRO-MAGNET: RENORMALIZATION GROUP TREATMENT.

by

Paulo Murilo Castro de OLIVEIRA* and

Constantino TSALLIS**

NOTAS DE FÍSICA é uma pré-publicação de trabalhos em Física do CBPF

NOTAS DE FISICA is a series of preprints from CBPF

Pedidos de cópias desta publicação devem ser enviados aos autores ou à:

Requests for free copies of these reports should be addressed to:

Divisão de Publicações do CBPF-CNPq
Av. Wenceslau Braz, 71 - Fundos
22.290 - Rio de Janeiro - RJ.
Brasil

CBPF-NF-036/81

ANISOTROPIC SQUARE LATTICE POTTS FERRO-MAGNET: RENORMALIZATION GROUP TREATMENT.

by

Paulo Murilo Castro de OLIVEIRA* and

Constantino TSALLIS**

* Departamento de Física - Universidade Federal Fluminense
C.P. 296 - 24000 Niterói - RJ - Brazil

** Centro Brasileiro de Pesquisas Físicas/CNPq
Av. Wenceslau Braz nº 71 - 22290 Rio de Janeiro
Brazil

ABSTRACT

The choice of a convenient self-dual cell within a real space renormalization group framework enables a satisfactory treatment of the anisotropic square lattice q-state Potts ferromagnet criticality. The *exact* critical frontier and dimensionality crossover exponent ϕ as well as the expected universality behaviour (renormalization flow sense) are recovered for any linear scaling factor b and all values of $q(q \leq 4)$. The $b = 2$ and $b = 3$ approximate correlation length critical exponent ν is calculated for all values of q and compared with den Nijs conjecture. The same calculation is performed, for all values of b, for the exponent $\nu(d=1)$ associated to the one-dimensional limit and the *exact* result $\nu(d=1) = 1$ is recovered in the limit $b \to \infty$.

I - INTRODUCTION

During recent years a considerable amount of effort has been dedicated to the construction of real space renormalization group (RG) frameworks suitable for the treatment of several models like the site and bond percolation, Ising and q-state Potts ones. A particular case which has frequently been focused is the anisotropic square lattice q-state Potts ferromagnet whose hamiltonian is given by

$$\mathcal{H} = -q \sum_{\langle i,j \rangle} J_{ij}\, \delta_{\sigma_i \sigma_j} \qquad (\sigma_i = 1, 2, \ldots, q \;\forall\, i) \tag{1}$$

where $J_{ij} = J_x \geq 0$ ($J_{ij} = J_y \geq 0$) if sites i and j are "horizontal" ("vertical") first neighbours (as a matter of fact, the present paper remains practically unchanged in the case where one or both coupling constants are negative). Any satisfactory RG proposal for this problem should recover the following facts:

(i) The transition is continuous (first order) if $0 \leq q \leq 4$ ($q > 4$) according to Baxter 1973, Straley and Fisher 1973, and Kim and Joseph 1975.

(ii) All properties of the system are invariant through $x \leftrightarrow y$ permutation.

(iii) The anisotropic square lattice is self-dual, therefore the dual transformation (Kim and Joseph 1975, Burkhardt and

Southern 1978, and Baxter et al 1978)

$$e^{-\frac{qJ_x}{k_BT}} \longleftrightarrow \frac{1 - e^{-\frac{qJ_y}{k_BT}}}{1 + (q-1)\, e^{-\frac{qJ_y}{k_BT}}} \qquad (2)$$

interchanges its para - and ferromagnetic phases, and consequently the critical frontier is given by

$$t_x = t_y^D \equiv \frac{1 - t_y}{1 + (q-1)t_y} \qquad (3)$$

where we have introduced convenient variables (hereafter referred to as <u>transmissivities</u>; see Tsallis 1981, and Tsallis and Levy 1981, and references therein), through

$$t_r \equiv \frac{1 - e^{-\frac{qJ_r}{k_BT}}}{1 + (q-1)\, e^{-\frac{qJ_r}{k_BT}}} \qquad (r = x,y) \qquad (4)$$

(iv) The system is <u>universal</u>, i.e. its critical behaviour for fixed q is one and the same for all non vanishing values of J_x and J_y (in particular, the correlation length critical exponent ν is the same along the critical frontier excepted both one-dimensional limits $J_x = 0$ or $J_y = 0$).

(v) The crossover exponent ϕ associated to the one-dimensional limits equals one; this fact means that if we consider, for instance, the limit $J_y/J_x \to 0$, the critical frontier satisfies $t_y \propto 1-t_x$. It is clear that this weak restriction is satisfied by Eq. (3) which implies $t_y \sim (1-t_x)/q$.

(vi) The correlation length critical exponent $\nu(d=1)$ associated to the one-dimensional limits equals one.

(vii) The q-dependance of the critical exponent ν (for $J_x, J_y \neq 0$) has not yet been rigorously established, however den Nijs 1979 conjecture, namely

$$\nu = \frac{2}{3[2 + \pi/(\arccos \sqrt{q}/2 - \pi)]} \tag{5}$$

$$\sim \frac{\pi}{3\sqrt{q}} \qquad \text{for } q \to 0 , \tag{5'}$$

is possibly exact.

