FISSIONING OF NONLINEAR ION-ACOUSTIC RAREFACTIVE PULSE IN A HOMOGENEOUS QUIESCENT PLASMA

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Abstract:

A finite amplitude rarefactive ion-acoustic wave is observed to fission resulting in two minima. After fissioning the two minima travel at different speeds, one at $0.8\, C_s$ and the other at $1.2\, C_s$, where C_s is the ion-acoustic speed.

1. Introduction.

Korteweg de Vries (KdV) equation is known [1] to describe weakly nonlinear ion-acoustic waves with dispersion. Compressive ion-acoustic pulse evolves into a set of ion-acoustic solitons, which result from the balance of nonlinearity and dispersion and are stationary solutions of the KdV equation. A finite amplitude rarefaction pulse is also subjected to nonlinear steepening, but unlike the compressive pulse case, the KdV equation does not give any stationary soliton solutions for the rarefactive initial condition.

Ion-acoustic solitons have been the subject of large number of theoretical and experimental studies [2, 3] because of their interesting characteristics. The experimental investigations on the rarefactive ion-acoustic pulses have been comparitively few [4, 5, 6]. Experiments in double plasma device [4, 5] have shown that a train of peaks develop on the rising part of the rarefaction pulse and no rarefactive solitons exist. While detailed measurements on the peaks generated at the rising part of the rarefactive pulse have been conducted, the behaviour of the rarefactive pulse itself has not been studied in details. In the present communication we present experimental results on the propagation of the rarefaction pulse and results of the numerical solution of KdV equation for rarefactive initial conditions.

2. Experimental System:

The experimental device consists of a cylindrical stainless steel chamber of 40 cm diameter and 1 m length evacuated to a base pressure of $< 10^{-5}$ Torr. Plasma is produced by electron impact ionisation of argon gas at a pressure of 10^{-4} Torr. The plasma is uniform over the chamber dimensions, except close to the walls, and has following parameters: the plasma density $n_e \simeq 2.10^8\ cm^{-3}$, $T_e \simeq 1$ eV; $T_i \lesssim T_e/20$. The ion-acoustic speed as measured through low amplitude ion-acoustic wave propagation is $\simeq 1.66 \times 10^5$ cm/sec.

A grid of diameter 20 cm, is immersed in the plasma and biased to a large negative voltage w.r.t. to plasma potential. The waves are launched by imposing a step voltage to the grid. For wave detection a Langmuir probe biased to collect electron saturation current is employed.

3. Experimental Results:

During the experiment both positive (compressive) and negative (rarefactive) pulses were launched. The compressive pulses evolved into well known solitons, displaying characteristic relationship between amplitude, width and the speed.

For the case of negative pulse no rarefactive solitons were observed. Close to the launching grid, but outside the grid sheath, the negative pulse was seen to be a symmetric single pulse with pulse width $\sim 10\lambda_D$. The propagation characteristics of pulses with initial amplitude $\lesssim 0.5$ % are shown in Fig. 1. The pulses are observed to broaden as they move away from launching grid with speed less than but close to ion acoustic speed.

Fig. 2 which shows density fluctuations as function of time at different axial positions, illustrates the typical propagation characteristics for larger amplitude (0.6 %–10 %) pulses. The rarefactive pulse starts out as a symmetric negative pulse together with a wavetrain with prominent compressive spike at the rising end of the negative pulse. Further propagation leads to (i) broadening of the trough and (ii) steepening of the rising part of the density and generation of wave train at the trailing edge. The broadened trough next fissions near the minimum resulting in the generation of two minima separated by a local maximum. After this fissioning or breaking, the two troughs travel with different speeds and move away from each other. The wavetrain at the trailing edge also separates along with newly created trough. The process of broadening of the trough and steepening at the trailing edge (near the rising part of the density) continues for both the troughs as they propagate further after the fissioning and for the case of pulses of larger initial amplitude ($\geqslant$ 5%) each of the troughs undergo fissioning process as the wave propagates further (Fig. 3).

In Fig. 4, we have plotted time of flight of the pulse minimum to different initial amplitude. The path followed by newly created trough after the fissioning is shown by the dashed line. The solid line in each plot represents the path which the trough would have followed if it travelled with ion-acoustic speed. An examination of these plots reveals the following.

(i) The rarefaction pulse travels with a speed $\lesssim C_s$ initially, and this speed depends upon the initial amplitude of the pulse. The speed decreases with increase of the initial amplitude. (ii) After the fissioning, the first trough acquires a speed $> C_s$ (branch 1) while the newly created trough and the wavetrain which separated along with newly created trough travel with a speed $< C_s$. (iii) The distance to which the rarefaction pulse propagates before fissioning reduces as initial amplitude decreases. (iv) For pulses of initial amplitudes between 2 % and 4.5 %, the speeds of the two troughs created through the fissioning of original trough are $1.2\ C_s$ and $0.8\ C_s$. This separation of $\pm 0.2\ C_s$ in the speeds of fast and slow troughs from the ion-acoustic speed C_s is, however, not maintained in case of pulses of higher initial amplitude, though after each fissioning, one of the troughs has sub-sonic speed while the other one is supersonic.

