THE COMPRESSIONAL ALFVÉN INSTABILITY IN ECRH PLASMAS

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ABSTRACT. It is shown that the hot electron component present in an electron cyclotron resonance heated plasma can destabilize the compressional Alfvén wave if β of the background plasma exceeds a certain limit. The relevance of the result to the Elmo Bumpy Torus experiment is discussed.

A renewed interest in the experimental and theoretical study of electron cyclotron resonance heated (ECRH) plasmas was prompted by the recent success in producing a stable plasma in Elmo Bumpy Torus (EBT). EBT consists mainly of a number of simple magnetic mirror machines which are longitudinally connected and bent into a torus. The injection of microwave power produces a finite-β annulus of resonantly heated electrons which provides a local diamagnetic well (β is the ratio of kinetic pressure to magnetic pressure). Within this minimum-average- B layer, a stable warm core plasma is obtained [1].

It is essential that the non-resonant fluid modes are stable in a successful fusion device. Previous work has shown that the stability of a finite-β plasma to the low frequency interchange mode is related to the presence of a minimum-average-B surface in the vacuum magnetic field and not to the diamagnetic well dug by the plasma [2,3]. For this mode $\omega/\Omega_{ci} \ll 1$ and $\vec{k}.\vec{B} = o$ where ω is the mode frequency, Ω_{ci} is the ion cyclotron frequency, $\vec{k}$ is the wave vector and $\vec{B}$ is the magnetic field. The effects of the presence of a hot electron component on the mode were later taken into account [4,5]. It was shown that the hot electrons can provide stability if

$$4\epsilon - \beta_h < \beta_w < 4\epsilon/(1+\beta_h) \qquad (1)$$

where $\epsilon = L_n/R_c$, $L_n = |\partial[\ln n(x)]/\partial x|^{-1}$ is the density scale length in the slab model with the density decreasing in the x-direction, R_c is the radius of curvature of the magnetic field and the subscripts w and h refer to the warm and hot plasma components respectively. We may note from the above condition that a reversal of the direction of the magnetic drift of the charged particles caused by the diamagnetic well is necessary for stability, contrary to the case when hot electrons were absent.

The high frequency interchange instability of the hot electron component has also been investigated in the zero and finite-β limits [6-9]. For this mode $\omega \sim k\, v_{dh}$ where v_{dh} is the average drift velocity of the hot electrons in the vacuum magnetic field. It was found that the instability disappears if the ratio of the warm electron density to the hot electron density, n_{ew}/n_h, exceeds a certain threshold. In effect, this sets a lower bound on β_w for stability.

The purpose of this work is to show that a new high frequency instability may be present in an ECRH plasma, driven by the destabilizing influence of the hot electrons on the compressional

Alfvén oscillations. The related stability threshhold is derived.

We begin by using the two fluid equations to derive the dispersion relation for the compressional Alfvén mode in a two component inhomogeneous cold plasma. The necessary equations are:

$$\frac{\partial n_{1j}}{\partial t} + \vec{\nabla} \cdot (n_{oj} \vec{v}_{1j}) = o \qquad (2)$$

$$m_j \frac{\partial \vec{v}_{1j}}{\partial t} = e_j \, (\vec{E}_1 + \vec{v}_{1j} \times \vec{B}_o) \qquad (3)$$

$$\sum_j n_{1j} \, e_j = o \qquad (4)$$

$$\vec{\nabla} \times \vec{B}_1 = \mu_o \sum_j n_{oj} \, e_j \, \vec{v}_{1j} \qquad (5)$$

$$\vec{\nabla} \times \vec{E}_1 = - \dot{\vec{B}}_1 \qquad (6)$$

The subscripts o and 1 refer to the equilibrium and perturbed values respectively, $\vec{E}$ is the electric field, e_j is the particle charge of species j and $\vec{v}$ is the particle velocity. Adopting a slab model in the local approximation with the equilibrium magnetic field pointing in the z-direction and the density decreasing in the x-direction and noting that the mode is characterized by the field components E_x, E_y and $B_z \sim \exp[-i(\omega t - ky)]$ we can solve for the dispersion relation to obtain

$$\omega^2 = k^2 \, V_A^2 \, (1 - \delta' \omega / \Omega_{ci}) \qquad (7)$$

where $V_A = [B_o^2 / (\mu_o \, n_o \, M)]^{1/2}$ is the Alfvén speed, M is the ion mass and $\delta' = 1/(kL_n)$. It is easy to show that the wave is always stable. The presence of a density gradient appears through the parameter δ' and can be important in some limits [10].

