PARTICLE ACCELERATION BY ELECTROMAGNETIC PULSES

H.M. LAI
Department of Physics,
The Chinese University of Hong Kong,
Hong Kong

Particle interaction with plane electromagnetic pulses is studied. It is shown that particle acceleration by a wavy pulse, depending on the shape of the pulse, may not be small. Further, a diffusive-type particle acceleration by multiple weak pulses is described and discussed.

I. INTRODUCTION

The study of particle interaction with an intense plane electromagnetic pulse in vacuum has attracted continued attention [1-12], not only because of its theoretical interest but also because of its relation to practical problems such as free electron laser, particle acceleration and plasma heating. A basic question concerned in this study is how much momentum a particle will gain after the passage of such a pulse. Studies [12] neglecting radiation damping have shown that the gain will be considerable if the pulse is solitary or non-oscillatory and that the gain will be negligible if the pulse is wavy or contains many oscillations. On the other hand, studies which include radiation damping [4,7,10,11] have indicated a finite amount of momentum gain even if the pulse is wavy, the reason being that radiation damping results a small phase shift of the particle velocity relative to the electric field and thus a forward averaged Lorentz force.

The present paper reports two recent results from our study on this problem. First we have found that a wavy pulse, under certain conditions, can transfer to the particle a finite amount of momentum which is not at all small (not to mention negligible), especially when compared with that due to radiation damping. This will be described in Section II. Furthermore, we have found an interesting mechanism of particle acceleration by multiple random weak pulses. Although each pulse transfers only a minute amount of momentum to a particle, multiple pulses can result in particles of finite momentum through a process similar to the random walk process in momentum space. This will be presented in Section III.

II. PARTICLE ACCELERATION BY A WAVY PULSE

For such a study, it is convenient to start with the two well-known constants of motion [2]: $\vec{p}_\perp+q\vec{A}/c$ and $K-p_\parallel c$, where $p_\perp$ and $p_\parallel$ are, respectively, the momentum components transverse and parallel to the direction of wave propagation (taken to be along the z-axis), K is the kinetic energy, $\vec{A}$ is the vector potential of the wave, q is the charge of the particle and c is the speed of light. To make it simple, we let the charge be initially stationary at position z=0 and the pulse interact with the particle only after time t=0, so that the two above expressions become

$$\vec{p}_\perp = -q\vec{A}/c \tag{1}$$

$$K = p_\parallel c = p_\perp^2/2m \tag{2}$$

where the vector potential is a function of $\xi \equiv t-z/c$ only and is related to the wave electric field $\vec{E}(\xi)$ by

$$\vec{A}(\xi) = -c \int_0^\xi \vec{E}(\eta)d\eta \,. \tag{3}$$

Note that the last equality in Eq. (2) comes from $K = (p^2+m^2c^2)^{\frac{1}{2}}c-mc^2$, with m being the rest mass. Also note that $p_\parallel/p_\perp$ is a small quantity for non-relativistic cases. From Eqs. (1) and (2), it is obvious that the final momentum of the particle after the passage of the pulse is uniquely determined by the asymptotic value of the vector potential: $\vec{A}_f \equiv \vec{A}(\xi\to\infty)$. Consider two electromagnetic (field) pulses of similar energy content, one is solitary or non-oscillatory and the other is wavy. The solitary pulse will give a vector potential which clearly increases monotonically with ξ to its asymptotic value at large ξ, while the wavy pulse will give a vector potential which increases and decreases alternatively and eventually may end up with a small value. We thus see that the momentum gain by the particle after an encounter with a solitary pulse will be most efficient.

