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THE PARTIAL WIDTHS OF BOSON RESONANCES IN THE QUARK-GLUON MODEL OF STRONG INTERACTIONS

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Abstract

The quark-gluon model of strong interactions based on the topological expansion and the string model is used for the calculation of the partial widths of boson resonances in the channels with two pseudoscalar mesons. With the help of the relations between the residues of the secondary reggeons obtained in the previous paper [10], the partial widths of mesons with arbitrary spins lying on the vector and tensor Regge trajectories are expressed in terms of the only one constant the ρ-meson width. We predict that the violation of SU(3) symmetry increases with the growth ofthe spin of the resonance. The theoretical predictions are in a good agreement with experimental data.

I. Introduction

The approaches to the description of the processes with low transverse momenta based on the quantum chromodynamics were proposed recently. One of the most promising approach to this problem is connected to the quark-gluon picture of the strong interactions [1-10], based on the topological expansion [11-14] and the string model [15-18]. This picture is universal and allows one to connect to each other the peripheral and hard processes. This approach has been applied very successfully to many problems of hadronic physics - from the calculation of masses of the stable particles and resonances [9] to the description of the basic properties of multiparticle production processes in hadron-hadron [3-7] and hadron-nucleus [3] collisions.

In the present paper we use this quark-gluon model for the calculation of the width of boson resonances lying on the vector $\rho, \omega, K^*, \varphi$ and the tensor A_2, f, K^{**}, f' Regge trajectories. We use here the relations between the reggeon residues which have been obtained using this approach in the previous paper [10]. These relations based on the topological expansion generalise the condition of universality to the arbitrary values of t and allow one to express the non spin-flip residues of the secondary reggeons for any hadrons in terms of the one function $g(t)$. In the framework of this approach the definite violation of SU(3) symmetry is predicted which is due to the mass differences of u-, d- and s-quarks and to the splitting of the $\rho(\omega)$, K^* and φ trajectories. We use the simple parametrization for the function $g(t)$ based on the duality.

After that all the partial widths of the resonances with arbitrary spins are expressed in terms of only one constant, which is fixed by the value of ρ meson width. The model predicts the systematic deviations from the predictions of the SU(3)-symmetry which increases with growth of spin of a resonance. The obtained predictions for all the measured widths of boson resonances (up to $J^P = 4^+$) are in a very good agreement with experimental data.

II. The relations between the residues of secondary Regge-poles

In this Section we discuss the relations between the residues of secondary Regge-poles which were obtained in the previous paper [10] with the help of topological expansion and show how to use these relation for the determination of the partial widths of boson resonances.

In the framework of the topological expansion (an expansion on $1/N_f \approx 1/N_c$, where N_f is the number of flavours and N_c is the number of colours) of the scattering amplitudes the main diagrams are the planar ones, which are shown in fig. 1 for the case of two-particle process. These diagrams have a simple space-time interpretation [9] the valence quarks i and $\bar{i}$ which compose the colliding hadrons annihilate and the colour tube is formed in the space between the two spectator quarks. This colour-tube breaks in two parts which corresponds to the final hadrons c and d with the pair of quarks j and $\bar{j}$ produced from the vacuum. The imaginary parts of elastic scattering amplitudes are connected by the optical theorem to the total cross-sections of interactions corresponding to the valence quark annihilation.

At the same time the diagrams in the fig.1 correspond at high energy to the exchange by the secondary Regge poles in t-channel. In the framework of this approach the part of the total cross-section, which corresponds to the Pomeron is due to the cylinder-type diagrams [11-14]. By comparing the contribution of the planar diagrams to the processes of elastic scattering of different hadrons with the contribution of the corresponding reggeons, it is possible to obtain the system of equations which connected the residues of different Regge poles [10]. This system has a factorisable solution. The helicity averaged residues of secondary Regge poles α_i $g_i^a(t) = \frac{1}{2S_a+1}\sum_{\lambda_a} g_{i\,\lambda_a\lambda_a}^{aa}(t)$ could be written in terms of the functions $g(t)$, $g_1(t)$ which correspond to the exchange in t-channel by u,d and s quarks.

