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RESUMO

Aplicando a teoria da mecânica do contínuo de misturas binárias de fluidos newtonianos incompressíveis, estudou-se escoamentos tipo Poiseuille e Couette com fins de verificar se ocorre difusão em tais escoamentos. Constata-se que a difusão não ocorre no caso Couette. Entretanto em escoamento Poiseuille há diferenças significantes entre as velocidades dos constituintes da mistura. Este resultado mostra-se plenamente concordante com aqueles de Mills para misturas similares de composição não uniforme. (autor)

ABSTRACT

Using the continuum theory of binary mixtures of incompressible Newtonian fluids, Poiseuille and Couette flows are studied with a view to determining whether diffusion occurs in such flows. It is shown that diffusion is absent in the Couette case. However, in Poiseuille flow there are significant differences between the velocities of the species comprising the mixture. This result is in broad agreement with that of Mills for similar mixtures of nonuniform composition. (author)

1. Introduction

It has been established that the three coefficients of viscosity of a binary mixture of incompressible Newtonian fluids can be determined in flows without diffusion and can then be utilized for cases where the relative velocity is not zero[1]. It is therefore of great interest to determine in which viscometric flows diffusion does not occur to a significant extent. These can then be used to determine the aforementioned coefficients of viscosity.

In this paper we consider four classical viscometric flows: planar Couette, circular Couette, planar Poiseuille, and circular Poiseuille flows. Assuming constant composition, the velocity of each species of the binary mixture is determined in each of these four flow configurations. Comparison is made with the results of Mills for similar mixtures of nonuniform composition[2].

2. Equations of motion

We consider a binary mixture of incompressible Newtonian fluids of uniform composition. The steady balance equations of mass and linear momentum may be written as[3]

$$\nabla \cdot \underset{\sim}{v}_\alpha = 0, \quad \alpha = 1,2 \qquad (1)$$

$$\rho_\alpha \varepsilon_\alpha \nabla \underset{\sim}{v}_\alpha \cdot \underset{\sim}{v}_\alpha = \nabla \cdot \underset{\sim}{T}_\alpha + \underset{\sim}{m}_\alpha + \rho_\alpha \varepsilon_\alpha \underset{\sim}{b}_\alpha, \qquad (2)$$

where $\underset{\sim}{v}$ denotes velocity, ρ material density, ε volumetric fraction, $\underset{\sim}{T}$ stress tensor, $\underset{\sim}{m}$ internal body force, and $\underset{\sim}{b}$ external body force. The subscript α denotes the species α. Since the materials of the species are incompressible and the mixture composition is uniform, we note that both ρ_α and ε_α are constant. Clearly,

$$\sum_{\alpha=1}^{2} \varepsilon_\alpha = 1. \qquad (3)$$

As the summation of the linear momentum balance equation of the species should give the corresponding equations for the mixture, then[3]

$$\sum_{\alpha=1}^{2} \underset{\sim}{m}_\alpha = \underset{\sim}{0}; \qquad (4)$$

i.e. $\underset{\sim}{m}_1 = - \underset{\sim}{m}_2 = \underset{\sim}{m}$, say.

We assume that the only external body force acting on the mixture is that due to gravity. Then,

$$\underset{\sim}{b}_\alpha = \underset{\sim}{g}, \quad \alpha = 1,2. \qquad (5)$$

Following Sampaio and Williams[1], we write the constitutive equations for the stress tensors of the species as

$$\underset{\sim}{T}_1 = - p_1 \underset{\sim}{1} + 2\mu_1 \nabla \underset{\sim}{v}_1^s + 2\mu_3 \nabla \underset{\sim}{v}_2^s , \qquad (6)$$

$$\underset{\sim}{T}_2 = - p_2 \underset{\sim}{1} + 2\mu_4 \nabla \underset{\sim}{v}_1^s + 2\mu_2 \nabla \underset{\sim}{v}_2^s , \qquad (7)$$

where

$$\mu_1 = \varepsilon^2 \eta_1 + \varepsilon(1-\varepsilon)\eta_{12}, \qquad (8)$$

$$\mu_2 = (1-\varepsilon)^2 \eta_2 + \varepsilon(1-\varepsilon)\eta_{12}, \qquad (9)$$

$$\mu_3 = \mu_4 = \varepsilon(1-\varepsilon)\ \mu_{12} \qquad (9a)$$

$$\eta_{12} = \sqrt{\eta_1 \eta_2} . \qquad (10)$$

Here, p denotes an indeterminate pressure, η_1 and η_2 denote the viscosity coefficients of the fluids in the unmixed state. The superscript s denotes the symmetric part of the tensor.