A RG treatment of the present problem consists in the construction of a two-dimensional recursive relation (generated by the renormalization of an appropriate cell into a smaller one) which we shall note

$$\begin{aligned} t'_x &= R^x_b(t_x, t_y) \\ t'_y &= R^y_b(t_x, t_y) \end{aligned} \tag{6}$$

where b > 1 is the linear scaling factor. This recursive relation is expected to provide fixed points (t_x^*, t_y^*) which satisfy

$$t_x^* = R_b^x(t_x^*,\ t_y^*)$$
$$t_y^* = R_b^y(t_x^*,\ t_y^*) \qquad (7)$$

as well as a Jacobian matrix

$$\begin{pmatrix} \dfrac{\partial t_x'}{\partial t_x} & \dfrac{\partial t_x'}{\partial t_y} \\ \\ \dfrac{\partial t_y'}{\partial t_x} & \dfrac{\partial t_y'}{\partial t_y} \end{pmatrix} \qquad (8)$$

whose eigenvalues and eigenvectors at each one of those fixed points are associated to relevant critical quantities. Let us note that it is by no means necessary (or even eventually convenient) to perform the renormalization in a two - dimensional space (t_x - t_y space in our case) and wider spaces can be used.

Let us now translate the restrictions (i) - (vii) into RG language:

(i') An anomaly must appear, at q = 4, in the topology of the flow diagram while q varies; by anomaly we refer for instance to a bifurcation, or terminal, or turning point in the path of the relevant fixed points. However it is not obvious that such anomaly can be observed without an enlargement of the renormalization space (see for example Nienhuis et al 1979 and Riedel 1981).

(ii') It must be

$$R_b^y(y, x) = R_b^x(x, y) \equiv R_b(x, y) \qquad (9)$$

This restriction leads to the invariance of the flow diagram through $t_x \longleftrightarrow t_y$ permutation, i.e. there is a mirror symmetry with respect to the isotropic $t_x = t_y$ axis. The most satisfatory way for obtaining relation (9) is to use cells which themselves preserve the equivalence between the "horizontal" and "vertical" directions.

(iii') It must be

$$R_b^x(x, y) = [R_b^y(y^D, x^D)]^D \equiv \frac{1 - R_b^y(y^D, x^D)}{1 + (q-1)R_b^y(y^D, x^D)} \qquad (10)$$

Where upperscript D denotes transformation (3) (see also Tsallis 1981, and Tsallis and Levy 1981). The most satisfactory way for obtaining relation (10) is to use self-dual cells (a cell is said to be self-dual if it can be superimposed to itself in such a way that each one of its bonds is cut by one and only one bond of the original cell). The exact critical frontier (Eq. (3)) must be recovered as a flow line which runs between the one-dimensional limit points.

(iv') A semi-stable fixed point must exist on the critical line in between the two one-dimensional limits, i.e. the eigen value, (of the Jacobian matrix (8)), noted λ_2, associated to the eigenvector tangential to the critical line must be less than one (the other eigenvalue, noted λ_1, clearly must be bigger than one).

(v') At both one-dimensional limits, unstable fixed points must exist, and the associated Jacobian matrix must be proportional to unity ($\lambda_x = \lambda_y \equiv \lambda$), at least in the limit $b \to \infty$.

(vi') The eigenvalue λ must be proportional to b in the limit $b \to \infty$ (we recall that $\nu(d=1) = \lim_{b\to\infty} \frac{\ln b}{\ln \lambda}$).

(vii') The eigenvalue λ_1 must be such that $\nu = \lim_{b\to\infty} \frac{\ln b}{\ln \lambda_1}$ agrees with the possibly exact result (Eq. (5)).

Let us now place in the preceding context the recent RG literature on the subject. To the best of our knowledge, no RG treatment of the anisotropic q-state Potts model is available. In what concerns the isotropic model ($t_x = t_y$), only restrictions (i'), (iii') and (vii') are to be considered. Nienhuis et al 1979 qualitatively (but not quantitatively) satisfy these three restrictions. Blöte et al 1981 do not satisfy (i') nor calculate the critical point (restriction (iii')), but obtain, for $q < 4$, a quite precise numerical approximation for ν (restriction (vii')). Tsallis and Levy 1981 do not satisfy (i'), but obtain the exact critical point ($t_c = 1/(1+\sqrt{q})$), and acceptable numerical approximations for ν ($q < 4$).

In what concerns the anisotropic system, some effort has been dedicated to the bond percolation problem (which corresponds to the particular case $q \to 1$, according to Kasteleyn and Fortuin 1969). In this case, restriction (i') is out of consideration. In what concerns restrictions (ii') - (vii'), Ikeda 1979 satisfies none of them, and Chaves et al 1979 and de Magalhães et al 1981 only satisfy (ii') and (iii'), and obtain acceptable numerical approximations for ν (restriction (vii')). Nakanishi et al 1981 only satisfy (ii'), (iv'),(v') and (vi'); it must however be pointed out that they satisfy restriction (ii') through an ad hoc procedure and not by considering a single cell whose "horizontal" and "vertical" spannings determine the corresponding recursive relations (Eq. (6)). Oliveira 1981a uses a suitable family of cells (Riera et al 1980, de Magalhães et al 1981, Curado et al 1981, Oliveira 1981b; see Fig. 1) and simultaneously satisfies restrictions (ii') to (vi'); the exact critical frontier $t_x + t_y = 1$ is obtained because, besides the fact that restrictions (ii') and (iii') are satisfied, each cell of this family reduces to a single linear chain in the one-dimensional limits (this important property is not satisfied by the cells used by Chaves et al 1979 and de Magalhães et al 1981; at the terminals of these cells different linear chains are being mixed).

