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PHONON-ASSISTED TRANSITIONS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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ABSTRACT

A theory of the effect of a crossed electric, E, and magnetic, H, fields in the indirect transitions in semiconductors is developed. A semi-classical treatment is adopted where the electric field is considered as a small perturbation. A numerical application to GaP gives the limiting values of E/H valid to this approach.

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It is shown by Zak et al. [1] that, for low values of E/H and by using a method due to Luttinger et al. [2], a magnetic type of transitions occur, while for a high E/H ratio, a procedure due to Kane [3] has been developed [4], which gives a continuous solution of the eigenvalue problem.

The purpose of the present paper is to develop a semiclassical theory of the phonon-assisted transitions in crossed electric and magnetic fields. The electric field has been treated as small perturbation. The behaviour of the absorption coefficient is essentially of magnetic type, i.e. has step functions corresponding to transitions between the Landau levels in the different bands considered, as in the case of zero electric field [5]. The effect of the electric field is to reduce the indirect energy gap by the amount $\frac{1}{2}(m_c^* + m_v^*)\, c^2 (E/H)^2$. The other effect of the electric field is to destroy the Landau selection rules, due to the shift of the centre of the cyclotron motion. Furthermore, the electric field reduces the Landau levels spacing. The effect of temperatures is to split each step function into two different steps. A numerical application to GaP gives the different values of the ratio E/H valid in this treatment.

From second-order time-dependent perturbation one obtains, for the transition rate of a process in which a photon of frequency ω is absorbed with the simultaneous absorption, (-) or emission (+) of phonon of frequency Ω by an electron moving in crossed fields

$$W_{cv}(\omega) = \frac{2\pi}{\hbar}\left(\frac{e}{mc}\right)^2 A_0^2 \sum_{\ell,n} |\vec{M}_1|^2\, \delta(\mathcal{E}_c - \mathcal{E}_v - \hbar\omega \pm \hbar\Omega) \quad , \qquad (1a)$$

where

$$\vec{M}_1 = \frac{\langle c|\vec{\varepsilon}\cdot\vec{P}|\ell\rangle\langle\ell|\vec{Q}|v\rangle}{\mathcal{E}_\ell - \mathcal{E}_v \pm \hbar\Omega} + \frac{\langle c|\vec{Q}|\ell\rangle\langle\ell|\vec{\varepsilon}\cdot\vec{P}|v\rangle}{\mathcal{E}_\ell - \mathcal{E}_v - \hbar\omega} \qquad (1b)$$

$\vec{P} = \vec{p} + \frac{e}{c}\vec{A}$ is the generalized momentum in the magnetic field and $\vec{Q}$ is the electron-phonon interaction operator. The summation $\sum_\ell$ is over the intermediate states ℓ and the Landau quantum number n.

The wave function Ψ and energy $\mathcal{E}$ of the electron in crossed fields can be written in the form [6]

$$\Psi_\ell(\vec{r}) = F(\vec{r})\,\psi_{\ell 0}(\vec{r}) \quad , \qquad (2a)$$

where $\psi_{\ell 0}$ is the periodic part of the Bloch function for the ℓ band taken at $\underline{k} = 0$, and the envelope function

$$F(\vec{r}) = \left(\frac{eH}{\hbar c \pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} \exp\left\{i(k_x x + k_z z) - \frac{eH}{2\hbar c}\left(y - k_x L_M^2 + \frac{eEL_M^2}{\hbar\omega_c}\right)\right\} \times$$

$$\times H_n\left\{\sqrt{\frac{eH}{\hbar c}}\left(y - k_x L_M^2 + \frac{eEL_M^2}{\hbar\omega_c}\right)\right\} , \qquad (2b)$$

where H_n is the Hermite polynomial, $L_M = \left(\frac{\hbar c}{eH}\right)^{1/2}$ is the magnetic radius and $\omega_c = \frac{eH}{m^* c}$ is the cyclotron frequency. The energy levels are

$$\mathcal{E}_{\ell, n, k_z} = \mathcal{E}_{0\ell} \pm \frac{\hbar^2 k_{z\ell}^2}{2m_\ell^*} \pm (n+\tfrac{1}{2})\hbar\omega_\ell - eEL_M^2 k_{x\ell} \mp \frac{m_\ell^* c^2}{2} , \qquad (3)$$

where $\mathcal{E}_{0\ell}$ is the difference in energy between the extremum of the ℓ band and the top of the valence band.

