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SIMILARITY FLOWS BETWEEN A ROTATING AND A STATIONARY DISK

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ABSTRACT

The radial distribution of fluid pressure on a stationary disk coaxial with a rotating disk is determined experimentally for various inter-disc spacings. The results show that similarity flows are only possible for both small and large values of this distance. In the former case, the flow faraway from the stationary disk appears to be that suggested by Batchelor, while in the latter case, the flow turns out to be in accordance with the assumption of Stewartson. (author)

1. Introduction

The flow of an incompressible Newtonian fluid between two coaxial disks, one of which is rotating and the other stationary, is of interest not only from an applications point of view, but also due to the fact that exact solutions of the Navier-Stokes equations can be obtained for any Reynolds number through the application of the Von Kármán similarity transformation. It is therefore not surprising that this problem has attracted a lot of attention in the way of theoretical analysis. An important feature of the theory is the multiplicity of solutions at high Reynolds numbers. In particular, for very large values of the Reynolds number, two flows have been found possible. In the first, orginally postulated by Batchelor [1], there is a boundary layer on each disk and the fluid in between rotates at an angular velocity less than that of the rotating disk. In the other flow, first suggested by Stewartson [2], the fluid between the boundary layers is in purely axial motion.

2. Theoretical Considerations

Consider the laminar flow of an incompressible Newtonian fluid between two coaxial circular disks of infinite radius, the disks lying in the planes $\bar{z} = 0$ and $\bar{z} = d$. Let the former disk be stationary and let the latter rotate with a constant angular speed Ω. Assuming azimuthal symmetry, the Navier-Stokes equations in cylindrical coordinates may be written in dimensionless form as

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{\partial^2 v}{\partial z^2}, \tag{3}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}. \tag{4}$$

The boundary conditions to be satisfied are

$$Z = 0, \quad U = v = w = 0, \tag{5}$$

$$Z = 1, u = w = 0, \quad v = Rr, \tag{6}$$

where $R \equiv \frac{\Omega d^2}{\nu}$ is the Reynolds number of this problem.

Applying the von Kármán similarity transformation

$$u = -\frac{1}{2} r f'(z), \quad v = r g(z), \quad w = f(z), \tag{7}$$

the continuity equation (1) is identically satisfied, and eliminating the pressure between equations (2) and (4), we obtain

$$f^{iv} - ff''' - 4gg' = 0, \tag{8}$$

$$fg' - f'g - g'' = 0, \tag{9}$$

with the boundary conditions

$$f(0) = f'(0) = g(0) = 0, \tag{10}$$

$$f(1) = f'(1) = 0, \quad g(1) = R, \tag{11}$$

It is easy to show that the pressure must be of the from

$$p = \frac{C}{2} r^2 + \phi(z), \tag{12}$$

which for $z = 0$ reduces to

$$p = a_0 + a_2 r^2. \tag{13}$$

From equation (12), we see that $C \neq 0$ imples that the flow between the two disks rotates like a rigid body as Batchelor predicted [1]. On the other hand, if $C = 0$ then we conclude that the fluid between the disks is in purely axial flow as suggested by Stewartson [2]. Thus, the radial distribution of pressure at the stationary disk serves to indicate whether the fluid between the two disks is in Batchelor-like or in Stewartson-type flow or is indeed neither.

3. Experimental

Experiments were performed in a configuration of two horizontal disks, 11.5 cm in diameter, and bounded laterally by a stationary cylinder. The distance between the disks was varied between 0.5 and 40 cm. The lower disk was rotated at speeds in the range 50 - 190 rps.

Static pressure measurements were made in the stationary disk. The pressure taps were at radial positions between 2.0 and 4.8 cm. Further details of the experimental arrangement and program can be found in Ref. [3].

4. Results

For each inter-disk spacing and speed of rotation of the lower disk, the measured values of pressure were fited as a quadratic function of the radial distance as suggested by equation (13). As the fit was not satisfactory, a linear term was added to give the form

$$p = a_0 + a_1 r + a_2 r^2, \quad (14)$$

the resulting fit being a great improvement on that without the linear term. Needless to say, the coefficients a_0, a_1, and a_2 are functions of d and Ω. As seen in Section 3, for the von Kármán similarity transformation to hold, $a_1 = 0$. A close examination of our data shows that $a_1 \longrightarrow 0$ for two cases: $d \longrightarrow 0$, and $d \longrightarrow \infty$. Furthermore, in the latter case, $a_2 \longrightarrow 0$. Typical variation of a_1 and a_2 with d are show in Figures 1 and 2 respectively.

5. Conclusions

Our results show that for very small inter-disk spacings, the fluid between the disks may well be in Batchelor-like flow. This is to be expected, since the influence of the rotating disk is large in this case and can cause the fluid to rotate as a rigid body. On the other hard, for very large inter-disk spacings, the flow between the disks appears to be Stewartson-type and this also is not surprising as the influence of the rotating disk is then minimal. For intermediate inter-disk spacings, similarity flow does not appear possible as $a_1 \neq 0$.