Fig. 5 shows the variation of the amplitude of the rarefactive pulses as they propagate in the plasma. The amplitude of the newly created trough after the fissioning is indicated by dashed line on each curve. It is seen that,

(i) the attenuation of the pulse amplitude is not characterised by a single attenuation length for pulses of different initial amplitude (ii) the slower trough generated after the fissioning attenuates faster and (iii) for pulses of larger initial amplitudes, the slower trough grows in amplitude over a certain length before attenuation sets in.

4. Numerical Solution of KdV equation.

KdV equation has been solved numerically for rarefactive initial conditions. The results of the calculations for the case of rarefactive initial conditions is shown in figure 6. The shape of the initial pulse is taken to be $\mathrm{Sech}^2\,(\xi/\xi_0)$ and the amplitude is taken to be negative. As seen in the figure the rarefactive pulse travels to the left in ion-acoustic wave frame, implying a speed less than the ion-acoustic speed in laboratory frame, as it evolves in time. Further while the leading edge gets stretched, the trailing edge steepens and a wave train is generated at this edge. However, the numerical calculations do not show any breaking of the rarefactive pulse near the minimum, which continuously reduces in amplitude . Calculations, further show that the number of peaks in the wave train grow as the wave propagates further. The speed of the wave depends upon the initial amplitude of the pulse and decreases with increasing amplitudes.

5. Discussion.

The experimental results reported in section 3, are difficult to understand on the basis of the KdV equation alone, as according to the theory of KdV equation [7, 8] and the results of numerical solutions presented above (section 4), a rarefactive pulse evolves into a wavetrain and no fissioning is expected to occur.

Okutsu and Nakamuro 5 have observed ion-acoustic soliton like pulses evolving from rarefactive initial conditions. The compressive pulses thus produced have been interpreted in terms of the solution of KdV equation. The evolution of the rarefactive part has not been discussed in details.

The behavior observed in present experiment prior to breaking is similar to the behavior observed by Okutsu & Nakamura [5] and as predicted by numerical calculation in all essential aspects. The present experiment, however, does not show as many peaks in the wave train as expected on the basis of numerical calculations. This is understandable in view of the fact that while slow wave train is subjected to Landau damping in actual experiment, the KdV equation does not account for this damping.

Additional features found in the present experiment is the fissioning resulting in the generation of two troughs from the original narrow pulse, and the difference in the speeds of these troughs.

Essential difference between the earlier experiment [5] and the present experiment is in the shape and width of the initial perturbation. While in former case, the initial perturbation is close to single sinusoidal pulse with widths $\gtrsim 100\lambda_D$ in the present experiment the initial perturbation is better approximated by a $\mathrm{Sech}^2(\xi/\lambda)$ with typical width $\sim 10\lambda_D$. The rarefactive pulse, in the present case, thus contains higher K- numbers and hence subjected to larger dispersive effects than the broader pulse in the earlier experiments. It thus appears that the steepening of these narrow pulses at their trailing edge results in generation of higher K-numbers and higher order dispersive term, neglected in deriving KdV equation, may start becoming important. Further, as the potential associated with density depletion is an attractive potential for the ions, for finite ion temperature the ion distribution function may be getting modified in presence of the wave.

It has been shown [9] that the kinetic equation for ion waves exhibit different properties, whether or not terms of order higher than quadratic were included. Further recent theories [10, 11] give significant corrections to the solutions of KdV equations arising out of the higher order dispersive and non-linear terms.

Preliminary numerical calculations including the contribution of higher order terms to KdV equation [10] but neglecting the finite ion temperature effects, show that these higher order terms do not contribute significantly [12] to the rarefactive part of the wave but only modify the wave train at the trailing edge. The higher order terms alone thus do not appear to explain the observed breaking of the trough.

Calculations based on kinetic equations for ions and taking into account higher order nonlinearity and dispersion terms are proposed to be carried out to examine the cause of the observed breaking. Experimental measurement of the ion distribution function within the wave is also envisaged in this connection.

6. Conclusions.

A rarefactive ion acoustic pulse is observed to fission into two, besides developing the expected wavetrain at its trailing edge, as it propagates in a homogeneous quiescent plasma. After the fissioning, the two troughs travel with different speeds, one of which is subsonic and the other one supersonic. KdV equation does not account for observed fissioning.

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Fig. 1 Low amplitude ($|\frac{\delta n}{n}| \simeq 0.5$ %) rarefaction pulse propagation in plasma. Successive traces are obtained at axial positions separated by 25 λ_D. Horizontal axis is time of flight and vertical axis is the amplitude.