For $\underset{\sim}{m}$, we use the form developed by Struminskii for binary mixtures of gases[4,5]

$$\underset{\sim}{m} = \frac{\rho_1 \varepsilon \rho_2 (1-\varepsilon)}{\rho_1 \varepsilon M_2 + \rho_2 (1-\varepsilon) M_1} \frac{K\theta}{D_{12}} (\underset{\sim}{v}_1 - \underset{\sim}{v}_2), \qquad (11)$$

where K is the Boltzmann constant, θ denotes absolute temperature, M molecular weight, and D_{12} binary diffusion coefficient.

3. Couette flows

Consider unidirectional horizontal flow between two infinite parallel plates, a distance 2L apart. Let there be no externally imposed pressure gradient and let one of the planes move at a speed V in the horizontal direction, the other plate remaining stationary. The equations of motion reduce to[6]

$$\frac{d^2 v_{1x}}{dy^2} = K_1\left[v_{1x} - \left(-\frac{K_2}{K_1} v_{1x} + AK_2 y + bK_2\right)\right], \quad (12)$$

$$v_{2x} = -\frac{K_2}{K_1} v_{1x} + AK_2 y + BK_2, \quad (13)$$

where

$$A = \frac{V}{2L}\left[\frac{K_1+K_2}{K_1 K_2}\right], \quad (14)$$

$$B = \frac{V}{2}\left[\frac{K_1+K_2}{K_1 K_2}\right], \quad (15)$$

v_{1x} and v_{2x} are velocity components in the horizontal direction, y is the vertical coordinate measured from the plane midway between the two infinite plates, and

$$K_1 = \frac{\rho_1 \rho_2 \frac{K\theta}{D_{12}}\left[\frac{\mu_2 + \mu_3}{\mu_1\mu_2 + \mu_3^2}\right]\varepsilon(1-\varepsilon)}{(\rho_1 M_2 - \rho_2 M_1)\,\varepsilon + \rho_2 M_1}, \quad (16)$$

$$K_2 = \frac{\rho_1 \rho_2 \frac{K\theta}{D_{12}}\left[\frac{\mu_1 + \mu_3}{\mu_1\mu_2 - \mu_3^2}\right]\varepsilon(1-\varepsilon)}{(\rho_1 M_2 - \rho_2 M_1)\,\varepsilon + \rho_2 M_1}, \quad (17)$$

The solution of equations (12) and (13) subject to the boundary conditions

$$y = -L,\ v_{1x} = v_{2x} = 0, \quad (18)$$

$$y = L,\ v_{1x} = v_{2x} = V, \quad (19)$$

is given by

$$v_{1x} = \frac{V}{2}\left(\frac{y}{L} + 1\right), \quad (20)$$

$$v_{2x} = \frac{V}{2}\left(\frac{y}{L} + 1\right); \quad (21)$$

Clearly, no diffusion occurs in this flow.