This trend is not shown by the experimental results of Nguyen et al [4], notwithstanding the fact that their rotational speeds and inter-disk spacings are such that the Reynolds number lies in the range 0 - 5000, while we have much higher Reynolds numbers due to the large inter-disk spacings we used. Their data are in agreement with Batchelŏrs prediction in the range $0 \leq R \leq 2000$. They do not detect any Stewartson-type flow. Furthermore, they do not find any linear term in the radial distance; i.e., $a_1 = 0$.

Nguyen et al [4] do not accept the reasoning that at large inter-disk spacings the flow should be Stewartson-like, because the argument invokes disks of finite radius. We find this questionable. In so far as the flow between any two given disks is concerned, the edge effect due to the finiteness of the disks is localized near the periphery [5] and hence the fluid away from the discs' periphery 'sees' an infinite disc. Thus, we believe it reasonable to expect Stewartson-type flow for large values of inter-disk spacing. Similarly, Batchelor-like flow is to be expected for small disk separations. As stated above, this is borne out by our data.

References

[1] Batchelor, G.K., "Note on a class of solutions of the Navier-Stokes equations representing steady rotationally symmetric flow", Quart. Journ. Mech. Appl. Math., Vol. 4 (1951), pp. 29-41.

[2] Stewartson, K., "On the flow between two rotating coaxial disks", Proc. Camb. Phil. Soc., Vol. 49 (1953), pp. 333-341.

[3] Buchmann, J.H., M.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, (1978).

[4] Nguyen, N.D. Ribault, J.P., and Florent, P., "Multiple solutions for flow between coaxial disks", J.Fluid Mech, Vol. 68 (1975), pp. 369-388.

[5] Greenspan, H.P., The Theory of Rotating Fluids, Camb. Univ. Press, Camb. (1968), p. 141.

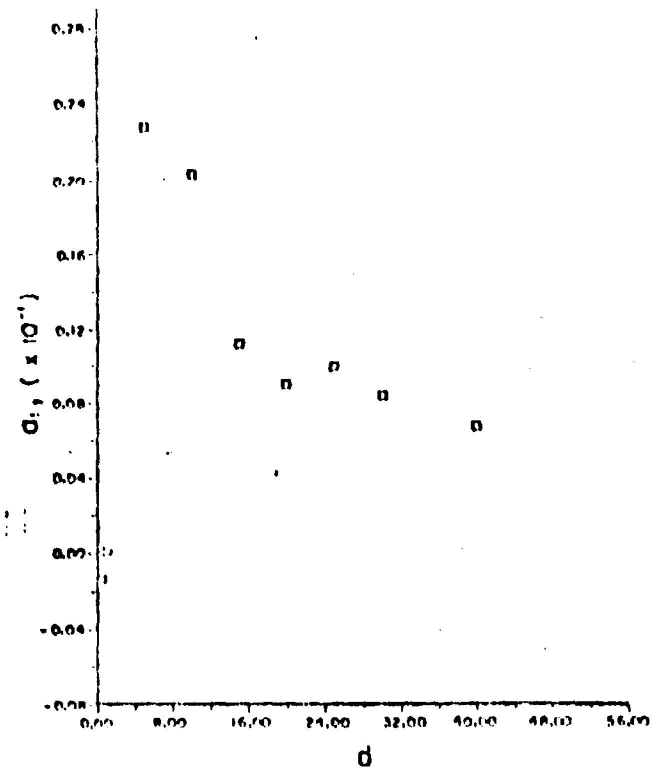

Figure 1 - Variation of parameter a_i with inter-disk Spacing d: Speed of rotation 190 rps .

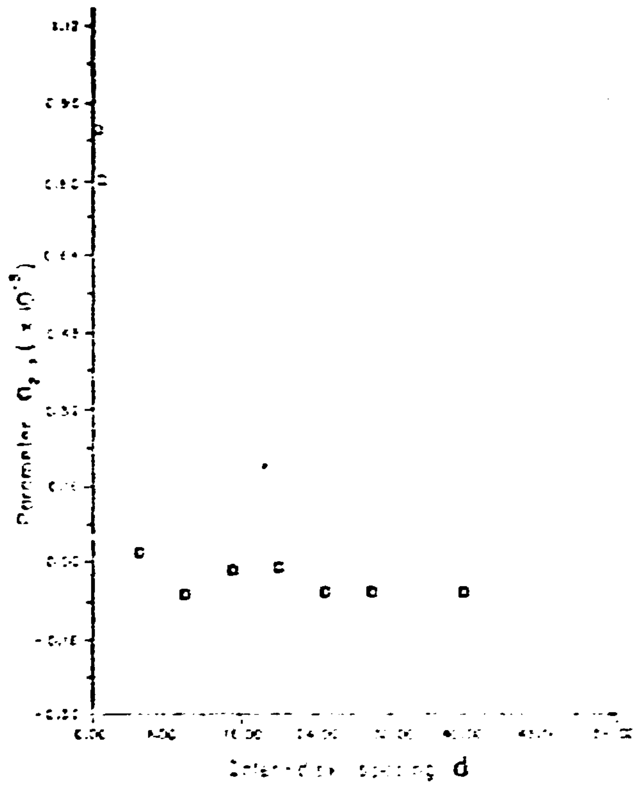

Figure 2- Variation of parameter a_2 with inter-disk Spacing **d**: Speed of rotation 180 rps.