APPLICATION OF THE DESCRIPTIVE FUNCTION ON NON-LINEAR ELECTROMAGNETIC PHENOMENA

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INTRODUCTION

Owing to present possibilities to succeed in obtaining intensive electromagnetic field, the circle of ondulating non-linear phenomena expands more and more, especially in the micro-waves range and in the laser frequency.

The differential equations that rule the behaviour of these processes must be normally solved by altgarithms. The problem is normally quasi-lined in order to effect later on corrections due to the non-linear ends.

Regarding the propagation of electromagnetic waves, either on a limitless medium or on guides, and when the flow makes harmonious oscillations with a W-frequency, the approximation of the prefixed field [1] is used, that is, the component of the field corresponding to the primary harmonic (W) is determined beginning with Maxwell's linear equations. Once this field is determined, for instance E (W) operates as a prefixed function in the equations that rule for the superior harmonic.

However, this method does not make provision for the response from the non-linear medium in regard to the frequential components of the initial flow or to the electromagnetic field free of primary frequency (W).

In this work the descriptive function technique used in non-linear control is stated as a tool that can be usable in ondulating processes and which allows to analyse the response from the medium against the O.E.M. propagation of primary frequency (W).

A plane transverse O.E.M. is considered in a limitless medium for the sake of simplicity in the solution and due to the utility that it offers in more complex problems.

I. MAIN EQUATIONS OF ELECTRODYNAMICS

1.1 Maxwell's Equations System and Constitutive Relationship.

The characterization of an electromagnetic field in a certain point in the space is given by the existing relationship among the vectors E (electric field); H (magnetic field); B (magnetic induction); D (electric displacement); J (current density) and P (charge density).

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad ; \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \cdot \vec{B} = 0 \quad ; \quad \nabla \cdot \vec{D} = \rho \qquad (1)$$

With the following functional relationships of the aforementioned vectors with the electric polarization (P), the magnetisation (M) and the current density equivalent to the flow J ext. [2].

$$\vec{D} = \vec{D}(\vec{E}) = \epsilon_0 \vec{E} + \vec{P}(\vec{E})$$
$$\vec{B} = \vec{B}(\vec{H}) = \mu_0 \vec{H} + \vec{M}(\vec{H})$$
$$\vec{J} = \vec{J}(\vec{E}) + \vec{J}_{ext} \qquad (2)$$

When the medium is linear, homogeneous and isotropous are:

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{J} = \sigma \vec{E} + \vec{J}_{ext} \qquad (3)$$

in that ϵ, μ, σ correspond to the permittivity, permeability and conductivity of the medium respectively.

Owing to the importance of the fields that oscillate harmonically, it is useful to represent the Maxwell's equations in their complex way where:
E (x, y, z, t) $\rightarrow \dot{E}$ (x, y, z) exp jwt...etc.
and developing the functions E (t)...etc...etc...
in Fourier series under type [3]
E (x, y, z, t) = $\sum \dot{E}n$ (nw) exp jwt
it is had for any non-linear medium, the infinite equations system.

$$\begin{aligned} \nabla \times \dot{H}_n &= jn\omega [\epsilon \dot{E}_n + \dot{P}_n] + \dot{J}_n ext \\ \nabla \times \dot{E}_n &= -jn\omega [\mu \dot{H}_n + M_n] \\ \nabla \cdot \dot{B}_n &= 0 \\ \nabla \cdot \dot{D}_n &= \rho_n \end{aligned} \qquad (4)$$

1.2 Wave Equations for a Non-linear Dielectric Medium.

If we consider an homogeneous and isotropous dielectric medium, the wave equations for E and H are obtained beginning with: $\rho_n = 0 \quad \forall n$

$$\nabla^2 \dot{E}_n = (jn\omega)^2 \mu \dot{D}_n + jn\omega\mu \dot{J}_n ext$$

$$\nabla^2 \dot{H}_n = (jn\omega) \nabla \times \dot{D}_n \qquad (5)$$

In order to simplify (5), the displacement complex vector is defined as:

$$\dot{D}_n (n\omega) = \epsilon N_n (E_n) \dot{E}_n (n\omega) \qquad (6)$$

where Nn is the complex factor that takes into account the response from the medium to the propagation of the wave (E). (N(E) = 1 for a linear dielectric).

That way, the wave equations for an electromagnetic field take the form of:

$$\begin{aligned} \nabla^2 \dot{E}_n &= (jn\omega)^2 \mu\epsilon [\dot{E}_n N_n] + \dot{F}_n \\ \nabla^2 \dot{H}_n &= (jn\omega)^2 \mu\epsilon [\dot{H}_n N_n] - jn\omega\epsilon (\nabla N_n \times \dot{E}_n) \end{aligned} \qquad (7)$$

where Fn is a complex function that takes into account the external flows and the prefixed functions. Normally for n = 1 F1 corresponds to the external flow of primary frequency, for n ≠ 1, Fn corresponds mainly to the prefixed functions to those of order n - 1, previously determined, for the case of magnetic non-linears, the function appears in the wave equation of the magnetic field.

