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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1980 MIRAMARE-TRIESTE

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and
United Nations Educational Scientific and Cultural Organization

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ABSTRACT

The polarizability for a two-dimensional electron gas is calculated in the presence of impurities by a Green function formalism. This leads to a system with finite mean free path due to electrons scattering off impurities. The calculated polarizability is found to be strongly dependent on the mean free path. The main feature is the suppression of the sharp corner at wave vector $2k_F$ for finite mean free paths, and the pure metal result is recovered for the infinite mean free path. A possible application of the results to the transport properties of semiconductor inversion layers is discussed.

MIRAMARE - TRIESTE
June 1980

* To be submitted for publication.

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I. INTRODUCTION

The two-dimensional electron gas (2 DEG) has been a subject of much theoretical study mainly due to the existence of effectively two-dimensional systems, for example inversion layers in semiconductors and electrons on a liquid helium surface amongst others. The polarizability of a 2 DEG was calculated by Stern (1967) in the self-consistent field approximation by considering the two-dimensional analogue of the Lindhard dielectric constant. Fetter (1974) has considered thermodynamic properties of the 2 DEG. Many-body corrections to the 2 DEG response have been studied by Jonson (1976) who compared the RPA, Hubbard approximation and the self-consistent approach by Singwi et al. (1968). The polarizability of the 2 DEG at finite temperature has been calculated by Maldague (1978).

The 2 DEG polarizability is known to have a sharp corner at the wave vector $2k_F$, where k_F is the Fermi wave vector, analogous to the corresponding phenomena in one and three dimensions (Kohn 1959, Kittel 1968). However consideration of impurity scattering has an effect of suppressing the anomaly at $2k_F$. Such a suppression in three dimensions has been discussed by de Gennes (1962) who showed that the polarizability is strongly dependent on the mean free path of conduction electrons. The predictions of De Gennes have been confirmed by a series of NMR experiments by Hegger et al. (1966). It is proposed in the present paper that impurity scattering effects round up the sharp corner at $2k_F$ for the 2 DEG polarizability as well.

The plan of this paper is as follows. In Sec.II we develop a Green function (GF) formulation to study the effect of electrons scattered by impurities. The GF is related to the mean free path, ℓ, of the electrons and a correlation function which is used in Sec.III to calculate the polarizability. It is found that when ℓ is finite the sharp corner at $2k_F$ is suppressed and the limit ℓ approaching infinity (pure metal) correctly reproduces the static limit of Stern's (1967) polarizability. Sec.IV is devoted to a discussion of our results. These results could be important for example in explaining impurity scattering effects on the transport properties in semiconductor inversion layers. Recently, Stern (1980) has pointed out the importance of scattering effects on the screening parameter and hence carrier mobility, and our theory here is relevant to that physical situation.

II. SCATTERING BY IMPURITIES

Consider an electron interacting with a set of N impurity ions randomly distributed in the 2 DEG. The Hamiltonian of the system is

$$H = H_0 + \sum_{i=1}^{N} W(\underline{r} - \underline{R}_i) \,, \tag{1}$$

where H_0 is the pure metal Hamiltonian and W is the impurity potential. $\underline{R}_i$ are the positions of the impurity ions. The single particle GF can be expanded in powers of W as shown in Fig.1. Techniques for dealing with scattering by impurities have been discussed by Abrikosov (1958), Rickayzen (1961) and de Gennes (1962) and we adopt these to our two-dimensional system without reviewing them here. The only significant change here is that the integrations are in two dimensions. The average GF (see Rickayzen 1961, for example) is given by a perturbation series expansion whose sum is

$$\langle G(k,\omega)\rangle = \frac{1}{\epsilon_k - \omega + \Delta(\omega)} \,, \tag{2}$$

where $\epsilon_k \approx k^2/2m$ and this includes the real part of the self energy. The imaginary part of the self energy gives

$$\Delta(\omega) = in \int \frac{d^2k'}{(2\pi)^2} |W(k-k')|^2 \pi \delta(\epsilon_{k'} - \omega) \tag{3a}$$

$$= -\frac{i}{2\tau} \,, \tag{3b}$$

where τ is the relaxation time which is related to the mean free path ℓ by

$$\ell = v\tau = \frac{p}{nm|W|^2} \,. \tag{4}$$

In (3) and (4) n is the number of impurities per unit area, p and v are the electron momentum and velocity, respectively. The GF in (1) can also be written as

$$\langle G(k,\omega)\rangle = \frac{2m}{k^2 - q^2} \,, \tag{5}$$

where

$$\frac{q^2}{2m} = \omega - \Delta(\omega) \tag{6a}$$

or equivalently

$$q^2 - \frac{iq}{\ell} - 2m\omega = 0 \,, \tag{6b}$$

where (3b) and (6) have been used. The preceding equation defines a complex wave vector, $q(\omega)$, whose solution is

$$q(\omega) = \left(2\omega m - \frac{1}{4\ell^2}\right)^{1/2} + \frac{i}{2\ell} \tag{7a}$$

$$= q' + iq'' \tag{7b}$$

and $q'' > 0$.

