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STATISTICAL WINDOWS IN ANGULAR MOMENTUM SPACE: THE BASIS OF HEAVY-ION COMPOUND CROSS SECTION†

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ABSTRACT

The concept of statistical windows in angular-momentum space is introduced and utilized to develop a practical model for the heavy-ion compound cross section. Closed expressions for the average differential cross-section are derived and compared with Hauser-Feshbach calculations. The effects of the statistical windows are isolated and discussed.

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It is known that at low bombarding energies heavy ion compound processes exhaust almost all the reaction cross section. At higher energies other processes e.g., pre-compound and deep inelastic, start competing and eventually, at still higher energies, dominate the total reaction cross section, especially for heavier systems. For light systems, however, compound processes seem to be present even at those energies where DIC processes are important. It is therefore clear that a detailed study of these statistical compound nuclear (CN) reactions is called for.

Compound reactions are usually described, with relative success, within the Hauser-Feshbach (H-F) theory based on the statistical model. This model requires the knowledge of the penetrabilities for all the energetically allowed exit channels. Such information encompasses a very large number of states which imposes a strong limitation for the evaluation of the absolute cross sections. Normally only a manageable number of exit channels are considered and they are described by optical potentials (adjusted for ground state scattering) and by parametrized model level densities.

In practice, when calculations are compared with experimental results, these quantities are treated as free parameters in order to reproduce the magnitude of the experimental cross section. On the other hand, the behavior of the experimental angular distribution of a given transition via the CN indicate that the contributions of the partial cross-sections (σ_J) are well localized in angular momentum space [1,2]. To illustrate this fact, we exhibit in figure 1 a typical statistical model calculation[3] of σ_J for a light heavy ion system; e.g. $^{12}C(^{16}O,\alpha)^{24}Mg$. It is therefore clear that the parameters used in the H-F calculations act merely as scaffolding; the resulting cross section seems to be sensitive to combinations of these parameters. These combinations define a centroide, $\underline{L_0}$, and a width, $\underline{\Delta L}$, of the above mentioned localized "J-window".

In the present Letter we propose a completely different approach to obtain the Heavy Ion Compound Cross-Section, based on the use of a parametrized statistical window in angular momentum space. Closed forms for the differential cross sections are derived. These expressions clearly separate the overall geometrical behaviour, associated with the $1/\sin\theta$ dependence, from the dynamical behaviour related to the J-window. As is shown below, this separation enables the direct extraction of the J-window parameters.

In order to obtain the closed forms for the cross section, it is more convenient to start with the scattering amplitude. Notice that the partial waves amplitudes fluctuate rapidly with energy due the presence of the compound nucleus resonances. However, the fact that large-ℓ waves are dominant, permits the use of techniques usually employed in the treatment of quasi-elastic processes. This entails the use of the asymptotic forms of the Clebsch-Gordan (C-G) coefficients and of the spherical harmonic functions.

We take, for simplicity the azimuthal angle $\phi=0$. The amplitude $f_{I,m}(\theta)$ becomes[4]:

$$f_{I,m}(\theta) = \frac{\sqrt{4\pi}}{2\sqrt{\sin\theta}} \left\{ \mathcal{F}^{+}_{I,m}(\theta) + \mathcal{F}^{-}_{I,m}(\theta) \right\} \qquad (1)$$

The amplitudes $\mathcal{F}^{+(-)}_{Im}(\theta)$ represent the contribution from the near side (far side) component of the spherical harmonic i.e.

$$\mathcal{F}^{(\pm)}_{I,m}(\theta) = (-1)^{m-|m|} \sum_{\ell\ell'} \sqrt{(2\ell'+1)}\; d^{I}_{mK}\left(\frac{\pi}{2}\right) \mathcal{U}^{J=\ell}_{\ell'\ell I'} \;\times$$

$$\times \;\exp\left(\mp\left[(\lambda'\theta - \frac{\pi}{4} - |m|\frac{\pi}{2})\right]\right) \qquad (2)$$

where $\mathcal{U}$ represents the partial wave amplitude, $\underline{d}$ the Wigner rotational function, $K\equiv\ell-\ell'$, and $\lambda'\equiv\ell'+1/2$. The assumption that $I\ll\ell,\ell'$ has been made throughout, and the z-axis is taken along $\hat{k}_i$ and the y-axis along $\hat{k}_i x\hat{k}_f$. Cases that reflect a significant loss of angular momentum alignment[5], namely $I\geq\ell$ and $I\sim J_{max}$, are more involved and will be treated elsewhere[6].

