RAPID RECONNECTION OF FLUX LINES

A. SAMAIN
Association Euratom-CEA sur la fusion,
Département de recherches sur la fusion contrôlée,
Centre d'études nucléaires,
Fontenay-aux-Roses,
France

ABSTRACT

The rapid reconnection of flux lines in an incompressible fluid /1/ through a singular layer of the current density is discussed. It is shown that the liberated magnetic energy must partially appear in the form of plasma kinetic energy. A laminar structure of the flow is possible, but Alfven velocity must be achieved in eddies of growing size at the ends of the layer. The gross structure of the flow and the magnetic configuration may be obtained from variational principles.

I. Introduction

The free reconnection of magnetic surfaces in a toroidal configuration has been proposed by Kadomtsev /1/ as a situation where the configuration liberates magnetic energy, while preserving the MHD constraint except in a thin layer near a separatrix delimiting a growing magnetic island. In this layer, controlled both by resistivity and inertia, a singularity of the current density and of the plasma velocity appears. A rapid growth of the island is possible because the liberated magnetic energy may be dissipated at a high rate in the layer, at least by Joule effect. It is not clear whether the liberated energy appears in the form of Joule energy only, or also in form of kinetic energy. The linear phase of tearing modes /2/, where the liberated magnetic energy is shared into Joule and kinetic energy, would favor the second case. Also the numerical simulations of the $q = 1$ reconnection in Tokamaks /3/ seem to indicate the production of substantial kinetic energy. However a kinetic energy of the order of the magnetic energy means that the plasma velocity is of the order of the Alfven velocity C_A in a large domain. It is not easy to understand how this can occur through the traditional flow of incompressible plasma along the thin singular layer at the velocity C_A /4/, if this flow simply expands into the island. The aim of this paper is to show that a laminar free reconnection does produce a kinetic energy comparable to the available magnetic energy, but that this is possible only through more complex magnetic and kinetic structures.

II. Slow and free reconnections

A slow reconnection /5/, /6/ consists of a sequence of quasi equilibria with regular current profiles. This implies a normal separatrix between the reconnecting domains I, II and the island III (see Fig. 1), with a finite angle of the two branches at the X point. It is convenient /7/ to follow the evolution of the magnetic field B through evolution in the regions I, II, III of the function $G(\phi,t)$ which relates the magnetic flux $\psi(x,y,t)$ ($B_x = \partial\psi/\partial y$, $B_y = -\partial\psi/\partial x$) on the magnetic line passing by (x,y) to the area $\phi(x,y,t)$ embraced by this line

$$\psi(x,y,t) = G(\phi(x,y,t),t)$$

The total electrical current $\mathfrak{I}$ (along z) inside the line ϕ is given by

$$-\frac{4\pi\mathfrak{I}}{c} = P(\phi)\,\frac{\partial G(\phi,t)}{\partial\phi} \qquad (1)$$

where $P(\phi)$ depends on the shape of the magnetic lines only

$$P(\phi) = \left(\oint_\phi ds\ \delta l\right)\left(\oint_\phi ds\ \delta l\right) \qquad (2)$$

being s the abscisse along the line ϕ and δl the distance at abscissa s between the lines ϕ, $\phi + \delta\phi$. With a normal separatrix the functions $P(\phi)$ and $G(\phi,t)$ tend logarithmically to ∞ and o at the separatrix value $\phi = \phi_s$, so that

$$\left(P\frac{\partial G}{\partial\phi}\right)_{\phi_{sII}} - \left(P\frac{\partial G}{\partial\phi}\right)_{\phi_{sI}} = \left(P\frac{\partial G}{\partial\phi}\right)_{\phi_{sIII}} \qquad (3)$$

The plasma equilibrium implies that the magnetic energy

$$\mathcal{E}_M = \left(\int_I + \int_{II} + \int_{III}\right)\left(P(\phi)\left(\frac{\partial G(\phi,t)}{\partial\phi}\right)^2 d\phi \qquad (4)$$

where the function $G(\phi,t)$ is held constant, is a minimum with respect to the shape of the magnetic lines (which appears in (4), through the function $P(\phi)$). On the other hand the generalized Ohm law $E + vxB/c = \eta I$ ($E = -\partial\psi/c\partial t$, $4\pi I/c = -\Delta\psi$) implies that

$$-\frac{1}{c}\frac{\partial G(\phi,t)}{\partial t} = \eta < I >_\phi \qquad (5)$$

where $\langle I \rangle_\phi$ is the space averaged value of the current density I between the lines ϕ, $\phi + \delta\phi$. In view of (1) this equation becomes

$$\frac{\partial G}{\partial t} = \frac{\eta c^2}{4\pi} \frac{\partial}{\partial \phi}\left(P \frac{\partial G}{\partial \phi}\right) \qquad (6)$$