II. DESCRIPTIVE FUNCTION AND ITS APPLICATION TO THE WAVE EQUATION SOLUTION.

2.1 Descriptive Function

The descriptive function [4] assumes that the high frequencies of the outlet signal have poor effects upon the system characteristics, when at the inlet of the non-linear system a sinusoidal signal is applied. It is defined as the complex relationship of the primary component of outlet signal at the inlet sinusoidal signal.

$$N = \frac{\dot{D}_o}{\dot{D}_i} = \left|\frac{D_o}{D_i}\right| \angle \phi \qquad (8)$$

in that $D_i$senwt is the inlet signal and D_1 is determined by the expression

$$\dot{D}_1 = \frac{1}{2\pi}\int_{-\pi}^{\pi} d_s(t)\, exp\,(-j\omega t)\; d(\omega t)$$

(9)

where ds(t) is expressed by the Fourier series:

$d_s\ (t) = \sum \dot{D}n\ exp\ (j\ n\ w\ t)$

(10)

Once N is determined, the non-linear system can be analysed in the same way that in a linear system.

If it is desired to apply the descriptive function to characterize a non-linear medium, the expression (6) can be considered with n = 1, we associate it with the expression (8) in that D_1 is considered as an "outlet" and ϵE_1 as the "inlet" resulting in:

$$N_1 = \frac{D_1(\omega)}{\epsilon\, E_1(\omega)}$$

(11)

Once the descriptive function is obtained explicitly, it is introduced into (7) and then we are in conditions to resolve the wave equation of the primary frequency.

2.2 Linear Polarization

If we consider the propagation of a linearly polarized transverse wave, for example, the electric field according to the axis X, the magnetic field according to the axis Y and the propagation direction according to the axis Z, the expressions (7) for n = 1 result in:

$$\frac{d^2\dot{E}_1}{dz^2} = \gamma^2\,\dot{E}_1\; N_1\,(E_1)$$

(12)

where γ is the propagation constant. Here J ext = o has been considered and the solution of (12) is obtained by applying edge condition in the separation interface (Z =0), between a limitless linear medium (Z < 0) and the non-linear medium (Z > 0). When it is possible to express $N_1(E_1)$ as:

$N_1(E_1) = 1 + R(E_1)$; the equality:

$$\frac{\partial}{\partial z}\left[\frac{\partial E}{\partial z}\right]^2 = 2\,\frac{dE}{dz}\,\frac{d^2E}{dz^2}$$

(13)

allows to reduce the second order differential equation to another of a first grade resulting in:

$$\frac{dE_1}{E_1\left[1 + 2\int E_1 (N_1 - 1)/E_1^2\right]^{1/2}} = \pm\,\gamma\, dz$$

(14)

obtaining that way solutions such as:

$$E_1(z) = A(z)\, exp\,(-\gamma z)$$

(15)

In this case, the descriptive function is not restricted to the field E be a slowly variable function with Z. In case that it were so, the method WKB enables to obtain solutions for E(Z) which show an effect similar to the called Kerr effect. The latter is produced by the interaction between a field of primary frequency and another of null frequency. Instead, the former is produced by the non-linear medium response in front of the $E_1(Z)$ wave propagation of primary frequency. Other solutions obtained are on a basis of elliptic exhaustive analysis is required [5] .

If we consider the particular case of a dielectric with a cubic type lineality,

Be $D = \epsilon E - c\, E^3$

(16)

using (9) and (11), we have

$$N_1(E_1) = 1 - \frac{3}{4}\frac{c}{\epsilon}E_1^2 \qquad (17)$$

integrating (13) it turns out [6].

$$E_1(Z) = \frac{2a\exp[-\mu z]}{K + \frac{1}{K}\exp[-2\mu z]}$$

$$K = \frac{a \pm [a^2 - E_0^2]^{1/2}}{E_0} \; ; \quad a = \left(\frac{8}{3}\frac{\epsilon}{c}\right)^{1/2} \qquad (18)$$

K depends on E_0 which is the value of the electric field in Z = 0. If $c \to 0$; a, $K \to \infty$; $E_1(Z) \to E_0 \exp(-\mu z)$ which is the solution for a wave that is propagated in a linear medium. The wave reflected in the second medium is not considered since this is limitless.