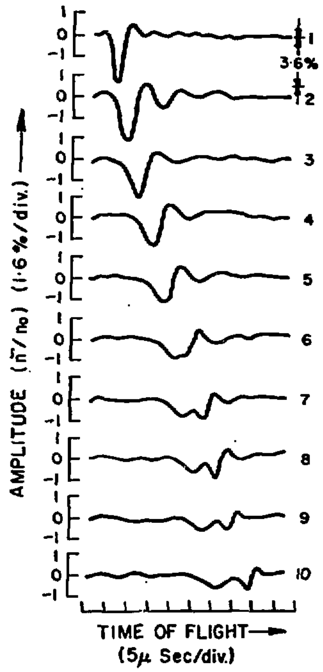

Fig. 2 Typical example of rarefaction pulse propagation in the plasma. Successive traces (1 to 10) are time records taken at axial position separated by 12.5 λ_D. Horizontal axis is time of flight in 5 μ sec/div and vertical axis gives amplitude ($\tilde{n}/n_0$) in 1.6 % per division. Amplitude of step pulse applied to grid for all the traces is 1 volt.

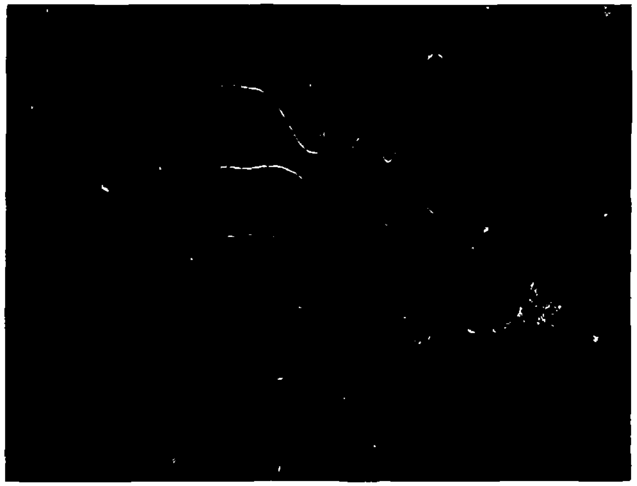

Fig. 3 Example of multiple fissioning of the rarefactive pulse . Axes and scales are same as for Fig. 2. The initial pulse amplitude in this case is 10 % and traces shown here are at positions away from the position at which first fissioning occurred.

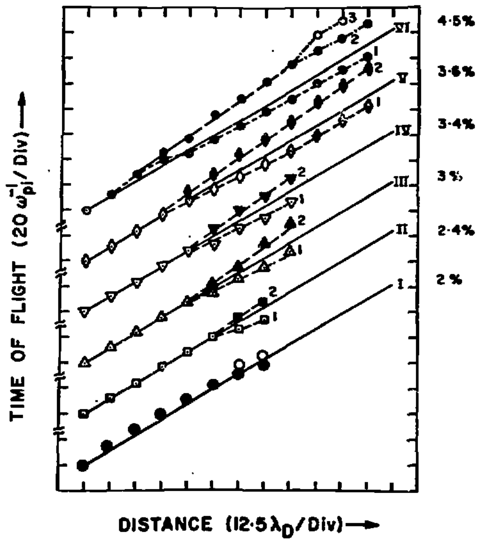

Fig. 4 Time of flight vs. Axial position for the rarefactive pulses of different initial amplitudes 2 % to 4.5 %. The initial amplitudes are indicated on right hand side.

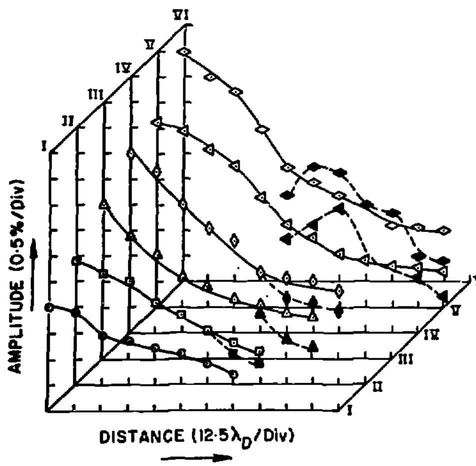

Fig. 5 Amplitude vs. axial position for rarefactive pulses of different initial amplitudes. Cases 1 to VI correspond to cases I to VI of Fig. 4 (a). Open points represent the amplitude of the original pulse and the darkened points denote the amplitude of new trough created after the fissioning.

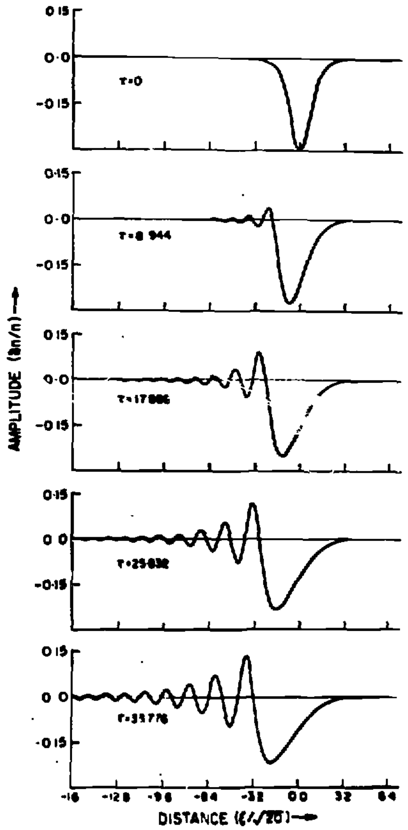

Fig.6 Evolution of the rarefactive pulse according to the calculations based on the KdV equation.