The equations (6), (3) and the variational principe (4) determine the evolution of the magnetic configuration. In the slow type of reconnection the island growth is possible only if the function $G(\phi,t)$ is smoothed by the resistive equation (6) so that the magnetic energy (4) decreases in the process. As long as this resistive effect is necessary in the bulk of the regions I, II, III, the scale time of the reconnection is a resistive time $\tau_R = (\eta c^2/4\pi)^{-1} a^2$, where a is a bulk scale. It is possible however that situations arise where the resistive effect is only necessary near the separatrix. The evolution then becomes more rapid. The function $G(\phi,t)$ becomes time invariant in the bulk of the regions I, II, III and the slope $\partial G/\partial\phi$ becomes finite (rather than to cancel logarithmically) near the separatrix in the reconnecting regions I, II. In view of (1) and (2) the function $P(\phi)$ becomes finite there and the flux lines tends to become regular near the X point, corresponding to a sharp variation of the magnetic field from I to II. A large value of the current density appears all along the separatrix. A free reconnection regime where inertia plays a role may be expected to emerge from such situations. However it is possible to show /7/ that a continuous transition between slow and free reconnection regimes is impossible. This is a first clue that free reconnection needs a change of magnetic and kinetic structure.

III. Simple free reconnection

A free reconnection takes place with a time scale τ which must increase with the resistivity η and the mass density ρ. We may assume that the ratio τ/τ_R and τ_A/τ ($\tau_A = a/C_A$) are extremely small. We will discuss the simple traditional pattern according to which the magnetic lines reconnect at the X point of a unique island by extracting the plasma along the separatrix. We exclude Petschek torsional shock waves /8/, because they cannot be consistent with the mechanical and resistive laws in an incompressible fluid. Then a singular layer of current density may only take place along the separatrix. We remark that the current density I_L in such a layer and its width δ_L are typically given by $E \sim \eta\, I_L$ with $E \sim Ba/\tau C$ and $4\pi I_L/C \sim B/\delta_L$, so that $\delta_L \sim a\sqrt{\tau/\tau_R}$. For the liberated magnetic power $\sim B^2a^2/\tau$ to be absorbed at least partially by Joule effect, the layer must extend over a length l of the order of the bulk scale a. It will be apparent that the velocity V in the layer may be at most of the order of C_A and the present configuration contains a negligible amount of kinetic energy.

At a distance $\delta > \delta_L$ from the separatrix the field B has a variation scale $\gtrsim \delta$ and we must have $I \lesssim I_L\, \delta_L/\delta$. Using small enough values of δ_L, the term η I in the generalized Ohm Law is therefore negligible as near as we want from the separatrix. The time variation of the function $G(\phi,t)$ given by (5) is then negligible. The function $G(\phi)$ in the reconnecting domains I, II maintain its initial value. In the island III it builds up through the continuity equation

$$G(\phi_{sI}) = G(\phi_{sII}) = G(\phi_{sIII}), \quad \phi_{sII} = \phi_{sI} + \phi_{sIII} \qquad (7)$$

These statements impose finite values of the derivative

$$\frac{\partial G}{\partial \phi} = -\oint_\phi \frac{ds}{B}$$

near the separatrix. By differentiation of (7) we obtain over contours near the separatrix

$$\oint_{\phi_{III}} \left|\frac{ds}{B}\right| = \oint_{\phi_I} \left|\frac{ds}{B}\right| + \oint_{\phi_{II}} \left|\frac{ds}{B}\right| \qquad (8)$$

The fact that the integral $\int ds/B$ has its bulk value up to the separatrix means that the propagation time of Alfven waves along flux lines is $\sim a/CA \ll \tau$. Plasma equilibrium (I constant on magnetic lines) is then achieved, as long as the fluid velocity along the flux lines is $\ll$ CA. With the present topology this may be not the case in the island III but, because of incompressibility, only within a domain the extension of which cancels with δ_L. Writing the dynamical equation in the form

$$\oint_R \rho \frac{dV}{dt} \cdot dM = \oint_R I \times B/c \cdot dM \qquad (9)$$

where the closed contour R consists of R_1 or R_2 as shown on fig. 2, we obtain that the velocity V in the layer is at most the Alfven velocity (as stated above) and on the other hand that the variation of the squared magnetic field B^2 across the separatrix cancels. This last statement and the equilibrium up to the separatrix implies that at a given time, i.e., for given area of the regions I, II, III and given $G(\phi)$ in these regions, the magnetic configuration, including the shape of the separatrix, is determined by making the magnetic energy (4) minimum.