Performing a rotation by $\frac{\pi}{2}$ such that the z-axis coincides with the $\hat{k}_i x\hat{k}_f$ direction and the x-axis with $\hat{k}_1$ we obtain an expression containing one unique sum. The cross section is invariant under the above rotation. The expression for the rotated amplitudes $\mathcal{F}^{(\pm)'}_{I,m}$ take the form

$$\mathcal{F}^{(\pm)'}_{Im} = (-1)^m \sum_{\ell'} \sqrt{(2\ell'+1)}\ \exp\left[\mp i(\lambda'\theta-\pi/4)\right] U^{J=\ell'\mp m}_{0,\ell'\mp m, I\ell'} \quad (3)$$

The average cross section can be evaluated by employing the following statistical properties of the partial amplitudes[7]:

$$\left\langle U^{J=\ell'\pm m}_{0,\ell'\pm m,\ I\ell'}\ U^{J'=\ell''*}_{0,\ell''\pm m,\ I\ell'} \right\rangle$$

$$= \left\langle \left| U^{J=\ell'\pm m}_{0,\ \ell'\pm m,\ I\ell'} \right|^2 \right\rangle \delta_{JJ'}\ \delta_{\ell'\ell''}\ \delta_{\ell'\pm m\ \ell''\pm m}$$

$$\left\langle U^{J=\ell'+m}_{0,\ell'+m,\ I\ell'}\ U^{*J'=\ell'-m}_{0,\ell'-m,\ I\ell'} \right\rangle \quad (4)$$

$$= \left\langle \left| U^{J=\ell'}_{0,\ell',\ I\ell'} \right|^2 \right\rangle \delta_{JJ'}\ \delta_{m0}\ \delta_{\ell'\ell''}$$

and $\left\langle U^{J}_{0,\ell'\pm m,\ I\ell'}\ U^{J'}_{0,\ell'\pm m,\ I\ell'} \right\rangle = 0$

The relations (4) constitute the dynamical input of the theory.

The average cross section calculated using the rotated amplitudes $\mathcal{F}^{(\pm)'}_{Im}$ and the equations (1), (3) and (4) is then given by

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin\theta}\left\{1 + \frac{(-1)^I}{(2I+1)}\left\langle \sin\left[(2\ell'+1)\theta\right]\right\rangle_{W_I(\ell')}\right\} \quad (5)$$

where

$$A_I = \frac{\pi}{k^2}\;\frac{2I+1}{2\pi^2}\;\sum_{\ell'}(2\ell'+1)W_I(\ell') \quad (6)$$

determines the magnitude of the cross section.

The average appearing in the second term of eq. 9 is performed over the windows W_I i.e.:

$$\left\langle \sin\left[(2\ell'+1)\theta\right]\right\rangle_{W_I(\ell')} \doteq \frac{\sum_{\ell'}(2\ell'+1)\sin\left[(2\ell'+1)\theta\right]W_I(\ell')}{\sum_{\ell'}(2\ell'+1)\;W_I(\ell')} \quad (7)$$

We have used the following relation to define the window:

$$2(2I+1)\;W_I(\ell') \equiv \sum_{m=-I}^{I}\left\{\left\langle\left|U^{J=\ell'-m}_{0,\ell'-m,\;I\ell'}\right|^2\right\rangle + \left\langle\left|U^{J=\ell'+m}_{0,\ell'+m,\;I\ell'}\right|^2\right\rangle\right\} \quad (8)$$

This window-function can be identified with the average H-F factors (σ_J) shown in fig. 1. (Although we write $W_I(\ell')$ as a function of ℓ', it should be understood as a function of J. We have transformed the J-dependence into an ℓ'-dependence when we employed the rotation described earlier, see Eq. (3)).

An important feature of Eq. (5) is the separation, in the expression of the cross section, of the overall $1/\sin\theta$ behaviour, associated with the angular phase space, from the dynamically generated oscillations contained in the second term which is determined completely by the window function. This second term should generate damped oscillations with a period determined by the center of gravity of the window. The damping of these oscillations is completely determined by the width of the window $W_I(\ell')$. Due to the $\frac{1}{2I+1}$ factor, the damped oscillations are more conspicuous in the I=0 transitions. The $(-1)^I$ phase is reminiscent of the Blair phase rule[8] obeyed by quasi-elastic cross sections.

Notice that the resulting expression for $\frac{d\sigma_I}{d\Omega}$ (eq.(5)) is symmetric about $\pi/2$.