$$
\begin{aligned}
g_\rho^a(t) &= g(t)\,\frac{N_u^a - N_d^a - N_{\bar u}^a + N_{\bar d}^a}{2} = g(t)\,T_3^a \\
g_f^a(t) &= g(t)\,\frac{N_u^a + N_d^a - N_{\bar u}^a - N_{\bar d}^a}{2} = g(t)\,\frac{Y^a + 2S^a}{2} \qquad (1)\\
g_\omega^a(t) &= g(t)\,\frac{N_u^a + N_d^a + N_{\bar u}^a + N_{\bar d}^a}{2} \\
g_{A_2}^a(t) &= g(t)\,\frac{N_u^a - N_d^a + N_{\bar u}^a - N_{\bar d}^a}{2} \\
g_\varphi^a(t) &= \frac{g_1(t)}{\sqrt{2}}\,(N_s^a - N_{\bar s}^a) = \frac{g_1(t)}{\sqrt{2}}\,S \\
g_{f'}^a(t) &= \frac{g_1(t)}{\sqrt{2}}\,(N_s^a + N_{\bar s}^a)
\end{aligned}
$$

here N_k^a is the number of valence quarks of the type k in the hadron a . In the limit of exact SU(3) symmetry, when $\alpha_\rho(t) = \alpha_{K^*}(t) = \alpha_\varphi(t)$ the functions $g(t)$ and $g_1(t)$ are equal. When the splitting of the trajectories is taken into account, these functions are different.

The elastic scattering amplitudes can be written in terms of the functions $g_i^a(t)$ in the following way (here and in what follows we consider the scattering of particles with zero spin)

$$T_{ab}^{(pl)}(s) = \frac{\pi s}{\bar{s}} \sum_i g_i^a(t)\, g_i^b(t) \left(\frac{s}{s_0^{ab}}\right)^{\alpha_i(t)-1} \sigma\, \eta(\alpha_i(t)) \qquad (2)$$

where $\eta(\alpha_i(t)) = -\frac{\exp[-i\pi\alpha_i(t)]+\sigma}{\sin[\pi\alpha_i(t)]}$ is a signature factor and the value of σ is equal to ± 1. As it was noted in the previous paper [10] the dependence on the type of the colliding hadrons is due to the value of s_0^{ab} , which satisfies to the following relations:

$$s_0^{ab} = \sqrt{s_0^{aa}\, s_0^{bb}}\ ; \qquad \frac{s_0^{ab}}{s_0^{ac}} = \frac{\bar{x}_c^i}{\bar{x}_b^i} \qquad (3)$$

here $\bar{x}_b^i$ is the mean value of the momentum fraction carried by the quark of the type i in the hadron b . The value of $\bar{x}_b^i$ can be determined independently from the analysis of the deep inelastic lepton-hadron scattering and the Drell-Yan processes. From the experimental data it follows that

$$\bar{x}_\pi^u / \bar{x}_N^u = 1.6 \pm 0.1 \quad \text{and} \quad \bar{x}_\pi^u / \bar{x}_K^u = 1.1 \pm 0.05$$

The relations (1)–(3) allow one to connect the contributions of the vector and tensor trajectories to the scattering amplitudes of pseudoscalar (π, K) mesons at arbitrary values of t. It is necessary to note that these amplitudes are universal and SU(3) symmetric (with the ideal mixing) only at the point $\alpha(t) = J = 1$, e.g. only for the vector mesons. At other values of t there exist the quite definite violation of the SU(3) symmetry and universality which are due to the fact that the strange quark is more massive and as a result $\bar{x}^u_\pi / \bar{x}^u_K \neq 1$.

The partial widths of decays of resonances with spin J to the two pseudoscalar mesons $a\bar{a}$ – Γ_a can be expressed in terms of the residues of the corresponding reggeons at the points $t_J = M_J^2$ $(\alpha(t_J) = J)$

$$\Gamma_a = \left(\frac{4 p_a^2(t_J)}{s_0^{aa}}\right)^J \frac{p_a(t_J)\, s_0^{aa}}{M_J^2\, \alpha' \bar{s}} \frac{\Gamma^2(J+1)}{4\pi^2\, \Gamma(2J+2)} \left(g_i^a(t_J)\right)^2 \qquad (4)$$

here Γ is the gamma-function and $p_a(t_J) = \frac{1}{2}\sqrt{M_J^2 - 4m_a^2}$ is the momentum of meson a in the rest system of the resonance M_J.