In the present paper we follow along the lines of Oliveira 1981a and, by formulating the problem in terms of the already mentioned transmissivities, extend the RG treatment to the Potts model. By doing so, we satisfy restrictions (ii') to (vi') for all q and obtain a qualitatively acceptable q-dependence of ν (restriction (vii')); we fail however in what concerns restriction (i').

II- REAL SPACE RENORMALIZATION GROUP TREATMENT

We shall use the family of self-dual cells indicated in Fig. 1. By using the Break-Collapse Method (BCM; Tsallis and Levy 1981) we calculate the recursive relation (Eq. (6)) which renormalizes the $b = 2$ cell (Fig. 1.c) into the $b = 1$ cell (Fig. 1.a) (remark that a single pair of cells provides both t_x - and t_y- recurrences: it is enough to appropriately choose the input and output points, as illustrated, for $b = 1$, in Figs. 1.a and 1.b) and obtain

$$\begin{aligned} t'_x &= R_2(t_x, t_y) \\ t'_y &= R_2(t_y, t_x) \end{aligned} \qquad (11)$$

with

$$\begin{aligned} R_2(t_x, t_y) \equiv [&t_x^3 + 4t_x^2 t_y + 3t_x t_y^2 + 2(q-2)t_x^3 t_y \\ &+ 4(q-2)t_x^2 t_y^2 + 2(q-2)t_x^4 t_y + (q^2 + 2q - 5)t_x^3 t_y^2 \\ &+ (4q-6)t_x^2 t_y^3 + (4q^2 - 13q + 10)t_x^4 t_y^2 \\ &+ (6q^2 - 18q + 12)t_x^3 t_y^3 + (q-2)t_x^2 t_y^4 \\ &+ (q^2 - 5q + 6)t_x^5 t_y^2 + (2q^3 - 6q^2 + 10)t_x^4 t_y^3 \\ &+ (3q^2 - 13q + 14)t_x^3 t_y^4 + (2q^3 - 12q^2 + 26q - 20)t_x^5 t_y^3 \\ &+ (3q^3 - 18q^2 + 38q - 28)t_x^4 t_y^4 + (q^4 - 7q^3 + 21q^2 - 30q + 17)t_x^5 t_y^4]/ \end{aligned}$$

$$[1 + 2(q-1)t_x t_y + 2(q-1)t_x^3 t_y + (q^2-1)t_x^2 t_y^2 + (2q^2-6q+4)t_x^3 t_y^2 + (2q^2-3q+1)$$
$$\cdot\, t_x^4 t_y^2 + 2q(q-1)t_x^3 t_y^3 + (q-1)t_x^2 t_y^4 + (q^2 - 3q + 2)t_x^5 t_y^2$$
$$+ (2q^3 - 4q^2 - 2q + 4)t_x^4 t_y^3 + (3q^2 - 9q + 6)t_x^3 t_y^4 + (2q^3-10q^2+16q-8)t_x^5 t_y^3$$
$$+ (3q^3 - 15q^2 + 24q - 12)t_x^4 t_y^4 + (q^4 - 7q^3 + 18q^2 - 20q + 8)t_x^5 t_y^4] \qquad (12)$$

This recursive relation (which, for $q = 1$, recovers that of Oliveira 1981a) presents two trivial stable fixed points (namely $(t_x^*, t_y^*) = (0,0)$ and $(t_x^*, t_y^*) = (1,1)$), two one-dimensional unstable fixed points (namely (1,0) and (0,1)) and one isotropic semi-stable fixed point (namely (t_c, t_c) with $t_c = 1/(\sqrt{q}+1)$ which is the exact value): see Fig. 2. As a matter of fact the same set of fixed points will be obtained for all values of b.

Let us first analize the isotropic fixed point . The Jacobian matrix (8) associated to Eqs. (11) and (12) presents an eigenvalue (bigger than unity for any finite q)

$$\lambda_1(b=2) = (2025+11160\sqrt{q} + 26580q + 35792q^{3/2} + 29852q^2 + 15816q^{5/2}$$
$$+ 5207q^3 + 976q^{7/2} + 80q^4)/(2025 + 8820\sqrt{q} + 16804q + 18290q^{3/2}$$
$$+ 12444q^2 + 5424q^{5/2} + 1481q^3 + 232q^{7/2} + 16q^4) \qquad (13)$$

associated to the eigenvector $(1,1)/\sqrt{2}$ (which in fact will be the same for all values of b), and an eigenvalue (less than unity for any finite q)

$$\lambda_2(b=2) = (10125 + 88650\sqrt{q} + 342860q + 781853q^{3/2} + 1178008q^2 + 1240724q^{5/2}$$
$$+ 939667q^3 + 516906q^{7/2} + 205408q^4 + 57611q^{9/2} + 10844q^5 + 1232q^{11/2}$$
$$+ 64q^6)/(91125 + 595350\sqrt{q} + 1782540q + 3234167q^{3/2} + 3960600q^2$$
$$+ 3449388q^{5/2} + 2191343q^3 + 1023534q^{7/2} + 349008q^4 + 84773q^{9/2}$$
$$+ 13932q^5 + 1392q^{11/2} + 64q^6) \qquad (14)$$

associated to the eigenvector $(-1, 1)/\sqrt{2}$ (the same for all values of b). The fact that $\lambda_2 < 1$ enables the satisfaction of restriction (iv'). The q-dependence of the approximate critical exponent $\nu(b = 2) = \ln 2/\ln\lambda_1(b = 2)$ is presented in Fig. 3 and Table 1. It is clear that $\lambda_1(b = 2)$ could have been obtained directly from the isotropic case ($t_x = t_y \equiv t$) whose recursive relation is given by