We now turn to the case when a hot electron component is present. For simplicity we assume all three components to posses the same density scale length and replace the magnetic curvature by an equivalent gravity $\vec{g}_j = (2 \, v_j^2 / R_c) \, \vec{e}_x$, where v_j is the thermal speed of species j [2]. The hot electrons will be assumed to be monoenergetic with a distribution function given by

$$f_{oh} = n_{oh} \, \delta(W_\perp - W_h) \, \delta(W_\parallel - W_h/2) \left[1 - \frac{1}{L_n}\left(x - \frac{v_y}{|\Omega_{ce}|}\right)\right] \qquad (8)$$

where $W_\perp$ and $W_\parallel$ are the kinetic energies perpendicular and parallel to the magnetic field respectively, W_h is a constant and δ is the Dirac delta-function. This choice eliminates the need to compute the residues and offers a good approximation to the more realistic maxwellian distribution [9]. The diamagnetic well can be obtained directly from Eq. 8 and Ampere's law

$$\vec{\nabla} B_o / B_o \simeq \frac{\beta_h}{2L_n} \, \vec{e}_x \qquad (9)$$

To obtain the dispersion relation, we solve the linearized Vlasov equation for the perturbed hot electron distribution function in the standard manner to obtain [2,3]

$$f_{1h} = \frac{i f_{oh}}{L_n B_o} \; \frac{\left(E_{y1} - i \frac{k v_h^2}{2|\Omega_{ce}|} B_{z1}\right)\left(1 - \frac{i k v_x}{|\Omega_{ce}|}\right)}{\omega + b \omega_{dh}} \qquad (10)$$

where $b = \beta_h / 4\epsilon - 1$ and $\omega_{dh} = k v_{dh}$

We may now evaluate the perturbation in the hot electron charge and current densities, to be added to the cold contributions present in Eqns. 4 and 5.

Assuming for simplicity that n_{oh}/n_{oi} is negligibly small but that β_h remains finite, we obtain finally the dispersion relation

$$\omega^2 = k^2 V_A^2 \left(\frac{\omega/\omega_{dh} - 1}{\omega/\omega_{dh} + b} \right)(1 - \delta'\omega/\Omega_{ci}) \quad (11)$$

In the practical EBT limit $\omega_{dh}\delta'/\Omega_{ci} \gg 1$, the stability condition becomes

$$k^2 V_A^2 / \Omega^2_{ci} \gtrsim 4\, b/\delta'^2$$

which can be written as

$$n_{oi} L^2_n \lesssim 1.3 \times 10^{14}/b \quad cm^{-1} \quad (12)$$

for a hydrogen plasma. Near the stability boundary $\omega \sim 2\Omega_{ci}/\delta'$ and thus the instability is a high frequency one since δ' must be < 1 to justify the use of the local approximation. An increase in the depth of the diamagnetic well obviously reduces the maximum achievable core plasma density.

It is not difficult to account for the finite temperature effects of the background plasma [11]. The stability condition for the magnetosonic wave then becomes

$$\beta_w \lesssim \frac{1}{\frac{1+\beta_h}{4\epsilon} + \frac{4\,b}{(1+\tau_w)\,\gamma^2}} \quad (13)$$

where τ_w is the ratio of the warm electron temperature to the ion temperature, $\gamma = a_i/L_n$ and a_i is the ion larmor radius. The maximum achievable β_w is then always lower than that obtained by considering the stability of the background plasma to the interchange mode, given by Eq. 1. It has also been shown that the local solution is a good approximation to the more realistic case when the radial variation of the perturbed quantities is taken into account [12].

In conclusion we have shown that the presence of the hot electron component in an ECRH plasma can lead to the destabilization of the compressional Alfvén instability. A simple expression for the maximum achievable warm plasma density was obtained. This may be useful in the design of future devices which incorporate electron cyclotron resonance heating, such as the Elmo Bumpy Torus and the Tandem mirror.

ACKNOWLEDGEMENT. The author wishes to thank C.L. Hedrick for suggesting the study of the high frequency effects and for many useful discussions. The helpful comments of H.L. Berk, M.N. Rosenbluth, D.A. Spong and J.W. Van Dam are gratefully acknowledged.

REFERENCES

[1] DANDL, R.A., et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc.7th Int. Conf. Innsbruck, 1978) Vol.2, IAEA,Vienna (1979).

[2] ROSENBLUTH, M.N., KRALL, N.A., ROSTOKER, N., Nucl. Fusion Suppl. Pt. 1 (1962) 143.

[3] EL NADI, A.M., EMMERT, G.A., Plasma Physics <u>16</u> (1974) 365.

[4] VAN DAM, J.W., LEE, Y.C., "Stability Analysis of a Hot Electron EBT Plasma", EBT Ring Physics (Proc. of the Workshop, Oak Ridge, 1979) ORNL CONF-791228, Oak Ridge (1980) 471.

[5] NELSON, D.B., Phys. Fluids 23 (1980)1850.

[6] KRALL, N.A., Phys. Fluids 9 (1966) 820.

[7] GUEST, G.E., HEDRICK, C.L., NELSON, D.B., Phys. Fluids 18 (1975) 871.

[8] BERK, H.L., Phys. Fluids 19 (1976) 1255.

[9] DOMINGUEZ, R.R., BERK, H.L., Phys. Fluids 21 (1978) 827.

[10] BERK, H.L. (private communication).

[11] EL NADI, A.M., "On the Fluid Stability of an EBT Plasma". (to be submitted to Phys.Fluids).

[12] EL NADI, A.M., HASSAN, H.F., TSANG, K.T., (to be published).