Consider now unidirectional circular flow between two vertical concentric cylinders, the inner one fixed while the other rotates at an angular speed Ω. Let the radii of the inner and the outer cylinders be nR and R respectively, where $0 < n < 1$. For this case, the equations of motion reduce to[6]

$$\frac{d^2 v_{1\theta}}{dr^2} + \frac{1}{r}\frac{dv_{1\theta}}{dr} - \frac{v_{1\theta}}{r^2} - (K_1 + K_2)\, v_{1\theta} + $$

$$+ \frac{AK_1K_2}{2} r + \frac{BK_1K_2}{} = 0, \tag{22}$$

$$v_{2\theta} = -\frac{K_2}{K_1} v_{1\theta} + \frac{AK_2}{2} r + \frac{BK_2}{r}, \tag{23}$$

where $v_{1\theta}$ and $v_{2\theta}$ are velocity components in the azimuthal direction, and

$$A = \left(\frac{2\Omega}{1-n^2}\right) x \left(\frac{K_1+K_2}{K_1K_2}\right), \tag{24}$$

$$B = -\left(\frac{\Omega n^2 R^2}{1-n^2}\right) x \left(\frac{K_1+K_2}{K_1K_2}\right). \tag{25}$$

The boundary conditions may be written as

$$r = nR, \; v_{1\theta} = v_{2\theta} = 0, \tag{26}$$

$$r = R, \; v_{1\theta} = v_{2\theta} = \Omega R. \tag{27}$$

The solution of equation (21) is then

$$v_{1\theta} = \frac{\Omega}{(1-n^2)}\left(r - \frac{n^2R^2}{r}\right), \tag{28}$$

and substitution into equation (22) yields

$$v_{2\theta} = \frac{\Omega}{(1-n^2)}\left(r - \frac{n^2R^2}{r}\right). \tag{29}$$

As in the planar case, this flow takes places without diffusion.

4. Poiseuille flows

Consider unidirectional horizontal flow between two stationary plates. Let the distance between them be 2 L. Assuming an external pressure gradient C, applied to both fluids, the equations of motion can be simplified to[6]

$$\frac{d^2 v_{1x}}{dy^2} = (K_1 + K_2)\, v_{1x} - \frac{CK_1K_2K_3}{2} y^2 - - \frac{C(\mu_2-\mu_3)}{(\mu_1\mu_2-\mu_3^2)} + \frac{CK_1K_2K_3L^2}{2}, \tag{30}$$

$$v_{2x} = -\frac{K_2}{K_1} v_{1x} + \frac{CK_2K_3}{2} y^2 - \frac{CK_2K_3L^2}{2}, \tag{31}$$

where

$$K_3 = \frac{(\mu_1 - \mu_3)}{K_2(\mu_1\mu_2 - \mu_3^2)} + \frac{(\mu_2 - \mu_3)}{K_1(\mu_1\mu_2 - \mu_3^2)}. \tag{32}$$

The appropriate boundary conditions are

$$y = -L,\ v_{1x} = v_{2x} = 0, \tag{33}$$

$$y = L,\ v_{1x} = v_{2x} = 0. \tag{34}$$

The solution is given by

$$v_{1x} = \frac{CK_1K_2K_3}{K_1+K_2}\left[\frac{1}{K_1 + K_2} - \frac{(\mu_2 - \mu_3)}{K_1K_2K_3(\mu_1\mu_2 - \mu_3^2)}\right] \times \left[1 - \left\{\frac{e^{-\sqrt{K_1+K_2}\,y} + e^{\sqrt{K_1+K_2}\,y}}{e^{\sqrt{K_1+K_2}\,L} + e^{-\sqrt{K_1+K_2}\,L}}\right\}\right] + + \frac{CK_1K_2K_3}{2(K_1+K_2)}\ (y^2-L^2), \tag{35}$$

$$v_{2x} = - \frac{CK_1K_2K_3}{K_1+K_2} \left[\frac{1}{K_1+K_2} - \frac{\mu_2 - \mu_3}{K_1K_2K_3(\mu_1\mu_2 - \mu_3^2)}\right] \times$$

$$\times \left[1 - \left[\frac{e^{-\sqrt{K_1+K_2}\,y} + e^{\sqrt{K_1+K_2}\,y}}{e^{-\sqrt{K_1+K_2}\,L} + e^{\sqrt{K_1+K_2}\,L}} \right]\right] +$$

$$+ C \left[\frac{K_2 \cdot K_3}{2} - \frac{K_2^2 - K_3}{2(K_1+K_2)}\right] (y^2 - r^2) . \qquad (36)$$

The occurrence of diffusion is clear from equations (35) and (36), depending in magnitude on several material parameters, pressure gradient, and channel width.