$$t' = R_2(t, t) = [8t^3 + 6(q-2)t^4 + (q^2 + 8q - 15)t^5 + (10q^2-30q-20)t^6$$
$$+ (2q^3 - 2q^2 - 18q + 30)t^7 + (5q^3 - 30q^2 + 64q - 48)t^8 + (q^4 - 7q^3 + 21q^2$$
$$- 30q + 17)t^9] /$$
$$[1 + 2(q-1)t^2 + (q^2 + 2q - 3)t^4 + (2q^2 - 6q + 4)t^5 + 4q(q-1)t^6$$
$$+ (2q^3 - 14q + 12)t^7 + (5q^3 - 25q^2 + 40q - 20)t^8 + (q^4 - 7q^3 + 18q^2 -$$
$$20q + 8)t^9] \qquad (15)$$

hence $\nu(b = 2) = \ln 2/\ln(dR_2(t, t)/dt)_{t = 1/(\sqrt{q}+1)} = \ln 2/\ln\lambda_1(b=2)$

For b = 3 we have calculated (by using the BCM) the isotropic case and have obtained

$$t' = R_3(t,t) = \frac{\sum_{i=5}^{25}\left(\sum_{j=0}^{12} n_j^{(i)} q^{12-j}\right)t^i}{1+4(q-1)t^4+\sum_{i=4}^{25}\left(\sum_{j=0}^{12} d_j^{(i)} q^{12-j}\right)t^i} \qquad (16)$$

where the integer coefficients $\{n_j^{(i)}\}$ and $\{d_j^{(i)}\}$ are presented in Table 2. From this expression we straightforwardly obtain

$$\lambda_1(b = 3) = \frac{dR_3(t,t)}{dt}\bigg|_{t=1/(\sqrt{q}+1)} = \frac{\sum_{j=0}^{24} \alpha_j q^{j/2}}{\sum_{j=0}^{24} \beta_j q^{j/2}} \qquad (17)$$

where the coefficients $\{\alpha_j\}$ and $\{\beta_j\}$ are presented in Table 3. The associated critical exponent $\nu(b = 3) = \ln 3/\ln\lambda_1(b = 3)$ is presented in Fig. 3 and Table 1.

Let us now turn our attention onto the one-dimensional fixed points. The jacobian matrix (8) associated to Eqs. (11) and (12) is degenerate (i.e. proportional to the unity matrix) therefore the dimensionality crossover exponent ϕ equals one, which is the exact result. The degenerate eigenvalue is $\lambda(b = 2) = 3$ (bigger than unity as expected). As a matter of fact, for any value of b, the recursive relation in the vicinity of a one-dimensional fixed point (let us say $(t_x^*, t_y^*) = (1,0)$) leads to an eigenvalue $\lambda(b)$ which is that of a linear chain (along the x-direction in our case). The recurrence is given by

$$t_x' = R_b(t_x, 0) = t_x^{2b-1} \qquad (18)$$

hence

$$\lambda(b) \equiv \frac{dR_b(t_x,\ 0)}{dt_x}\Bigg|_{t_x=1} = 2b - 1$$

and finally

$$\nu(d = 1) = \lim_{b\to\infty} \frac{\ln b}{\ln\lambda(b)} = \lim_{b\to\infty} \frac{\ln b}{\ln(2b-1)} = 1 \qquad (19)$$

which is the exact result.

III- THE s-VARIABLE

In order to make a remark let us introduce a new variable (Tsallis 1981 and Tsallis and de Magalhães 1981) namely

$$s_r \equiv s(t_r) \equiv \frac{\ln[1 + (q-1)t_r]}{\ln q} \quad (r = x,\ y) \quad (20)$$

It is straightforward, through use of

$$t_r^D = \frac{1-t_r}{1 + (q-1)t_r} \qquad (r = x,\ y) \qquad (21)$$

to verify that

$$s^D(t_r) \equiv s(t_r^D) = 1 - s(t_r) \qquad (r = x,\ y) \qquad (22)$$

and that the critical frontier (3) can be rewritten in an universal form (the same for all values of q) namely

$$s_x + s_y = 1 \qquad (23)$$

which is precisely that of bond percolation ($q \to 1$). Consequently we can define the RG in an alternative manner, namely

$$s'_x \equiv s(t'_x) = s(R_b(t_x, t_y)) = s(R_b(\frac{q^{s_x}-1}{q-1}, \frac{q^{s_y}-1}{q-1}))$$

(24)

$$s'_y \equiv s(t'_y) = s(R_b(t_y, t_x)) = s(R_b(\frac{q^{s_y}-1}{q-1}, \frac{q^{s_x}-1}{q-1}))$$

The flow diagram presents, for <u>all values of q</u>, one and the same set of fixed points (namely $(s^*_x, s^*_y) = (0,0)$, $(1,1)$, $(1,0)$, $(0,1)$ and $(1/2, 1/2)$) and critical flow line (namely that of Eq. (23)), i.e. it presents the RG topology of the bond percolation problem. In what concerns the critical exponents nothing is changed with respect to the RG in the t-variables as, for any fixed point, we have

$$\begin{pmatrix} \frac{\partial s'_x}{\partial s_x} & \frac{\partial s'_x}{\partial s_y} \\ \frac{\partial s'_y}{\partial s_x} & \frac{\partial s'_y}{\partial s_y} \end{pmatrix} = \begin{pmatrix} \frac{\partial t'_x}{\partial t_x} & \frac{\partial t'_x}{\partial t_y} \\ \frac{\partial t'_y}{\partial t_x} & \frac{\partial t'_y}{\partial t_y} \end{pmatrix} \qquad (25)$$