Equation (4) can be obtained by the comparison of equation (2) near the point $\alpha(t) = J$ with the expression for the contribution of the resonance with spin J in the amplitude of elastic aa scattering

$$T_{aa}(s,t) = \frac{8\pi\sqrt{t}}{p_a(t)}(2J+1) f^J(t)\, P_J(z_t) \qquad (5)$$

when $t \approx t_J = M_J^2$

$$f^J(t) = \frac{\Gamma_a M_J}{(t - M_J^2) + i M_J \Gamma} \qquad (6)$$

Taking into account that at $z_t \gg 1$

$$(2J+1)\, P_J(z_t) \approx \frac{\Gamma(2J+2)}{\Gamma^2(J+1)}\left(\frac{z_t}{2}\right)^J = \frac{\Gamma(2J+2)}{\Gamma^2(J+1)}\left[\frac{s}{4p_a^2(t)}\right]^J$$

One get equation (4).

So Eqs. (1), (3), (4) allow one to express the partial width of all the resonances on the vector and tensor trajectories in terms of the universal functions $g(t)$ and $g_1(t)$ at the points $t=t_J$.

III. Parametrization of the functions $g(t)$ and $g_1(t)$ and comparison of the prediction for partial width with experimental data

In order to determine the rates of resonance decay into $\pi\pi$ and $K\bar{K}$ channels it is necessary to make a hypothesis on the behaviour of the functions $g(t)$ and $g_1(t)$. These functions must be equal to zero at all integer non positive values of $\alpha(t) = 0, -1, \ldots$ because of the strong exchange degeneracy of the poles with opposite signature. At large positive t these functions must decrease sufficiently fast $g^2(t) \sim \exp\left\{-\frac{t}{s_0^{aa}} \ln \frac{t}{s_0^{aa}}\right\}$ in order to compensate the rapid growth of the factor $(4p_a^2(t)/s_0^{aa})^J$ and give the asymptotic behaviour of the partial amplitudes compatiable with analycity and unitarity.

All these conditions can be satisfied by the expression *)

*) We suppose that the trajectories in the planar approximation are linear $\alpha(t) = \alpha(0) + \alpha' t$ and the planar approximation is a good one (at least at $t \gtrsim 1\ \mathrm{GeV}^2$).

$$g^2(t) = \frac{g_0^2}{\Gamma[\alpha_\rho(t)]} \qquad (7)$$

where g_0 is a constant.

Such a behaviour of the Regge pole residues arises in dual models [19,20], where the value of $s_0^{\pi\pi}$ is equal to $(\alpha'_\rho)^{-1} \approx$ ≈ 1.1 GeV^2.

For $s_0^{K\bar{K}}$ in correspondence with Eq. (3) we shall use the value

$$s_0^{K\bar{K}} = s_0^{\pi\pi} \left(\frac{\bar{\lambda}_\pi^{-4}}{\bar{\lambda}_K^{-4}} \right)^2 = 1.3 \text{ GeV}^2 \qquad (8)$$

It is worth noting that this value is very close to $(\alpha'_\varphi)^{-1}$ – the slope of the trajectory of s-channel resonances in elastic $K\bar{K}$ scattering [9].