We now consider unidirectional flow through a circular horizontal tube of radius R. Let there be an externally imposed pressure gradient C. The equations of motion reduce to[6]

$$\frac{d^2v_{1z}}{dr^2} + \frac{1}{r}\frac{dv_{1z}}{dr} - (K_1 + K_2)\ v_1 = \frac{-CK_1K_2K_3}{4} r^2 +$$

$$+ \frac{CK_3R^2}{4} K_1K_2 + CK_3 \left[\frac{\mu_2 - \mu_3}{\mu_1\mu_2 - \mu_3^2}\right], \qquad (37)$$

$$v_{2z} = - \frac{K_2}{K_1} v_{1z} + \frac{CK_2K_3r^2}{4} - \frac{CK_3R^2}{4} K_2, \qquad (38)$$

subject to the boundary conditions

$$r = 0,\ \frac{dv_{1z}}{dr} = \frac{dv_{2z}}{dr} = 0, \qquad (39)$$

$$r = R,\ v_{1z} = v_{2z} = 0, \qquad (40)$$

the solution is given by

$$v_{1z} = \left\{ \frac{\left[\dfrac{C}{K_1 + K_2}\right]\left[\left(\dfrac{\mu_2 - \mu_3}{\mu_1\mu_2 - \mu_3^2}\right) - \dfrac{K_1K_2K_3}{K_1 + K_2}\right]}{I_o\left[(K_1 + K_2)R\right]} \right\} I_o\left[(K_1+K_2)r\right] +$$

$$+ \frac{CK_1K_2K_3}{(K_1+K_2)} r^2 + \frac{C}{K_1+K_2}\left[\frac{K_1K_2K_3}{K_1+K_2} -\right.$$

$$\left. - \frac{K_1K_2K_3}{4} R^2 - \left(\frac{\mu_2 - \mu_3}{\mu_1\mu_2 - \mu_3^2}\right)\right], \qquad (41)$$

$$v_{2z} = -\frac{K_2}{K_1} \frac{\left[\dfrac{C}{K_1 + K_2}\right]\left[\left(\dfrac{\mu_2 - \mu_3}{\mu_1\mu_2 - \mu_3^2}\right) - \dfrac{K_1K_2K_3}{(K_1+K_2)}\right]}{I_o\left[(K_1 + K_2)\ R\right]} \ x$$

$$x\ I_o\left[(K_1+K_2)\ r\right] + C\left[\frac{K_2K_3}{4} - \frac{K_2}{K_1}\ x\right.$$

$$\left. x\ \frac{K_1K_2K_3}{4(K_1+K_2)}\right] r^2 - \frac{CK_2K_3}{4} R^2 -$$

$$- \frac{C}{(K_1+K_2)}\left[\frac{K_1K_2K_3}{K_1+K_2} - \frac{K_1K_2K_3}{4} R^2 -\right.$$

$$\left. - \left(\frac{\mu_2 - \mu_3}{\mu_1\mu_2 - \mu_3}\right)\right] \frac{K_2}{K_1}, \qquad (42)$$

where I_o is the modified Bessel function of order zero and of the first kind. Again, diffusion occurs to an extent which depends on fluid material parameters, pressure gradient, and tube radius.

5. Conclusions

We have shown that for a diffusion - free flow of a binary mixture of incompressible Newtonian fluids, Couette flow, whether planar or circular, is to be adopted as a viscometric flow. This is for any apparatus dimensions and for any fluid pair. This is not the case for Poiseuille flow, planar or circular. These results are in agreement with those of Mills[2] who considered mixtures of varying composition.

Our work complements that of Sampaio and Williams[1] in that they showed that measurements in a none - diffusing flow are sufficient to determine all viscosity coefficients and we have established that Couette flows are such flows. They, rather than Poiseuille flows, suggest themselves as appropriate viscometric flows for binary mixtures of incompressible Newtonian fluids.

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