A first possibility (which could result by continuity from a slow reconnection regime) is that the separatrix consists of two regular curves tangent at the X point, as shown on fig. 3a.

As we have seen above, the current singularity must extend over a finite length to insure Joule consumption of the magnetic energy and a constant discontinuity of B^2 across each branch of the separatrix is necessary. Of course the value of B is smaller in the island region III. This contradict the relation (8). The other possible topology is the one given by the fig. 3b, with two Syrovatskii points S,S' [9]. The angle of the diverging flux lines at these points is obtained from the principle (4) and cannot cancel without returning to the first possibility. In these conditions the magnetic field cancels in all directions at the Syrovatskii points, and is continuous except through the interval SS'. The major difficulty with this scheme may be expressed by writing the dynamical condition (9) over the closed contour R shown on fig. 3b. The left hand side cancels while the right hand side is at least equal to the finite variation of the magnetic pressure $B^2/8\pi$ from inside to outside the current layer at the X point. One may consider also the flux of the Poynting vector through the closed contour Λ shown on the fig. 3b. As long as the equilibrium and the continuity of the magnetic configuration is achieved outside Δ, this flux is equal to the variation rate of the available magnetic energy. On the other hand, it is equal to $E\iint_\Delta I\,dx\,dy$.

But inside the I profile in the layer, the term $v \times B/C$ in the generalized Ohm Law is of the same importance as the term ηI, so that the quantity $\iint_\Delta E\,I\,dx\,dy$ is significantly larger than the Joule power $\iint_\Delta \eta\,I^2\,dx\,dy$. Therefore the magnetic energy cannot be consumed by Joule effect only. A part of this energy must appear in the form of kinetic energy, in contradiction with the present pattern. It may be noticed that the above arguments would not be valid if the ions experienced a drag force larger than the inertia force $\varrho\,V.\nabla V$ in the layer. Such a force would allow mechanical equilibrium in the domain Δ. But again in that case the magnetic energy would not appear only in the form of Joule energy but as well in the form of ion energy induced by the drag force.

IV. Free reconnection with kinetic Eddies

It seems that the simplest scheme which escapes the above difficulties is the one shown on fig. 4 : the kinetic energy at Alfven velocity provided in the layer SS' accumulates in the growing magnetokinetic eddies IV, V, VI, VII. The current density is regular except near the separatrix and again the function $G(\phi,t)$ is time invariant in each domain I, ..., VII, being specified by (7) in the growing domains III, ..., VII. The transverse velocity must be $\ll C_A$ to preserve the bulk value of the electric field, and the stream function U (x,y,t) ($V_x = \partial U/\partial y$, $V_y = -\partial U/\partial x$) is nearly a function of $\phi(x,y,t)$: $U = H(\phi,t)$. The function $H(\phi,t)$ is not time invariant, but so is the Kelvin integral

$$\oint_\phi v\,ds = -P(\phi)\,\frac{\partial H(\phi,t)}{\partial\phi}$$

Being given at a given time the functions $G(\phi)$ and $H(\phi)$ in each region I, ..., VII, and the area of these domains, the magnetic and kinetic configuration may be determined by making the quantity

$$\int_{I,\ldots VII} P(\phi)\left(\left(\frac{\partial G}{\partial\phi}\right)^2 - 4\pi n\varrho\left(\frac{\partial U}{\partial\phi}\right)^2\right) d\phi$$

extremum with respect to the shape of the magnetic lines. The rate by which the normal island III and the streaming domains IV, ..., VII increase may be determined by expressing that the variations of the magnetic energy and of the kinetic energy are respectively equal to the quantities

$$\iint \eta\,I^2\,dx\,dy \quad , \quad \iint \frac{I\times B}{c}\cdot V\,dx\,dy$$

in the layer near the separatrix.