In order to find an explicit form for the average $\left\langle \sin(2\ell'+\theta)\right\rangle_{W_I(\ell)}$ we have to specify the functional form of the window. Empirical observations suggest the following shape for the window

$$W_I(\ell') = \frac{d}{d\lambda'}\left[\frac{w_I}{1+\exp\frac{L(I)-\lambda'}{\Delta L(I)}}\right] \qquad (9)$$

with $\lambda' = \ell' + 1/2$ considered as a continuous variable. The average $\left\langle \sin(2\lambda'\theta)\right\rangle_{W_I(\ell')}$ may now be calculated by replacing the sum $\sum_{\ell'}$ by a integral with limits $\pm\infty$.

We thus find

$$\left\langle \sin(2\lambda'\theta) \right\rangle_{W_I(\lambda')} = \sin(2L(I)\theta) \frac{2\pi \frac{\Delta L(I)}{\alpha} \theta}{\sinh 2\pi (\frac{\Delta L(I)}{\alpha}) \theta} \qquad (10a)$$

where the parameter α is inserted to account for the fact that the profile of the ℓ-window is not a symmetric Gaussian function. The corresponding expression for A_I Eq. 6 becomes

$$A_I = w_I \frac{2I+1}{2\pi k^2} \left[1 - e^{-L(I)/\Delta L(I)} \right] \qquad (10b)$$

Due to the approximations used to obtain the above form for the average (eq. 10a), the symmetry about 90° is lost. It is however restored by writing the following identity:

$$\left\langle \sin(2\lambda'\theta) \right\rangle_{W_I(\lambda')} \equiv \frac{1}{2} \left[\left\langle \sin(2\lambda'\theta) \right\rangle_{W_I(\lambda')} - \left\langle \sin(2\lambda'(\pi-\theta)) \right\rangle_{W_I(\lambda')} \right] \qquad (11)$$

The second term in eq. 11 is evaluated in exactly the same manner as in eq. 10a.

Inserting equation (11) into equation (5) we obtain

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin\theta} \left\{ 1 + \frac{(-1)^I}{2I+1} \left[\sin(2L(I)\theta) \frac{2\pi \frac{\Delta L(I)}{\alpha}\theta}{\sinh 2\pi \frac{\Delta L(I)}{\alpha}\theta} - \sin(2L(I)\ (\pi-\theta)) \frac{2\pi \frac{\Delta L(I)}{\alpha}(\pi-\theta)}{\sinh 2\pi \frac{\Delta L(I)}{\alpha}(\pi-\theta)} \right] \right\} \qquad (12)$$

Expression (12) is valid for angles $\theta > \frac{I}{\ell_g}$ and contains four parameters; A_I, $L(I)$, $\Delta L(I)$ and α. In ref. (2) the value of α is fixed so that the coherence angles, θ_c, of the angular cross-correlation function agrees with the empirical value[1])

$$\theta_c = \frac{\Delta L_G}{1.4} \tag{13}$$

where L_G corresponds to the width of a Gaussian window. An equivalent width obtained from the derivative of a Fermi function (see Eq. 9) is given by $\Delta L \cong \frac{\Delta L_G}{2}$. Using these relations for ΔL and ΔL_G we obtain the value $\alpha=1.45$

In fig. II we present the results for $\frac{d\sigma_I}{d\Omega}$ calculated using expression Eq. (12) compared to the results obtained by calculations based on the Hauser-Feshbach theory[3]).

The adjusted J-window parameters, Eq. (9), namely its centroid $L(I)$ and width $\Delta L(I)$ were found to be very close to the expected values:

$L(I) \sim \ell_g + I + s$ and $\Delta L(I) \sim \frac{1}{2}\left(\frac{\sigma}{\sqrt{2}} + \Delta\ell\right)$ where $\underline{\ell_g}$ represents the grazing angular momentum of the emitted particle, $\underline{I}$ the final state spin, s the emitted particle spin, σ the spin-cutoff parameter and $\Delta\ell$ is the ℓ-diffusness of the exit channel transmission coefficients.

A comparison of the experimental angular distributions with eq. 12, especially for reactions with spinless entrance and exit channels where the differential cross sections clearly display a diffraction pattern, readily furnishes experimental values for the J-window parameters.

In conclusion, we have developed a simple approach to the compound nucleus cross section in heavy ion reactions. By recognizing the importance of the localization, in angular momentum space, of the compound partial cross sections, in determining the characteristics of the angular distribution, we were able to derive closed expressions for $\frac{d\sigma_I}{d\Omega}$. These expressions enables one to "read off" the dynamics directly from the data.