The Fourier transform of the GF in (5) is

$$G(r) = \frac{1}{(2\pi)^2} \int e^{i\underline{k}\cdot\underline{r}} G(\underline{k},\omega)\, d^2\underline{k} \tag{8a}$$

$$= \frac{m}{\pi} \int_0^\infty \frac{J_0(kr)}{k^2 - q^2} k\,dk \,, \tag{8b}$$

where $J_0(kr)$ is a Bessel function. The integration in (8b) enables the GF to be expressed in terms of the modified Bessel function, $K_0(rz)$ (Gradshteyn 1965).

$$G(r) = \frac{m}{\pi} K_0(rz) \,, \tag{9}$$

where $z = -iq$. To obtain the polarizability we shall need to evaluate the correlation function

$$\int \overline{G(r0)\, G(0r)}\, e^{i\underline{p}\cdot\underline{r}} d^2\underline{r} \tag{10}$$

which we shall denote by L(p). From (9) and (10) we obtain

$$L(p) = \frac{m^2}{\pi}\int_0^{\infty}\left[K_0(rz)\right]^2 J_0(pr)\, r\, dr \tag{11}$$

and the integral in (11) is evaluated in Appx.A to obtain

$$L(p) = -\frac{4m^2}{\pi p}\,\frac{\mathrm{ArcSin}(p/2q)}{(p^2-4q^2)^{1/2}} \,. \tag{12}$$

III. THE 2DEG POLARIZABILITY

Analogous to the three-dimensional calculation of De Gennes (1962) the polarizability is related to a quantity

$$H(p) = \frac{L(p)}{1 - n|W|^2 L(p)} \,, \tag{13}$$

where L(p) has been given in (12). The polarizability is given by integration over frequencies and noting that

$$d\omega = \left(q - \frac{i}{2\ell}\right)\frac{dq}{m} \tag{14}$$

we obtain

$$\chi(p) = -\frac{4m^2}{\pi}\int_{-k_F}^{k_F}\frac{q'dq'}{mp}\,\frac{F}{\left[1+\frac{4F}{\pi\ell}\right]} \,, \tag{15}$$

where F is given by

$$F = \frac{\mathrm{ArcSin}(p/2q)}{(p^2-4q^2)^{1/2}} \tag{16}$$

and is a complex quantity by virtue of (7). We are interested in $\chi''(p)$ which is obtained by taking the imaginary part of (15)

$$\chi''(p) = -\left(\frac{4m}{\pi}\right)\int_{-k_F}^{k_F}\frac{q'dq'\quad F''}{p\left\{\left(1+\frac{4F'}{\pi\ell}\right)^2+\left(\frac{4F''}{\pi\ell}\right)^2\right\}} \,, \tag{17}$$

where F' and F" are real and imaginary parts of F and these are evaluated in App.B as

$$F' = \frac{T'B' - T''B''}{B'^2 + B''^2} \,, \tag{18a}$$

$$F'' = -\left(\frac{T''B' + T'B''}{B'^2 + B''^2}\right) . \tag{18b}$$

Eq.(17) for a 2 DEG is the main result of this paper and is analogous to (37) of de Gennes (1962) but not identical for obvious reasons. To proceed, numerical integration for (17) is performed for values of mean free paths ℓk_F = 2.5, 5.0, 10.0 and ∞ . The results are illustrated in Fig.2. In the limit

$$\ell \to \infty \qquad \text{(pure metal)} \tag{19}$$

there is considerable simplification since

$$q'' \to 0, \quad T'' \to 0 \quad \text{and} \quad B'' \to 0 \tag{20}$$

and Eq.(17) reduces to

$$\chi''(p) = m\left\{\left(1 - \left[1 - \left(\frac{2k_F}{p}\right)^2\right]^{1/2}\right\} \quad \text{for } p > 2k_F \tag{21}$$

and

$$\chi''(p) = m \qquad \text{for} \quad p < 2k_F \tag{22}$$

which agree with Stern (1967), Kittel (1968) and the sharp corner at $2k_F$ is recovered. It is possible to calculate the Fourier transform of $\chi(p)$ and obtain the range function

$$\chi(r) = \frac{1}{2\pi}\int_0^{\infty} \chi(p)\, J_0(pr)\, p\, dp \tag{23}$$

which has oscillations dependent on ℓ as the corresponding observation in three dimensions, but this study will not be done here.