Exactly how small the momentum gain from a wavy pulse will be depends on the exact shape of the electric field and this in turn depends on the emission mechanism of the wave. Suppose the wave is emitted from a damped harmonic oscillator, initially excited by a collision, say. The electric field of the wave may then be given by

$$\vec{E}(\xi) = \vec{E}_0 e^{-\xi/\tau}\sin\omega\xi \tag{4}$$

and the asymptotic vector potential can be easily calculated. This readily leads to a momentum gain, in units of mc,

$$\Delta\vec{p}_\perp/mc = (q\vec{E}_0\tau/mc)\,[\omega\tau/(1+\omega^2\tau^2)] \tag{5}$$

which becomes, for a wavy pulse (i.e., $\omega\tau>>1$),

$$\Delta\vec{p}_\perp/mc = q\vec{E}_0/mc\omega \,. \tag{6}$$

We immediately recognize that this is the same expression for the maximum momentum that a charge q will have during the time of interaction with a harmonic wave of amplitude E_o and frequency ω! At this point, we like to emphasize that the momentum gain depends on the pulse shape in a very sensitive way. Should the function $\sin(\omega\xi)$ in Eq. (4) be replaced by $\cos(\omega\xi)$ or $\xi\cos(\omega\xi)$, the gain $\Delta p_\perp$ in Eq. (6) would have been proportional to $E_o\tau/(\omega\tau)^2$ [redacted] in the limit $\omega\tau >> 1$. Nevertheless, the momentum gain from a general wavy pulse may be given by

$$\Delta \vec{p}_\perp/mc = (q\vec{E}_o\tau/mc)\cdot S(\omega\tau) \tag{7}$$

where $S(\omega\tau)$, a function of $\omega\tau$ and whose functional form depends on the shape of the field pulse, approaches zero as $\omega\tau$ tends to infinity. For many cases in which $\omega\tau$ is not too large, the gain in momentum may not be negligible. Note that Eq. (7) applies for waves of linear polarization, results for other states of polarization can be obtained via linear superposition.

We now compare Eq. (7) with the momentum gain due to the mechanism through radiation damping. A recent calculation [10] has shown that such a momentum gain, for an initially stationary electron and to the lowest order, may be written as

$$(\Delta \vec{p})_{R.D.}/mc = (r_o^2 E_o^2 \tau/3mc)\hat{e}_z \tag{8}$$

where $r_o \equiv e^2/mc^2$ is the classical electron radius. The ratio of the two values in Eqs. (7) and (8) is thus

$$\Delta p_\perp/(\Delta p)_{R.D.} = 3(E_c/E_o)\, S(\omega\tau) \tag{9}$$

where $E_c \equiv e/r_o^2 \doteq 10^{19}$ V/cm is the electric field produced by an electron at distance r_o. Since E_c is a huge quantity, E_c/E_o is generally very large. Even for an intense wavy pulse of radiation power density of the order of 10^{16} W/cm^2, the ratio E_c/E_o is still of the order of 10^{10}! We therefore see that, although $S(\omega\tau)$ is a small quantity, the ratio in Eq. (9) may very well be a large quantity. This is especially true for a weak pulse or for a short wavy pulse.

III. ACCELERATION BY MULTIPLE PULSES

In this section, we shall discuss a mechanism of particle acceleration by multiple pulses. The pulses are assumed to be random so that any two pulses are completely uncorrelated in terms of phase, polarization and direction. Furthermore, they are also assumed to be weak so that the effect of radiation damping may be neglected. Since $\vec{p}_\perp + q\vec{A}/c$ and $K - cp_\parallel$ are constants of motion, every pulse, after overtaking a particle, will transfer to the particle an amount of momentum Δp:

$$\Delta \vec{p}_\perp = -q\Delta\vec{A}/c \tag{10}$$

$$\Delta p_\parallel = \Delta K/c \tag{11}$$

where

$$\Delta\vec{A} = -c\int_{-\infty}^{\infty} \vec{E}(\eta)\,d\eta \tag{12}$$

with $\vec{E}$ being the electric field for that particular propagating field pulse. For simplicity, we shall restrict to nonrelativistic cases so that $\Delta p_\parallel$ may be neglected compared to $\Delta p_\perp$ and $\Delta p \doteq \Delta p_\perp$. Since the pulses are random, the momentum gains $\Delta\vec{p}$'s by a particle in successive encounters with the pulses are also random and it is then impossible to predict precisely the total momentum of the particle after a large number of encounters. However, if we consider a system of many non-interacting particles, each encountering with a large number of pulses, it is possible to get the averaged value over these particles. This is exactly like the problem of diffusion of test particles in momentum space [13]. The root-mean-square value of the total particle momentum gain after N encounters is thus

$$\Delta P_{rms} = \sqrt{N}\,\Delta p_{rms} \tag{13}$$

where Δp_{rms} is the root-mean-square value of the momentum gain in one encounter. We thus see that, even if Δp_{rms} is a minute quantity, a good portion of the particles can eventually gain a finite amount of momentum provided N is sufficiently large.