After substituting from (2) and (3) into (1) and performing the integration over the $\vec{k}$ space (taking into account that the valence and intermediate states lie at $\vec{\underline{k}} = 0$, while the conduction state lies at $\vec{\underline{k}} = \vec{k}'$) we obtain for the absorption coefficient *)

$$\alpha(\omega) = C_1 |H|^2 \Gamma |\vec{M}_1|^2 \times$$

$$\times \left[\frac{\delta(\Lambda_-)}{1 - \exp\left(-\frac{\hbar\Omega}{kT}\right)} + \frac{\delta(\Lambda_+)}{\exp\left(\frac{\hbar\Omega}{kT}\right) - 1}\right] . \qquad (4)$$

where

$$\Gamma = \exp\left\{-\frac{e|H|}{\hbar c}\left(\frac{eEL_M^2}{\hbar\omega_c} + \frac{eEL_M^2}{\hbar\omega_v}\right)^2\right\} \times$$

$$\times \left| H_n^*\left(\sqrt{\frac{eH}{\hbar c}} \frac{eEL_M^2}{\hbar\omega_c}\right) H_{n'}\left(\sqrt{\frac{eH}{\hbar c}} \frac{eEL_M^2}{\hbar\omega_v}\right)\right| ,$$

*) A more detailed calculation will be published elsewhere.

$$\Lambda_\pm = \hbar\omega \mp \hbar\Omega - \mathcal{E}_g - (n+\tfrac{1}{2})(\hbar\omega_{c_c} + \hbar\omega_{c_v}) + \tfrac{1}{2} mc^2 \left(\frac{\alpha_c + \alpha_v}{\alpha_c \alpha_v}\right)\left(\frac{E}{H}\right)^2$$

and

$$C_1 = \frac{4\pi^2 e^5}{(2^n n!)^2 n_0 \omega m \hbar^5 c^4 L^3 \sqrt{\alpha_c \alpha_v}}$$

n_0 is the refractive index and the step function $\delta(\Lambda) = 0$ for $\Lambda < 0$ and $\delta(\Lambda) = 1$ for $\Lambda > 0$.

Performing a numerical calculation of $\alpha(\omega)$ for GaP we have the following results:

1) At $E = 10^3$ V/cm and $H = 5 \times 10^6$ G, the absorption edge starts at 2.1 eV (for $n = 0$, $T > 100^\circ$K) i.e. the indirect energy gap has been reduced by the value of 0.12 eV.

2) As E/H decreases by reducing E, the absorption starts at higher energy. At $E \leqslant 10$ V/cm, the effect of the electric field becomes negligible and the absorption starts at the indirect edge of GaP, $\mathcal{E}_g = 2.22$ eV at 77°K. As $E \to 0$, we recover the indirect magneto-absorption process [7),5)].

3) As E/H increases by reducing H, the absorption edge rises more slowly than the above case. As $H < 1.25 \times 10^5$ G, the present theory is not valid, and we have to take into account the Airy function behaviour of the electro-absorption at a relatively high electric field.

4) At temperature $T \leqslant 100^\circ$K, there is only phonon emission, while at $T > 100^\circ$K we have both phonon emission and absorption. The difference in energy between the two steps is 0.1 eV, i.e. twice the energy of the transverse optical phonon of GaP.

In conclusion the absorption coefficient in a crossed electric and magnetic field has a step function behaviour. The effect of temperature splits each step into two. The first step at low temperature corresponds to phonon emission, while that at higher temperatures corresponds to phonon absorption. The transitions are of magnetic type for $E/H < (\varepsilon_g/2m_c^* c^2)^{1/2}$ (i.e. $= 2.4 \times 10^{-3}$ for GaP) and of electic type for $E/H > (\varepsilon_g/2m_c^* c^2)^{1/2}$.