IV. DISCUSSION

The main result of this paper is Eq.(17) describing the polarizability of a 2 DEG in the presence of impurities. The analogous discussion for a 3 DEG was given several years ago by de Gennes (1962). From Fig.2, it can be observed that the pure metal polarizability ($\ell k_F = \infty$) has the usual sharp corner at $2k_F$. This anomaly is suppressed by scattering due to impurities when the mean free path is finite and the polarizability is a smoothly varying function approaching the pure metal result only at large k_F.

Eq.(17) can be applied to several problems of physical interest. For example, transport properties of electrons in an inversion layer can be described by a screening parameter (Stern and Howard 1967) which is dependent on the polarizability. Up to recently, the effect of impurities scattering has not been considered, though Stern (1970) has suggested the importance of this mechanism in addition to Coulomb scattering and surface roughness scattering. We propose to pursue this problem in a separate paper.

Finally, some experiments analogous to those of Heeger et al. (1966) should be possible to probe the effect of the mean free path in the 2 DEG.

ACKNOWLEDGMENTS

The author would like to thank Professor F. Stern for introducing him to the inversion layer problem that led to the work discussed here and to Professor E. Tosatti for his interest and comments regarding this work. Thanks are also due to Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

APPENDIX A

Eq.(11) is of the general form

$$\int_0^{\infty} x^{\nu+1} K_\mu(ax)\, K_\mu(bx)\, J_\nu(cx)\, dx \ , \tag{A.1}$$

where $K_\mu(ax)$, $K_\mu(bx)$ are modified Bessel functions and $J_\nu(cx)$ is a Bessel function. For our present problem

$$\left.\begin{array}{l} \nu = \mu = 0 \\ a = b = z \\ c = p \\ x = r \end{array}\right\} \ . \tag{A.2}$$

According to Gradshteyn (1965) we obtain

$$\int_0^{\infty} \left[K_0(rz)\right]^2 J_0(pr)\, r\, dr = \frac{\pi^{1/2}}{2^{3/2} z^2} \left(u^2-1\right)^{-1/4} P_{-1/2}^{-1/2}(u) \ , \tag{A.3}$$

where $P_{-1/2}^{-1/2}(u)$ is an associated Legendre function of the first kind. This is related to the Gauss hypergeometric series $F(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{1-u}{2})$ by

$$P_{-1/2}^{-1/2}(u) = \frac{1}{\Gamma(3/2)} \left(\frac{u+1}{u-1}\right)^{-1/4} F\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{1-u}{2}\right) . \tag{A.4}$$

For

$$u = 1 + \frac{p^2}{2z^2} \tag{A.5}$$

and using (A.4) in (A.3) (Gradshteyn 1965), we obtain

$$\int_0^{\infty} \left[K_0(rz)\right]^2 J_0(pr)\, r\, dr = \frac{2\, \mathrm{ArcSin}(p/2q)}{p\left(p^2-4q^2\right)^{1/2}} \tag{A.6}$$

which is used in (11) to obtain (12).