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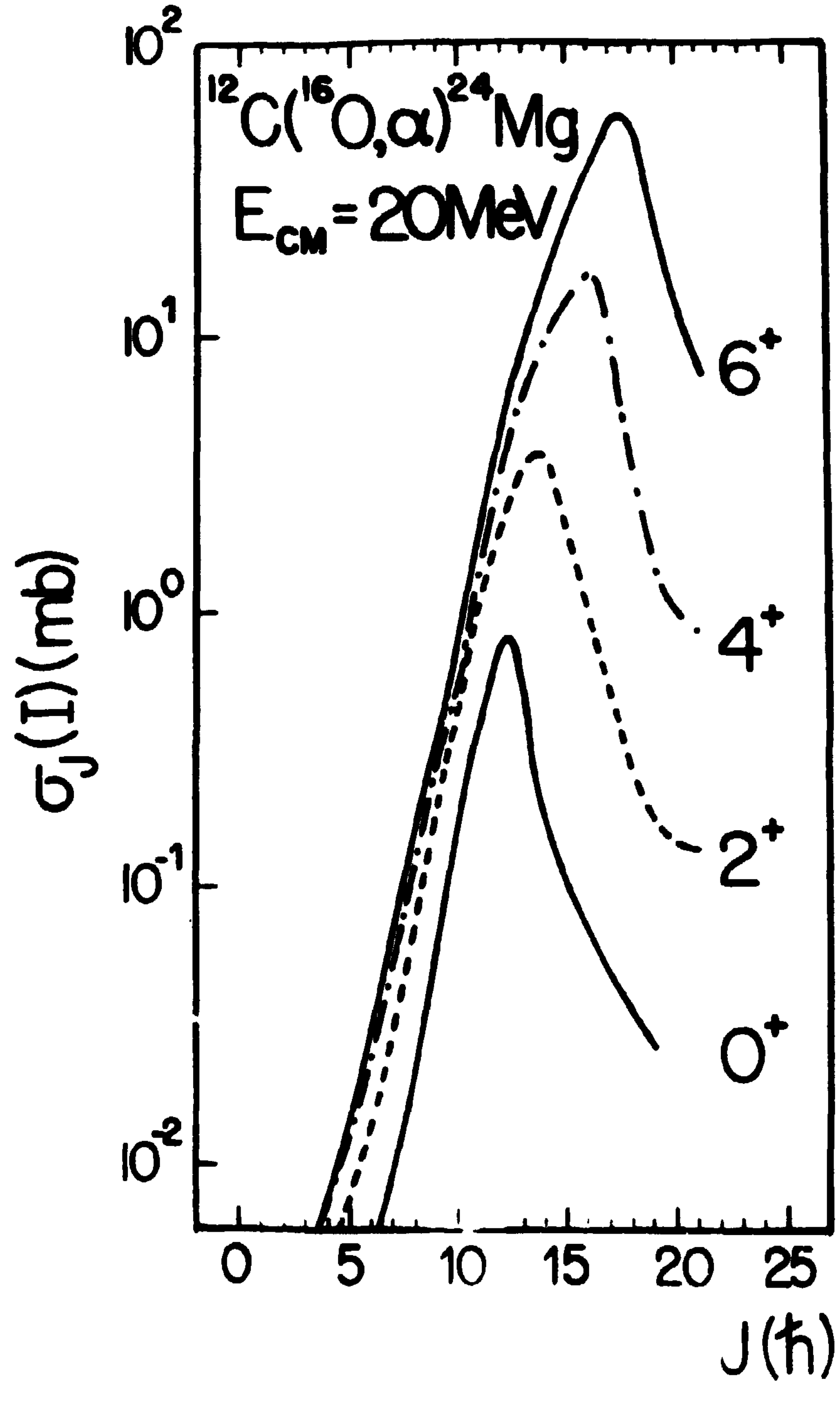
FIGURE CAPTIONS