As an example, we discuss the effect the above mechanism will bring to the charged particles in our Galactic space which is permeated by electromagnetic pulses of various kinds from the stars. The pulses may be considered random. Suppose we have a set of charges of one specie initially at rest. After encountering with N pulses, the root-mean-square momentum gain is given by Eq. (13) and the corresponding root-mean-square value of the particle displacement may be described by

$$\Delta R_{rms} = NT\Delta P_{rms}/m \tag{14}$$

where T is the averaged time between two consecutive encounters by a particle. We want ΔR_{rms} to be limited by the size of the Galaxy so that a large portion of the particles will still remain in the Galactic space after N encounters. This gives a limit to N, using Eqs. (13) and (14),

$$N \lesssim (R_G/cT)^{2/3}(\Delta p_{rms}/mc)^{-2/3} \tag{15}$$

where R_G is the radius of the Galaxy. This further gives a limit to the momentum

$$\Delta P_{rms}/mc \lesssim (R_G/cT)^{1/3}(\Delta p_{rms}/mc)^{2/3} \tag{16}$$

Since Δp_{rms} is independent of the mass we see that the momentum gain is proportional to the cube root of the particle mass. We may make a rough estimate of the gain by taking the size of our Galaxy to be 10^4 light-years, the averaged radiation energy density to be 1 eV/cm^3 [14], the averaged pulse width to be $\tau \simeq 10^{-8}$ sec and $T \simeq \tau$, the averaged frequency to be 10^{15} sec^{-1} and assuming the pulse shape given by Eq. (4). The result is

$$\Delta P_{rms}/mc \lesssim 10^{-2} . \tag{17}$$

This value of course cannot account for the existence of energetic cosmic rays, but nevertheless indicates a certain amount of particle momentum possessed by the background particles. Furthermore, we want to stress again that the value in Eq. (17) depends very much on the pulse shape chosen. Had we assumed a different pulse shape, the value obtained would have been quite different. Finally, we like to mention that the above consideration is classical. Quantum consideration is of interest and should provide a deeper understanding of the problem.

Acknowledgment- The author is grateful to Dr. K. Young for fruitful discussions.

References

[1] McMillan, E.M., Phys. Rev. 79 (1950) 498.

[2] Landau, L.D., Lifshitz, E.M., "Classical Theory of Fields", Addison-Wesley, Reading, Massachusetts (1962) 128.

[3] Morozov, A.I., Solovev, L.S., in "Reviews of Plasma Physics" (Leontovich, M.A., Ed.), Consultants Bureau, New York (1966), Vol. II, 203.

[4] Kibble, W.B., Phys. Rev. Lett. 16 (1966) 1054, 1233.

[5] Boyd, T.J. M., Sanderson, J.J., "Plasma Dynamics", Barnes and Noble, New York (1969) 28.

[6] Chan, Y.W., Phys. Lett. 30A (1969) 270, 35A (1971) 305.

[7] Steiger, A.D., Woods, C.H., Phys. Rev. A5 (1972) 1467.

[8] Schmidt, G., Wilcox, T., Phys. Rev. Lett. 31 (1973) 1380.

[9] Kaw, P.K., Kulsrud, R.M., Phys. Fluids 16 (1973) 321.

[10] Fradkin, D.M., Phys. Rev. Lett. 42 (1979) 1209; Phys. Rev. D 22 (1980) 1018.

[11] Schmidt, G., Phys. Lett. 74A (1979) 222.

[12] Lai, H.M., Phys. Fluids 23 (1980) 2373.

[13] Spitzer, L., Jr., "Physics of Fully Ionized Gases", 2nd Ed., Interscience Publ. (1962) 123-124.

[14] Ginzburg, V.L., Ptuskin, V.S., Rev. Mod. Phys. 48 (1976) 161.