IV- <u>CONCLUSION</u>

The use of appropriate cells (which are <u>self-dual</u> and in the one-dimensional limits reduce to <u>single</u> chains) enables to reproduce, within a simple real space renormalization group, a considerable quantity of <u>exact</u> results (points (ii) to (vi) of Section I) concerning the criticality of the aniso tropic square lattice q-state Potts model. In what concerns the q-dependance of the correlation length critical exponent ν (point (vii) of Section I) we obtain results which are compatible with den Nijs 1979 conjecture and which improve with increasing cell size as long as q is not too close to 4; on the whole they are quite similar to those obtained by Tsallis and Levy 1981 and reinforce den Nijs 1979 conjecture in the limit $q \to 0$ (tree-like percolation) as they all provide $\nu \propto 1/\sqrt{q}$. In what concerns point (i) of Section I we have failed, i.e. nothing special occurs at $q = 4$ (nor at any other finite value of q); the fact that we have not enlarged the parameter space (our renormalization is restricted to the (t_x, t_y) - space) is, according to the ideas contained in Nienhuis et al 1979 work, quite probably at the origin of this failure.

Incidentally we present (in Section III) a renormalization group (constructed in the (s_x, s_y) - space instead of the (t_x, t_y) one) which has interesting universal properties: the set of fixed points and critical flow line (critical frontier) independs from q and is that of bond percolation.

We acknowledge with pleasure computational assistance from G. Schwachheim and E.M.F. Curado.

REFERENCES

Baxter RJ 1973, J.Phys. C 6 L 445

Baxter RJ, Temperley HNV and Ashley S E 1978, Proc. R. Soc. A 358 535

Blöte HW, Nightingale MP and Derrida B 1981, J.Phys. A 14 L 45

Burkhardt TW and Southern BW 1978, J. Phys A 11 L 247

Chaves CM, Oliveira PMC, Riera R and de Queiros SLA 1979, Progr. Theor. Phys. 62 1550

Curado EMF, Tsallis C and Schwachheim G 1981, to be published

Ikeda H 1979, Progr. Theor. Phys. 61 842

Kasteleyn PW and Fortuin CM 1969, J. Phys. Soc.Japan (suppl.) 26 11

Kim D and Joseph RI 1975, J. Phys. A 8 891

de Magalhães ACN, Tsallis C and Schwachheim G 1981, J. Phys. C 14 1393

Nakanishi H, Reynolds PJ, Redner S 1981, J. Phys. A 14 855

Nienhuis B, Berker AN, Riedel EK and Schick M 1979, Phys. Rev. Lett. 43 737

den Nijs MPM 1979, Physica 95 A 449

Oliveira PMC 1981, (a) Phys. Rev. B to appear; (b) Phys. Rev. Lett. to appear

Riedel E K 1981, Physica 106 A 110

Riera R, Oliveira P M C , Chaves CM and de Queiroz SLA 1980, Phys. Rev. B 22 3481

Straley JP and Fisher ME 1973, J. Phys. A 6 1310

Tsallis C 1981, J. Phys. C 14 L 85

Tsallis C and Levy SVF 1981, Phys. Rev. Lett. 47 950

Tsallis C and de Magalhães ACN 1981, J. de Physique/Lettres 42 L 227

CAPTION FOR FIGURES AND TABLES

Fig. 1 - Self-dual cells (the arrows represent the possible entrances to and exits from the cell) and their two-rooted graph representation (• denotes internal site and o denotes root or terminal site); they provide $R_1(t_x, t_y) = t_x$ (a), $R_1(t_y, t_x) = t_y$ (b), $R_2(t_x, t_y)$ (c) and $R_3(t_x, t_y)$ (d).

Fig. 2 - Flow diagram associated to Eq. (11). The dots (heavy line), represent(s) fixed points (the critical flow line; it coincides with the exact result $t_x = t_y^D$ (Eq. (3))). (P) ((F)) denotes the paramagnetic (ferromagnetic) phase.

(a) complete b = 2 flow diagram for q = 2; (b) critical flow lines associated to various values of q and any value of b (the limit $q \to 0$ corresponds to tree-like percolation; the q - and q^{-1}- frontiers are, for all values of q, symmetric with respect to the straight line $t_x + t_y = 1$)

Fig. 3 - q-dependence of the correlation length critical exponent ν; the full (dashed) lines correspond to the present RG results (to den Nijs 1979 conjecture).

By b = 3/2 we mean the value obtained by renormalizing the b = 3 cell into the b = 2 one, hence $\nu(b=3/2) = (\ln 3/2)/\ln[(\lambda_1(b=3))/(\lambda_1(b=2))]$.

TABLE 1 - RG and conjectural values of the critical exponent ν. (a) see caption of Fig. 3; (b) these values coincide with those appearing in Oliveira 1981a.

TABLE 2 - Coefficients of the numerator ($\{n_j^{(i)}\}$; top value) and denominator ($\{d_j^{(i)}\}$; bottom value) of $R_3(t,t)$ (Eq. (16)); all missing coefficients vanish.