Let us choose the function $g_1(t)$ which characterizes the residues of φ and f' trajectories in the same form

$$g_1^2(t) = \frac{g_{01}^2}{\Gamma[\alpha_\varphi(t)]} \qquad (9)$$

In the limit when $\alpha_\rho(t) = \alpha_\varphi(t)$ the functions $g(t)$ and $g_1(t)$ coincide. So in our parametrization it is naturally to suppose that $g_0^2 = g_{01}^2$ and all the difference between $g(t)$ and $g_1(t)$ is due to the factor $\{\Gamma[\alpha_i(t)]\}^{-1}$

Up to now we have considered the elastic scattering processes, which were defined by the diagonal residues $g_i^{aa}(t)$. In the framework of this approach the factorization relations in s-channel are fulfilled [9] which allow one to express the nondiagonal residues $g_i^{ab}(t)$ in terms of the diagonal ones $g_i^{aa}(t)$ and $g_i^{bb}(t)$. In the case of interest the factorization relations give the possibility to express the residues of K^* and K^{**} trajectories in terms of the same constants g_0 and g_{01}:

$$g^2_{K^{*0}\to K^-\pi^+}(t) = g^2_{K^{**0}\to K^-\pi^+}(t) = \frac{1}{2}\, g_2^2(t)$$

$$g_2(t) = \frac{g_{02}^2}{\Gamma\left[\alpha_{K^*}(t)\right]}\,, \qquad g_{02}^2 = g_0\cdot g_{01} = g_0^2 \tag{10}$$

The value of $S_0^{\pi K}$ can be expressed with the help of factorization relations in terms of $S_0^{\pi\pi}$ and $S_0^{K\bar{K}}$

$$S_0^{\pi K} \approx \left(S_0^{\pi\pi}\cdot S_0^{K\bar{K}}\right)^{1/2} \approx 1.2\ \mathrm{GeV}^2 \tag{11}$$

The formulae (7)–(11) and the equation (4) allow one to express the partial decay widths to two pseudoscalar mesons of resonances with arbitrary spin J, composed from u,d and s-quarks and lying at the $\rho, f, \omega, A_2, K^*, K^{**}, \varphi$ and f' Regge trajectories in terms of the only one constant g_0. We fix the value of g_0 from the experimental width of the ρ-meson ($\Gamma_\rho = 158 \pm 5$ MeV) [21].

The calculated partial widths are given in the Table 1. The errors shown for the theoretical values correspond to the experimental (error) in the width of the ρ-meson.

A good agreement with experiment of the theoretical prediction on the width of the g-meson ($J^P = 3^-$) in $\pi\pi$ channel confirms our parametrization of the function $g^2(t)$ in the form (7) and the value of $S_0^{\pi\pi} = 1.1$ GeV^2. For the strange meson the analogous conclusion follows from the comparison of the width of the K^{*} (890) and K^{***}(1780) mesons which lie on the same trajectory.

Table 1 demonstrates a surprisingly good agreement between the predictions of this model and the experimental data [21]. The values with the partial width, which are predicted by SU(3) symmetry with ideal mixing are shown in the Table 1 in bra - ckets *). As it was noted before these values differ from the predictions of our model because of $s_0^{\pi\pi} \neq s_0^{\pi K} \neq s_0^{K\bar{K}}$ The difference increases with the growth of the resonance spin J proportionally to $(s_0^{\pi\pi}/s_0^{K\bar{K}})^{J-1}$. The experimental data are in a better agreement with the prediction of our model which takes into account the violation of SU(3) symmetry than with the prediction of the unbroken SU(3). Unfortunately for the ρ- -meson decay to $K\bar{K}$ (where the difference between the two predictions is of order of 50%) the experimental situation is controversal (see for example [21]). By this reason in the Table 1 the two different variants of experimental data fits on the $\rho \to K\bar{K}$ decay from the ref. [21] are given. We must note that both of these numbers disagree with the predictions of the unbroken SU(3) symmetry. It would be even more interesting to compare the experimental data on the $h \to K\bar{K}$ decay with theoretical predictions, but such a data do not exist at present.

By the help of equations (4), (7)-(11) it is easy to calculate the width of decay of resonances with the spins higher then that shown in the Table 1. But the experimental information on such a resonances is practically absent.