APPENDIX B

Since the wave vector q(ω) in 7 is complex, then F in 15 is complex

$$F = \frac{\mathrm{ArcSin}\,(p/2q)}{(p^2 - 4q^2)^{1/2}} \tag{B.1a}$$

$$= \frac{T' - iT''}{B'' + iB''} = F' + iF'', \tag{B.1b}$$

where

$$T' = \mathrm{ArcSin}\left(\frac{2x}{\alpha + \beta}\right) \tag{B.2}$$

$$T'' = \mathrm{Arc\,cosh}\left(\frac{\alpha + \beta}{2}\right) \tag{B.3}$$

$$\alpha = \sqrt{(1+x)^2 + y^2} \tag{B.4}$$

$$\beta = \sqrt{(1-x)^2 + y^2} \tag{B.5}$$

$$x = \frac{pq'}{2(q'^2 + q''^2)} \tag{B.6}$$

$$y = \frac{pq'}{2(q'^2 + q''^2)} \tag{B.7}$$

$$B' = \left\{\frac{\sqrt{a^2 + b^2} + a}{2}\right\}^{1/2} \tag{B.8}$$

$$B'' = \left\{\sqrt{a^2 + b^2} - a\right\}^{1/2} \tag{B.9}$$

$$a = p^2 - 4(q'^2 - q''^2) \tag{B.10}$$

$$b = -8q'q''. \tag{B.11}$$

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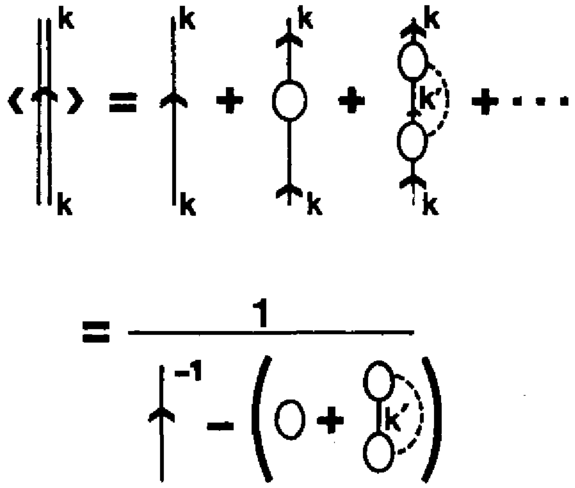

Fig1

Perturbation series expansion of the GF in the presence of scattering by impurities.

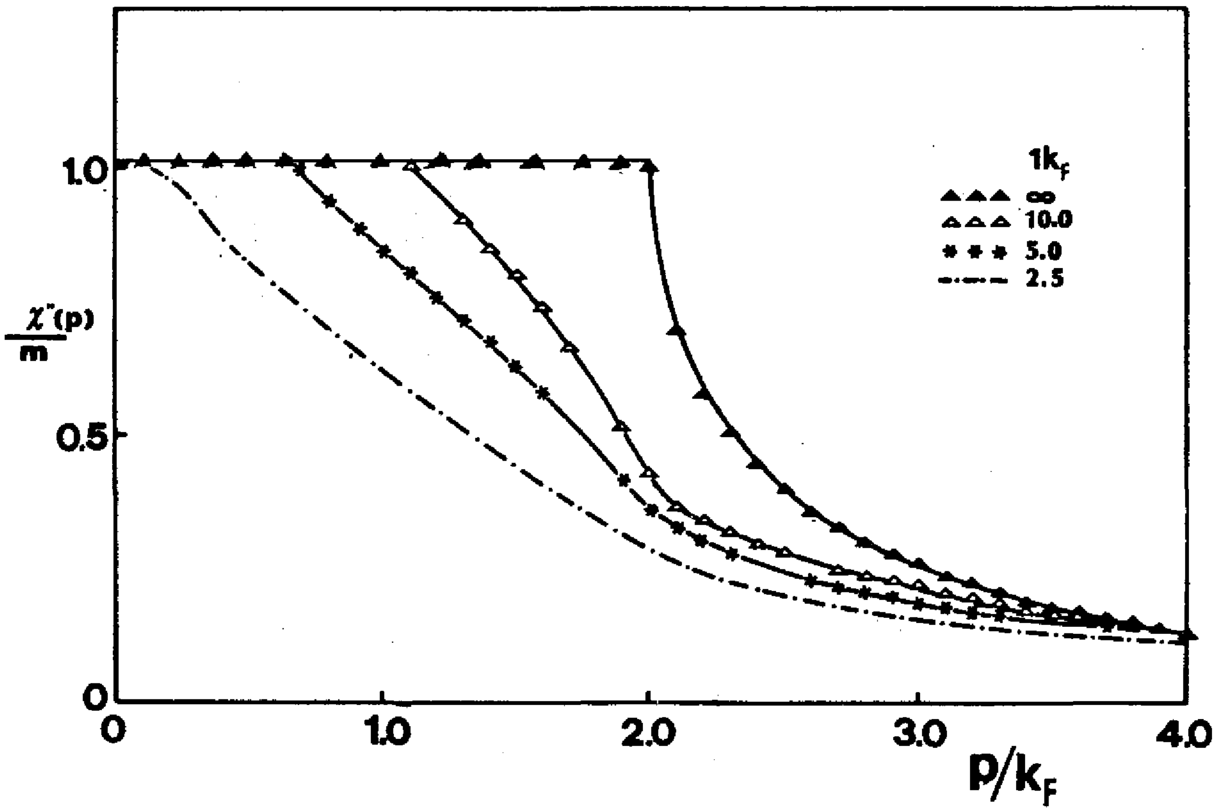

Fig2

Polarizability of a 2 DEG as a function of wave vector p for values of the mean free path ℓk_F = 2.5, 5.0, 10.0 and ∞.

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