TABLE 3 - Coefficients of the numerator ($\{\alpha_j\}$) and denominator ($\{\beta_j\}$) of $\lambda_1(b = 3)$ (Eq. (17))

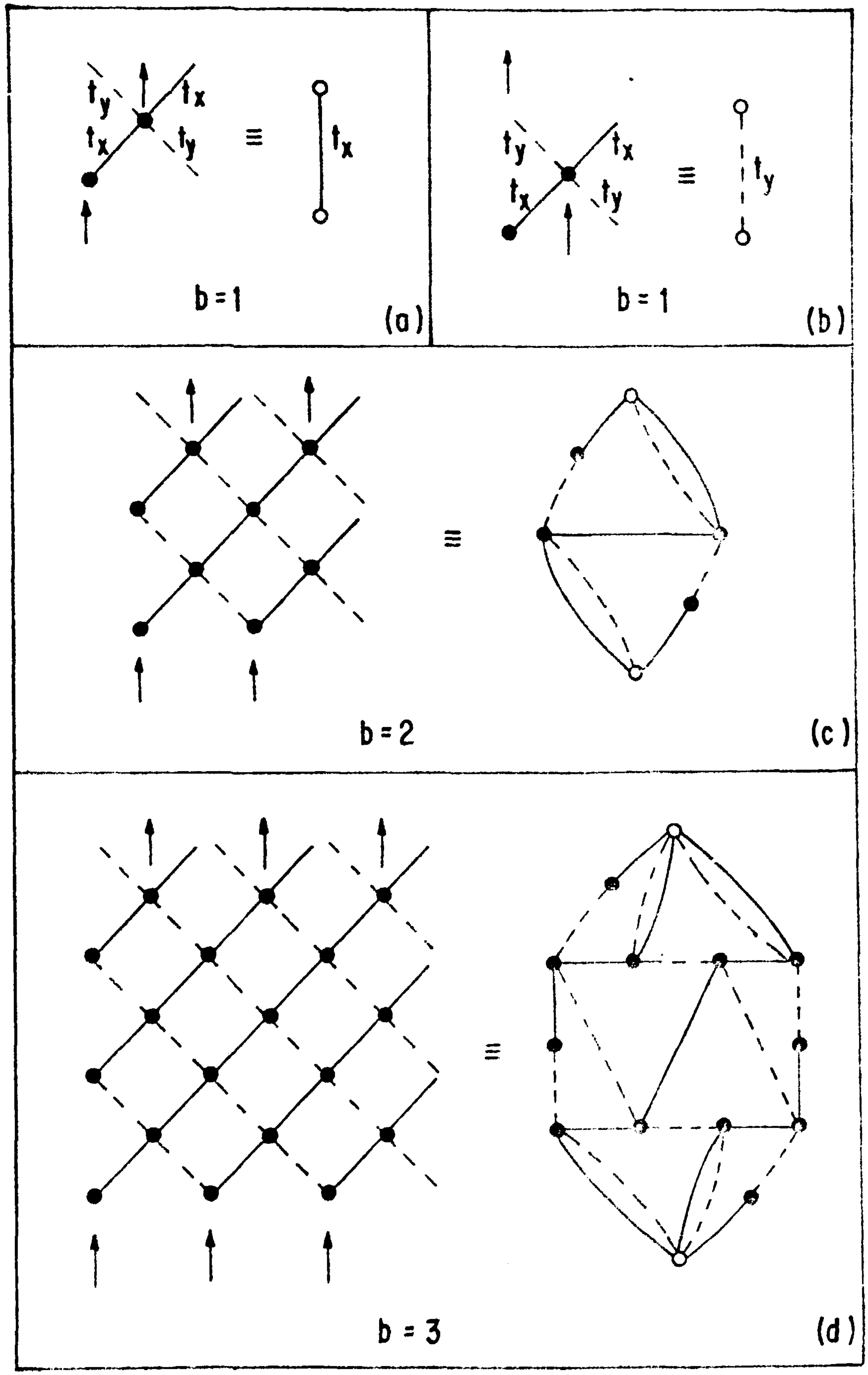

FIG. 1

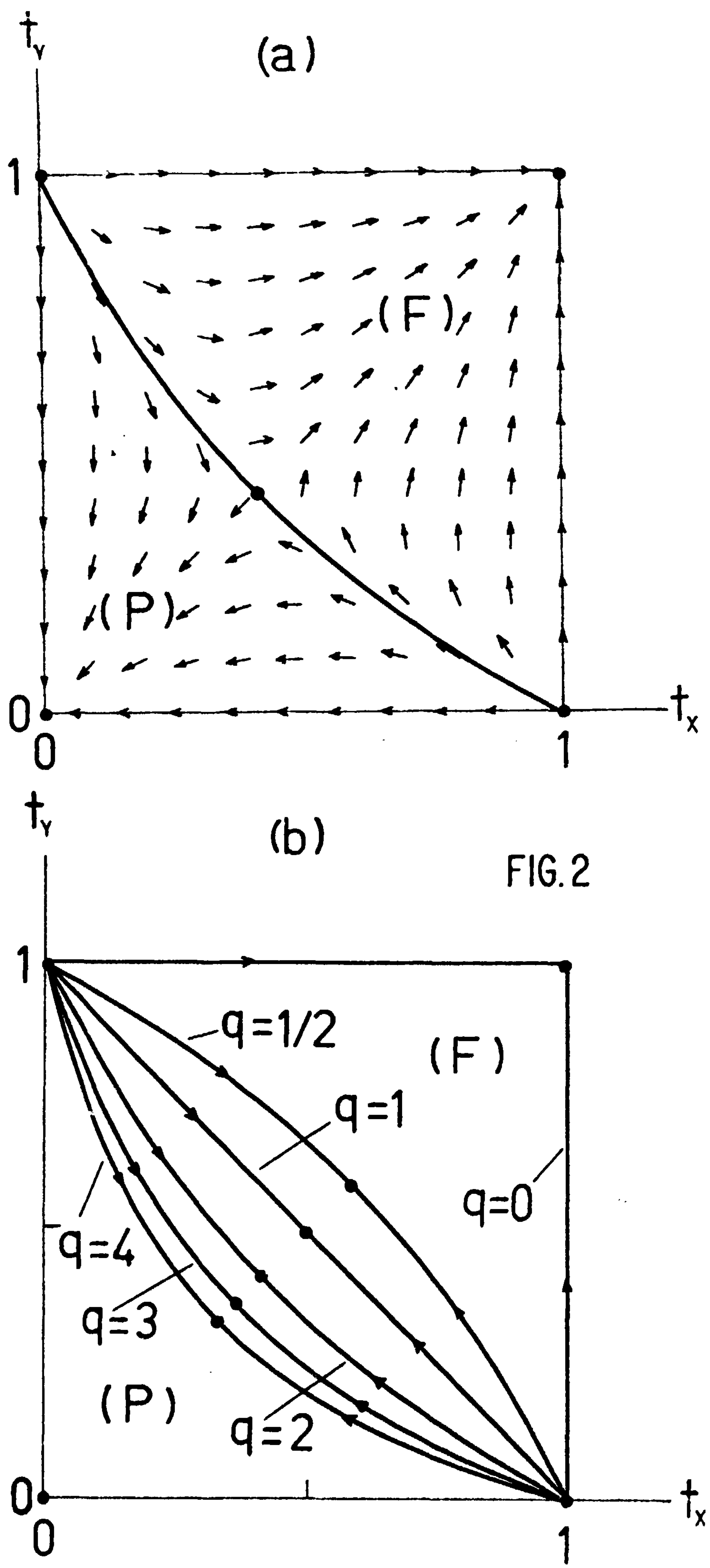

FIG. 2

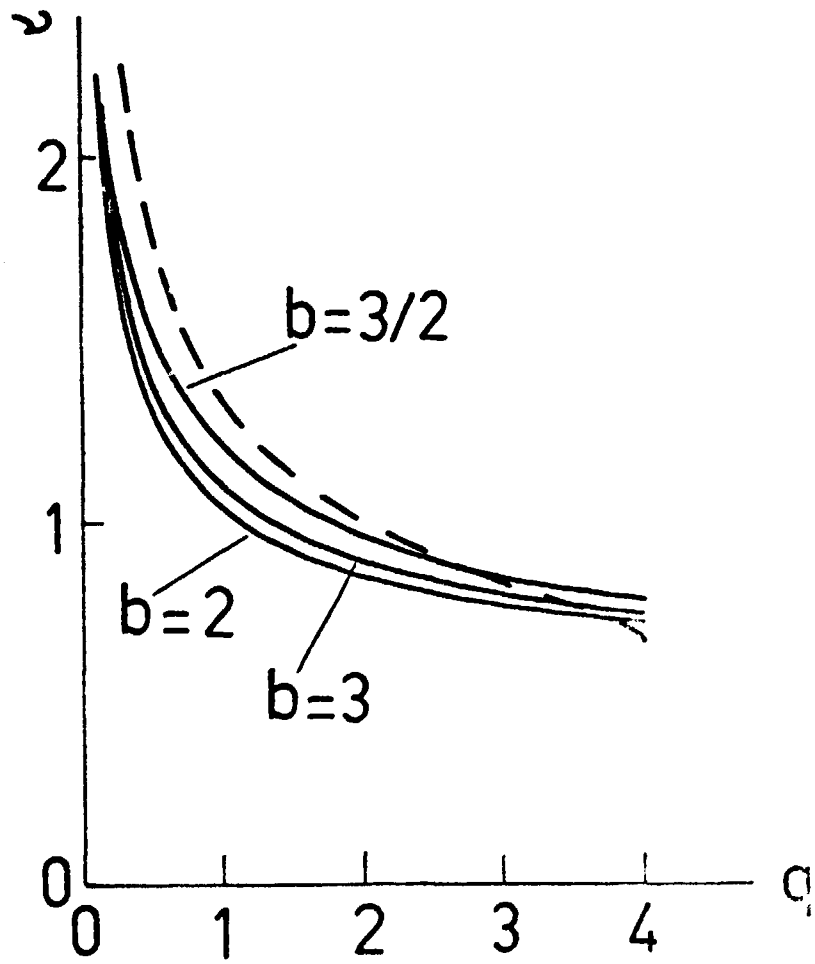

FIG. 3

	$q \to 0$	$q = 1$	$q = 2$	$q = 3$	$q = 4$
$b = 2$	$\frac{45 \ln 2}{52} \frac{1}{\sqrt{q}}$ $\simeq 0.600/\sqrt{q}$	$\frac{\ln 2}{\ln \frac{249}{2^7}}$ $\simeq 1.042$ (b)	$\frac{\ln 2}{\ln \frac{29}{13}}$ $\simeq 0.864$	$\frac{\ln 2}{\ln \frac{82917+47872\sqrt{3}}{34286+19795\sqrt{3}}}$ $\simeq 0.785$	$\frac{\ln 2}{\ln \frac{2193}{857}}$ $\simeq 0.738$
$b = 3$	$\frac{3625 \ln 3}{5996} \frac{1}{\sqrt{q}}$ $\simeq 0.664/\sqrt{q}$	$\frac{\ln 3}{\ln \frac{5700575}{2^{21}}}$ $\simeq 1.099$ (b)	$\frac{\ln 3}{\ln 3.3921}$ $\simeq 0.899$	$\frac{\ln 3}{\ln 3.8777}$ $\simeq 0.811$	$\frac{\ln 3}{\ln 4.2643}$ $\simeq 0.758$
(a) $b = 3/2$	$\frac{32625 \ln(3/2)}{16264} \frac{1}{\sqrt{q}}$ $\simeq 0.813/\sqrt{q}$	$\frac{\ln(3/2)}{\ln \frac{5700575}{249 \times 2^{14}}}$ $\simeq 1.212$	$\frac{\ln(3/2)}{\ln 1.5206}$ $\simeq 0.967$	$\frac{\ln(3/2)}{\ln 1.6034}$ $\simeq 0.859$	$\frac{\ln(3/2)}{\ln 1.6664}$ $\simeq 0.794$
Conjecture (den Nijs 1979)	$\frac{\pi}{3} \frac{1}{\sqrt{q}}$ $\simeq 1.047/\sqrt{q}$	$\frac{4}{3}$ $\simeq 1.333$	1	$\frac{5}{6}$ $\simeq 0.833$	$\frac{2}{3}$ $\simeq 0.667$

TABLE 1

	q^{12}	q^{11}	q^{10}	q^9	q^8	q^7	q^6	q^5	q^4	q^3	q^2	q	q^0
t^4											0	0	0
											6	4	-10
t^5											0	0	52
											10	-30	20
t^6										0	0	46	-92