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*) These values are calculated for a given SU(3) multiplet from the width $\Gamma_{a \to \pi\pi}$

The formulae (7)-(11) could be easily generalized to decays of resonances composed of the heavy c- and b-quarks. Unfortunately there are no experimental data now on the decays of heavy c- and b-flavoured mesons, lying on the leading Regge trajectories with corresponding quantum numbers and having the strong decay due to the creation $q\bar{q}$ pair from the vacuum *) . The exception is the D^* meson ($J^P = 1^-$) with decays to the $D\pi$ system. For the D^* and D^{**} trajectories we have the relation analogous to the equation (10)

$$g^2_{D^{*0}\to D^-\pi^+}(t) = g^2_{D^{**0}\to D^-\pi^+}(t) = \frac{1}{2} g^2_3(t)$$

$$g^2_3(t) = \frac{g^2_{03}}{\Gamma[\alpha_{D^*}(t)]}, \qquad g^2_{03} = g^2_0 \tag{12}$$

The partial width of decay $\Gamma_{D^{*+}\to D\pi}$ calculated with the help of eq. (4), (12) is equal to 0.02 MeV, that is substantionally lower than the existing experimental bound $\Gamma_{D^{*+}\to D\pi} < 2$ MeV.

IV. Conclusion

The comparison of the predictions of the quark-gluon model based on the topological expansion with the experimental data performed in the previous section shows that this approach can

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*) The decays of J/ψ, ψ', Υ, Υ' and Υ'' mesons to hadrons is due to the production and hadronization of gluons and the resonances ψ'' and Υ''' which have masses higher than the $D\bar{D}$ and $B\bar{B}$ thresholds are lying on the daughter Regge trajectories.

can describe the large amount of the experimental data on bosonic resonances decays. The deviations of the theoretical values of Γ from the experimental ones as a rule are less then 10%. Such a high accuracy of calculation based on the planar approximation is connected from our point of view to the "asymptotic planarity". Due to this asymptotic planarity the relations between the Regge residues (1) must be fulfilled at the region $t \gtrsim 1\ GeV^2$ with the accuracy better then 5-10% [10] . For the critical verification of our scheme it is very interesting to measure the partial widths of decay of resonances with high spins ($J \geqslant 4$) to π and K mesons. Such a data could give a good check of the considered mechanism of SU(3) breaking.

We have discussed the partial decay rates of resonances only to two pseudoscalar mesons. This is connected with a fact that the relations between residues (1) are valid for the helicity non-flip transitions. By this reason for the decays of mesons to $N\bar{N}$ system it is possible to obtain now only the lower bounds based on the account of one helicity amplitude.

An account of spin in the framework of our scheme would make possible not only the determination of partial widths of meson decays to $N\bar{N}$ pair, but to define the values of widths of baryon resonances.

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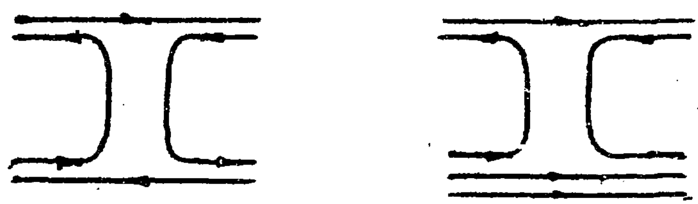
Table 1

resonance	channel	J^P	Γ (Mev) theor	$\Gamma_i/\Gamma_{\pi^+\pi^-}$ (%) theor	Γ (Mev) exp	$\Gamma_i/\Gamma_{\pi^+\pi^-}$ (%) exp
f(1270)	$\pi^+\pi^-$	2^+	83±3		98±12	
	$K\bar{K}$			4.3 (5.0)		5.1±.8
A_2(1310)	$K\bar{K}$	2^+	5.1±.3 (6.2)		4.9±.5	
g(1700)	$\pi^+\pi^-$	3^-	54±2		48±5.8	
	$K\bar{K}$			9.0 (13)		8±3 6.3±1.4
h(2040)	$\pi^+\pi^-$	4^+	30±5		30±6	
	$K\bar{K}$			10 (17)		
φ(1020)	$K\bar{K}$	1^-	2.0±.06		2.0±.05	
f'(1515)	$K\bar{K}$	2^+	52±2		$<$67±10	
K^*(890)	$K\pi$	1^-	44±2		49.5±.8	
K^{**}(1430)	$K\pi$	2^+	45±2		49.1±1.6	
K^{***}(1780)	$K\pi$	3^-	26±1.5		25.6±6.5	



a) b)

Fig.1

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