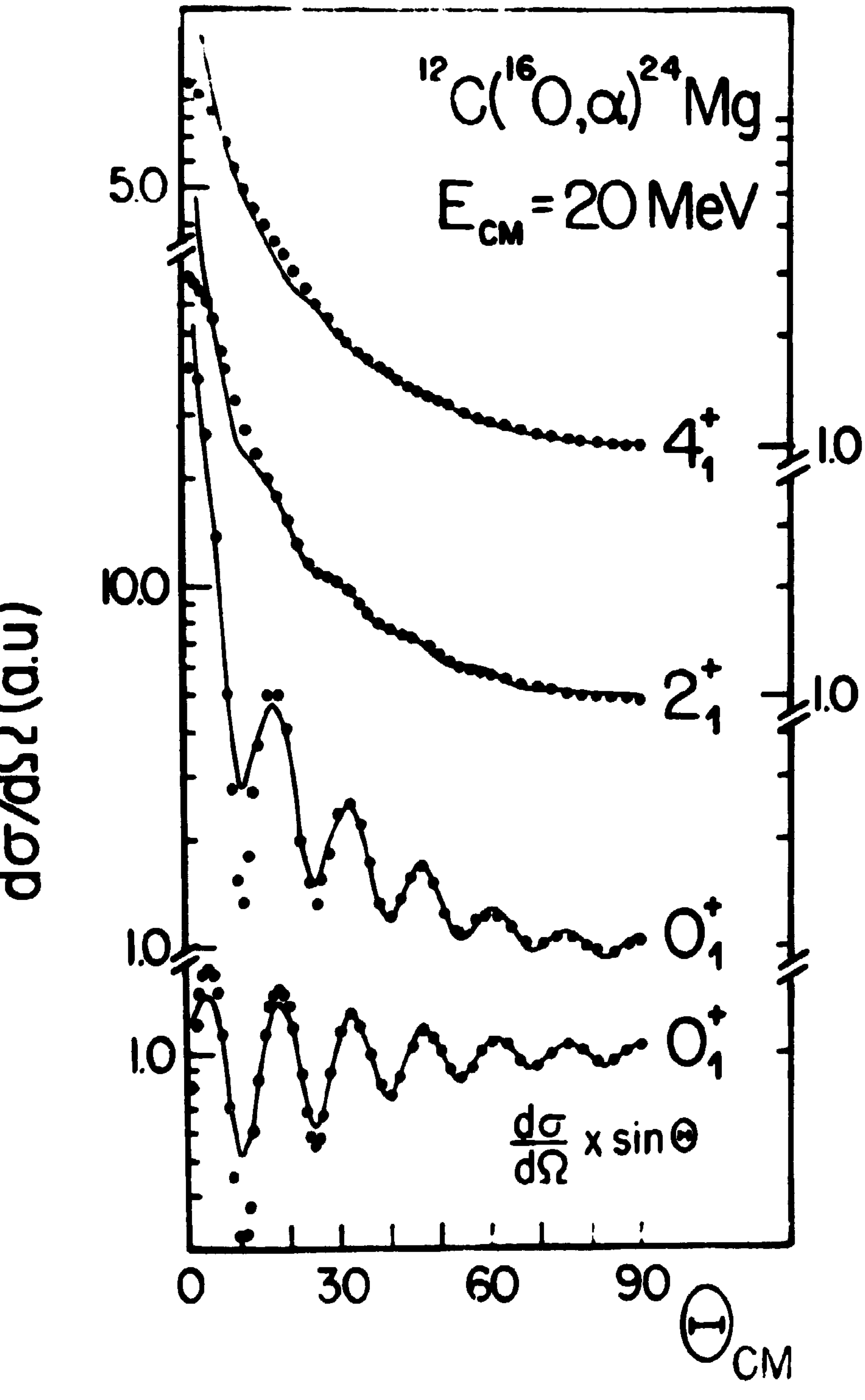
Figure 1: Partial cross sections $\sigma_J(I)$ for the excitation of final states with spin I=0,2,4 and 6, (calculated with the code STATIS[3]). The excitation energy $E^*=0$ has been considered in all the cases (J represents the C.N angular momentum).

Figure 2: Differential cross sections for the $^{12}C(^{16}O,\alpha)^{24}Mg$ (E_{cm}=20.0 MeV) calculated using Eq. (12) (solid line) and the code STATIS[3] (points). The J-window parameters used were: - ground-state (α=1.45, L_0= 12.5, ΔL_0=2.7, I=0)- first excited state (α=1.45, L_2=13.0, ΔL_2=2.7, I=2); second excited state (α=1.45; L_4=15.0; ΔL_4= 2.7; I=4). Curves were normalized to unity at θ_{cm} = 90°.



$^{12}C(^{16}O,\alpha)^{24}Mg$
$E_{CM} = 20MeV$
10^2
10^1
10^0
10^{-1}
10^{-2}
$\sigma_J(I)$ (mb)
6+
4+
2+
0+
0
5
10
15
20
25
$J(\hbar)$

FIGURE 1



$^{12}C(^{16}O,\alpha)^{24}Mg$
$E_{CM} = 20$ MeV
5.0
10.0
1.0
1.0
$d\sigma/d\Omega$ (a.u)
4_1^+
2_1^+
0_1^+
0_1^+
1.0
1.0
$\frac{d\sigma}{d\Omega} \times \sin\Theta$
0
30
60
90
Θ_{CM}

FIGURE 2