When the K factor $\to 1\angle\phi$; $\phi = 0 \pm \pi/2, \pi$ etc ... which means that $E_0 = \pm a; E_0 \gg a$; it is obtained:

$$E_1(z) = \pm a \,\mathrm{sech}\, \mu z \qquad \text{or}$$

$$E_1(z) = \pm j a \,\mathrm{cosech}\, \mu z \qquad (19)$$

In the case that the propagation constant be approximately equal to the face constant

$$E_1(z,t) = \pm \frac{a \cos \omega t}{\cos \beta z} \qquad \text{or}$$

$$E_1(z,t) = \pm \frac{a \cos \omega t}{\mathrm{sen}\, \beta z} \qquad (20)$$

not having now the argument $(\omega t - \beta z)$, characteristic of the electromagnetic wave propagation, but rather something similar to stationary waves.

If it is considered the relationship (16) with symbol plus

$$E_1(z,t) = \pm a \frac{\mathrm{sen}\, \omega t}{\mathrm{sen}\, \beta z} \qquad \text{or}$$

$$E_1(z,t) = \pm a \frac{\mathrm{sen}\, \omega t}{\cos \beta z} \qquad (21)$$

in that $E_0 = \pm J$; $E_0 \gg a$

This kind of solutions would correspond apparently to quasi-solitones.

2.3 Circular Polarization

If we superpose another linearly polarized wave propagating in the same direction Z and of equal range but whose corresponding fields E and H are in space quadrature and 90% face in regard to the first wave, and then the resulting wave incident to the non-linear medium is said to be polarized in a circular way. Such wave is described mathematically in Z = 0 as:

$$\vec{E} = [\hat{x} + j\hat{y}]\, E_0 \exp[j\omega t]$$

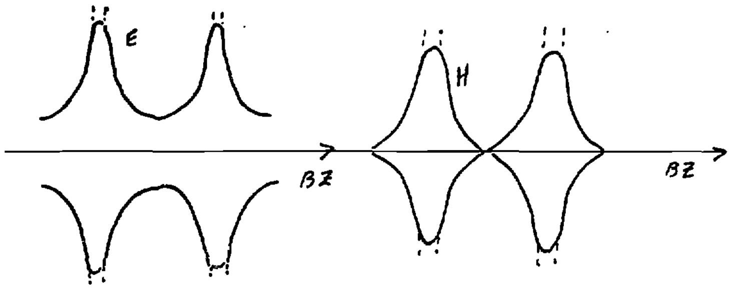
$$\vec{H} = [-j\hat{x} + \hat{y}]\, H_0 \exp[j\omega t] \qquad (22)$$

with a positive helix, one of the possible solutions for the non-linear medium will be:

$$E_1(z) = [\hat{x} + j\hat{y}]\, a \,\mathrm{sech}\, \mu z$$

$$H_1(z) = [-j\hat{x} + \hat{y}]\, \frac{a}{(\mu/\epsilon)^{1/2}} \,\mathrm{sech}\, \mu z \,\mathrm{tgh}\, \mu z \qquad (23)$$

The field distributions that are obtained are those shown in Fig. 1 for the case in that $\gamma \doteq j\beta$ with E (z) in the plane XZ and H (Z) in the plane YZ, the vectors E, H rotate in the plane XY at the speed of W for a Z constant, obtaining that way a rotatory/stationary field.



3. Qualitative Discussion on Hybrid Confinement of Plasmas

The usual confinement of plasmas for purposes of nuclear fusion are by magnetic bottles and through the inertial method with high power lasers. The confinement systems by magnetic mirrors and those called theta pinch, despite the fact they present a better pressure rate of the plasma to the magnetic pressure, they have the disadvantage that the plasma leaks through the opened ends.

The elementary idea that is discussed below is as follows: if it were possible to obtain, through powerful lasers, distributions of stationary field as those described by the equation (23), perhaps it could be applied to the open ends of a cylinder of plasma magnetically confined for the theta pinch. By that, it could be obtained a combined confinement, taking advantage of the straight magnetic field of the azimuthal current and the wheelcart type electromagnetic field covering the cylinder ends.

In the case of having a high temperature plasma, with few collisions between particles dynamics, in the cylinder ends, it indicates that it could have a short-circuit effect for the particles, controling that way the gross instabilities and some kinetic microinstabilities.

Anyway, there is a need for a much more exhaustive analysis to exploit that idea.

CONCLUSIONS

It has been described the way of solution of the non-linear plane wave equation. The descriptive function appears as a useful tool to describe the medium response when it is possible to consider intensive electromagnetic response. Despite it has been considered an ideal situation of a limitless non-linear medium, the results constitute a solid basis to mold more complex processes, such as those which take place in the plasma physics.

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