The present configuration is energetically less favourable than the traditional pattern, due to the presence of the kinetic energy and to the fact that the slopes $\partial G/\partial\phi$ in the domains III, ... VII are larger, resulting in larger magnetic energy in these domains. It is plausible that it is continuously distroyed by instability. An open question is to know whether the resulting turbulence simply incorporates in the island III or if it changes the mechanisms controlling the reconnection layer. In that second case the turbulent field δB, δV superimposing in a finite domain to the averaged field B, V responsible for a magnetic structure of the type given on fig. 1, would induce an evolution law for the function $G(\phi,t)$ associated to B

$$\frac{\partial G(\phi,t)}{\partial t} = -\langle \delta V\times\delta B/c \rangle = -\frac{\partial}{\partial\phi}\left(\oint_\phi \delta U\ \delta B\times dM\right)$$

The reconnection of the regions I, II into the island III would be specified by this equation (replacing (6)) and the extremum principle (4). Such a mechanism could in principle explain the rapid reconnections near the q = 2 surfaces which take place during soft disruptions in Tokamaks.

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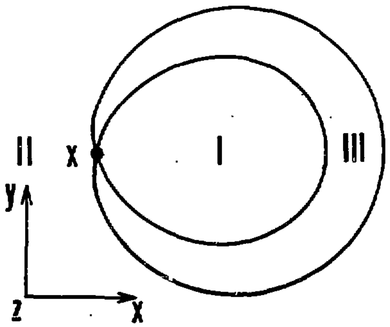

Figure 1 - Island Geometry

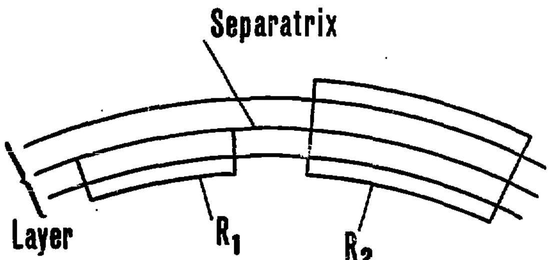

Figure 2 - Contours R for Eq.(9)

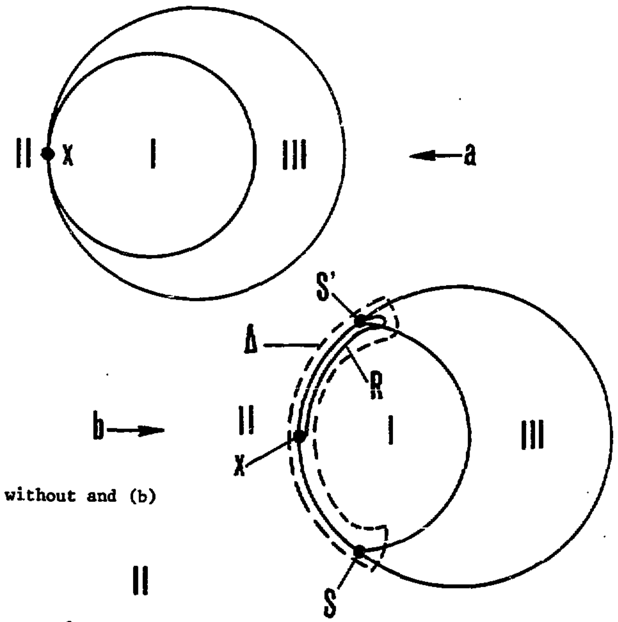

Figure 3 - Reconnection geometries (a) without and (b) with Syrovatskii points

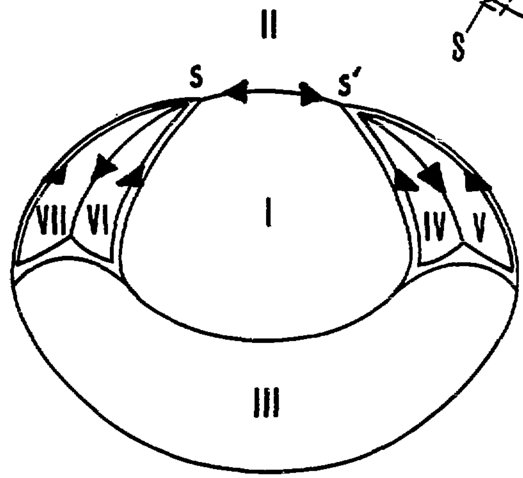

Figure 4 - Reconnection geometry with magnetokinetic eddies IV,..., VII.