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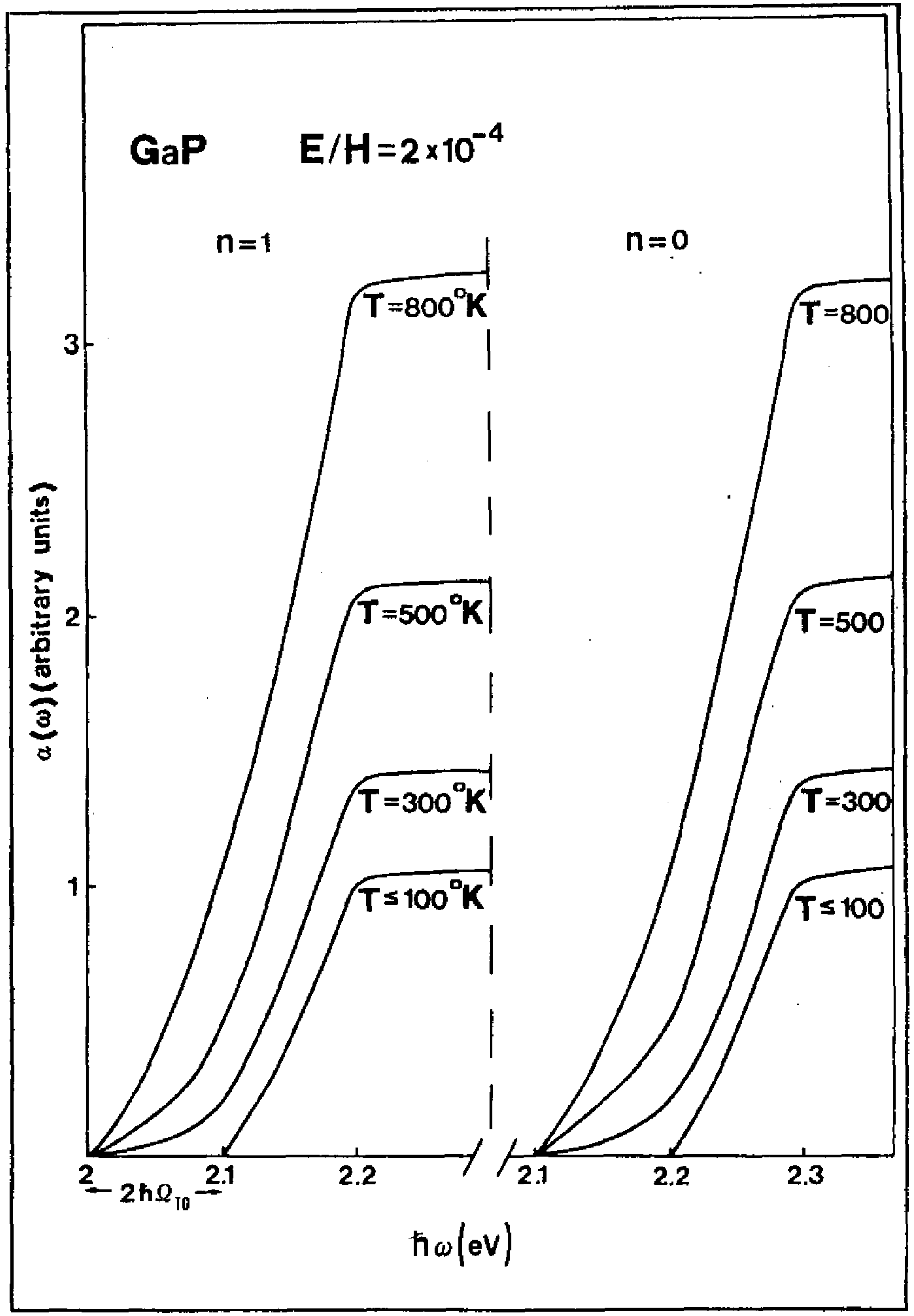
FIGURE CAPTION

Fig.1 Absorption coefficient $\alpha(\omega)$ as a function of the photon energy $\hbar\omega$ for GaP at $E/H = 2 \times 10^{-4}$ V/cm G. The effect of temperatures is shown at $T = 100^{\circ}K$, $300^{\circ}K$, $500^{\circ}K$ and $800^{\circ}K$. n is the magnetic quantum number.



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