										6	22	-29	1
t^7										0	10	184	-217
										22	-35	-49	62
t^8									0	0	228	-480	48
									5	46	-57	36	-30
t^9									0	70	319	-1102	893
									20	118	-558	548	-128
t^{10}								0	6	515	-1064	-525	1090
								2	121	-386	721	-469	11
t^{11}								0	183	723	-4258	4617	23
								26	78	136	-1297	963	94
t^{12}							0	24	944	-2276	-4238	14970	-10652
							1	84	439	-2342	3204	-1316	-70
t^{13}							1	322	1545	-13171	23993	-9728	-5082
							12	402	-879	-295	-1548	6676	-4368
t^{14}							44	1684	-6948	-2235	43626	-64269	26378
							107	698	-2763	-4616	24486	-27143	9231
t^{15}						2	565	691	-18558	42975	8663	-118459	97735
						8	495	273	-12816	30030	-9185	-31079	22274
t^{16}						85	1720	-8275	-24764	226588	-562754	620598	-262820
						95	1545	-8963	-2419	94854	-221797	206676	-69991
t^{17}					5	613	1317	-47733	238848	-541629	602256	-264552	-4770
					5	661	-387	-28119	134983	-260803	222492	-55752	-13080
t^{18}					96	2049	-23010	73686	-9690	-502699	1333460	-1483919	638470
					106	1643	-17981	48560	27403	-385954	790677	-692555	228001
t^{19}				6	670	-3482	-21155	253992	-1059645	2125249	-3247444	2397699	-755935
				6	646	-3136	-20232	216118	-803290	1592252	-1765127	1064401	-261638
t^{20}				100	822	-21875	156168	-593712	1369776	-1049916	1606006	-616482	34252
				100	808	-20885	141030	-490388	989517	-1153738	686392	-113320	-39516
t^{21}			6	426	-6242	35850	-100740	86250	344761	-1365061	2258237	-1923936	691514
			6	426	-6162	34040	-85830	29262	431471	-1280621	1769922	-1236910	352396
t^{22}			68	-616	-292	31567	-223782	862934	-2145912	3570355	-3832278	2513723	-738486
			68	-616	-342	31949	-221094	818160	-1906617	2390198	-2773986	1530688	-368408
t^{23}		4	56	-1798	17902	-102164	387934	-1038883	1997700	-2727555	2528469	-1433673	376314
		4	56	-1798	17862	-100988	375229	-966137	1746876	-2184472	1800428	-878332	191272
t^{24}		17	-340	3204	-18802	76432	-226564	499499	-819252	978516	-808826	415122	-99836
		17	-340	3204	-18748	75486	-218840	464259	-717462	787596	-581732	258640	-52080
t^{25}	1	-21	210	-1322	5841	-19107	47502	-90479	131006	-140449	105595	-49834	11121
	1	-21	210	-1322	5825	-18873	45945	-84337	115333	-114031	76962	-31652	5960

TABLE 2

j	α_j	β_j
0	26609765625	26609765625
1	345165975000	301151587500
2	2136160842300	1632004524900
3	8398260105840	5637003575850
4	23558927138490	13935002902530
5	50208537095364	26244158374710
6	84507606761853	39134969393899
7	115275290061296	47406621773750
8	129749686604187	47487604806763
9	122049367890464	39833254423768
10	96809133215685	28227060346311
11	65144830142464	16998851657994
12	37326398614887	8730873263387
13	18238527887576	3830174165004
14	7595924956680	1434397590136
15	2689378468288	457379903606
16	805456074350	123565884406
17	202466608404	28065470706
18	42226821982	5298478858
19	7186504632	817846452
20	973987016	100745336
21	101289672	9544940
22	7605120	654600
23	367800	29000
24	8625	625

TABLE 3

Secretaria de Planejamento da Presidência da República
CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico
CBPF - Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro, RJ
BRASIL