IC/80/43

INTERNATIONAL CENTRE FOR
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INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1980 MIRAMARE-TRIESTE

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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CLASSICAL SOLUTIONS AND EXTENDED SUPERGRAVITY *

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MIRAMARE - TRIESTE
March 1980

* Submitted for publication.

1. In some previous papers[1,2] we have investigated the existence and properties of classical solutions for gravity coupled to matter fields. Our discussion has been limited to conformally flat solutions of the form, in Euclidean notation,

$$g_{\mu\nu}(x) = \delta_{\mu\nu}\, h^2(x) \tag{1}$$

where the function $h(x)$ is further determined by requiring invariance under a given subgroup of the conformal group. In particular, invariance under the $O(4) \times O(2)$ group of four dimensional rotations and dilatations leads to the so-called meron solutions, whose simplest expression is $h(x) \propto 1/\sqrt{x^2}$ (*). As shown in our work these solutions exist only if matter fields are present with non-vanishing classical values. For a given Lagrangian density the matter solutions can be directly expressed through $h(x)$ and further their existence implies simple, though not trivial, relations

(*) An important rôle is of course played in these considerations by the dimensionality d to be ascribed to the field functions: in our approach it is determined by the tensor character of the field i.e. in units of length $d = -2, -1, 0$ for tensor ($g_{\mu\nu}$), vector (A_μ) and scalar fields respectively. With these assignements the gravitational action turns out to be perfectly dilatational invariant and contains only dimensionless parameters. In particular the Newton constant K does not appear in the Lagrangian, it is rather related to the flat limit of the theory. K can be reintroduced in the explicit expression of $\mathcal{L}$ via a suitable redefinition of the fields i.e. $(g_{\mu\nu})_{curv} \to \ell^{-2}(g_{\mu\nu})_{conv.}$, $A_\mu \to A_\mu$, $(\phi)_{curv} \to \ell(\phi)_{conv}$, $\ell^2 = 4\pi K$.

among the parameters of the theory. The possible meaning of those conditions is not yet understood (by us at least) and also the choice of a given Lagrangian is performed on a more or less empirical basis.

In the search of a guiding criterion to determine the form of the coupling among the fields one is naturally led to consider supersymmetric theories and the question arises whether classical solutions of the above-mentioned type persist in these models. The aim of this note is to offer a contribution to this line of research.

A first comparison between the constraints on the coupling constants required by the existence of meron solutions and the structure of supersymmetric Lagrangians including gravity and local internal symmetries is not particularly encouraging.

Let us consider as starting example the case of $N = 3$ extended supergravity with local $SO(3)$ gauge invariance[3]. The particle content of such a theory is the graviton, a triplet of spin 3/2, a triplet of spin 1 and a spin 1/2 singlet but for the sake of deriving classical solutions only the boson fields have to be included (we assume that the fermion fields have vanishing classical values). The corresponding Lagrangian density is(*)

$$\mathcal{L} = -\frac{1}{4}\sqrt{g}\,R - \frac{3}{8}\sqrt{g}\,\lambda^2 - \frac{\sqrt{g}}{4e^2}\left(\Sigma_\alpha\, F^\alpha_{\mu\nu} F^\alpha_{\mu'\nu'}\, g^{\mu\mu'} g^{\nu\nu'}\right). \tag{2}$$

Supersymmetry requires the cosmological term λ^2 to be related to the Yang-Mills charge e :

(*) Following our previous discussion the dimensions of the fields are such that the Lagrangian(2) has dimension -4 (λ^2 is dimensionless) and the Newton constant has not to be included.

$$\lambda^2 = -4e^2 \tag{3}$$

On the other hand, as discussed in Ref. (1), the Lagrangian of eq. (2) admits a general class of solutions of the form

$$\begin{aligned} g_{\mu\nu} &= \delta_{\mu\nu}\, h^2(x) \\ A_\mu &= i\,\sigma_{\mu\nu}\partial_\nu \ln h(x) \end{aligned} \tag{4}$$

provided $h(x)$ is solution of

$$\Box h + \tfrac{1}{2}\lambda^2 h^3 = 0 \tag{5}$$

and

$$\lambda^2 = e^2. \tag{6}$$

This is somehow disappointing: both approaches, the one based on general symmetry properties and the other dealing with explicit solutions, require the presence of a cosmological constant and lead to a simple constraint between the two parameters λ and e but for the numerical relation (including the sign), which turns out to be different.

Anyway both results require an enormous cosmological constant(*), whose physical significance is far from being clarified. A possible way out of this (well known) difficulty is to replace the λ^2 cosmological term by a function of a new scalar field $\phi(x)$. From the symmetry point of view this leads naturally to consider the $N = 4$ supergravity multiplet, which contains one spin 2 graviton,

(*) The present limit is $|\lambda^2| < 10^{-120}$.

four Majorana spin 3/2 gravitinos,six spin 1 mesons of opposite parities, four spin 1/2 Majorana neutrinos and a spin 0 parity doublet. This global internal symmetry can be further gauged to a local SU(2) x SU(2) invariance[(4)]. Since only boson fields of natural parity are assumed to exhibit non-vanishing classical configurations, the part of the supergravity Lagrangian of Ref.(4) relevant to our discussion is of the (non-polynomial) form(*)

$$\mathcal{L} = -\tfrac{1}{4}\sqrt{g}\,R - \tfrac{3}{8}\sqrt{g}\,\lambda^2\varphi^2 - \frac{\xi}{2\varphi^2}\sqrt{g}\, g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \frac{\sqrt{g}}{4e^2\varphi^2}\left(\Sigma_\alpha\, F^\alpha_{\mu\nu}\, F^\alpha_{\mu'\nu'}\, g^{\mu\mu'} g^{\nu\nu'}\right), \tag{7}$$

where

$$\lambda^2 = -\frac{e^2}{3}, \quad \xi = 1. \tag{8}$$

Again we can compare this result with what follows from a study of classical solutions for the Lagrangian (7). One finds in this case that when λ and e are constrained by the numerical relation (6),

$$\lambda^2 = e^2 \tag{6}$$

the following meronic solution exists:

(*) The expression (7) of the effective part of the $N = 4$ supergravity Lagrangian reduces to the one of Ref. (4) after redefining the scalar field $\varphi(x)$ i.e. $\varphi(x) = e^{\phi(x)}$. A single gauge coupling constant e has also been introduced to avoid parity violating effects in the next approximation. In the notation of ref. (4) $e_A = e$, $e_B = 0$.

$$
\begin{aligned}
g_{\mu\nu} &= \frac{1}{c^2}\,\frac{\delta_{\mu\nu}}{x^2} \\
A_\mu &= -i\,\frac{\sigma_{\mu\nu}x_\nu}{x^2} \\
\varphi &= c\sqrt{\frac{2}{e^2}}
\end{aligned}
\tag{9}
$$

c is an arbitrary (dimensionless) constant, which is not determined by the theory.

Thus a discrepancy persists between supergravity (i.e. eq. (8)) and standard meron solutions (i.e. eq. (6)). In this paper we want to point out that owing to the appearance of the scalar field a new set of meron solutions exist for Lagrangian models of the structure given in eq. (7).

In particular one can easily check that when the "supersymmetric" relation (8) holds the following solution exists:

$$
\begin{aligned}
g_{\mu\nu} &= \frac{1}{c^2}\,\delta_{\mu\nu} \\
A_\mu &= -i\,\frac{\sigma_{\mu\nu}x_\nu}{x^2} \\
\varphi &= c\sqrt{\frac{2}{e^2}}\,\frac{1}{r}
\end{aligned}
\tag{10}
$$

where c is a free constant, which now has the dimension of a length in order to match the dimensionality of the fields.

We proceed now to describe briefly a new class of solutions of which eq. (10) represents a particularly beautiful example(*). Their origin can be understood from the structure of the Lagrangian in eq. (7) which exhibits a simple covariance property under the following rescaling of the fields:

(*) A more detailed discussion of these solutions can be found in Ref. 5 and we mention them here for sake of completeness.

$$
g_{\mu\nu} \to \alpha^2 g_{\mu\nu}, \quad A_\mu \to A_\mu, \quad \varphi \to \alpha^{-1}\varphi
\tag{11}
$$

namely

$$
\mathcal{L} \to \alpha^2 \mathcal{L}
\tag{12}
$$

The Euler-Lagrange equations of motion remain unchanged under transformation (11).(*)

A first consequence of this fact is represented by the presence in the solution (10) of the arbitrary constant c. Secondly, one is led to guess a general family of solutions whose symmetry group contains a suitable combination of dilatation and of the transformation (11).

More specifically we propose the ansatz

$$
\begin{aligned}
g_{\mu\nu} &= \frac{1}{c^2}\,\frac{\delta_{\mu\nu}}{x^2}\,r^{2\gamma} = g^{(0)}_{\mu\nu}\,r^{2\gamma} \\
A_\mu &= -i\,\sigma_{\mu\nu}x_\nu/x^2 \\
\varphi &= c\,a\,r^{-\gamma} = \varphi^{(0)}\,r^{-\gamma}
\end{aligned}
\tag{13}
$$

"a" is a normalization constant which is fixed by the theory, while c remains arbitrary.

It is then easy to check that the space-time dependence of the equations of motion is matched by the above choice of powers, so

(*) It is not hard to verify that the covariance property eq. 12 of the full supergravity Lagrangian persists also when all the fields of the $N = 4$ supermultiplet are included. In particular if we denote the $3/2,\ 1^+,\ 1/2,\ 0^-$ fields by ψ_μ, B_μ, χ, B the transformation properties extending eq. 11 are

$$
\psi_\mu \to \alpha^{1/2}\psi_\mu, \quad B_\mu \to B_\mu, \quad \chi \to \alpha^{-1/2}\chi, \quad B \to \alpha^2 B .
$$

that the differential equations reduce to the following algebraic relations:

$$\frac{3}{2} a^2\lambda^2 = -\xi\gamma^2 - 3(\gamma^2 - 1) \, , \qquad (14)$$
$$2/e^2 a^2 = \xi\gamma^2 + 1 - \gamma^2 \, .$$

These formulae reproduce for $\gamma = 0$ the meron relation $\lambda^2 = e^2$ (independently of ξ) while for $\gamma = 2$ we find $\lambda^2 = -\frac{e^2}{3}\xi^2$ in agreement with supergravity.

As a final step let us discuss the form of the above solutions in Minkowski space. This is usually obtained by performing a conformal transformation, which maps origin and infinity into $\pm a_\mu$, $a_\mu = (0,0,0,1)$ and then performing the Wick rotation. If we do so the "old" parts of the solution $g_{\mu\nu}^{(0)}, A_\mu, \varphi^{(0)}$ take on the customary forms. Thus one has to deal with the new factors $r^{2\gamma}, r^{-\gamma}$. Since the previous operations amount to having

$$r \to e^{-i\rho} \qquad (15)$$

with ($t_\pm = t \pm \sqrt{\vec{x}^2}$)

$$\rho = \operatorname{arctg}(t_+) + \operatorname{arctg}(t_-) \qquad (16)$$

we are finally led to complex Minkowski solutions of the form

$$g_{\mu\nu} = \frac{\delta_{\mu\nu}}{c^2} \frac{1}{(1+t_+^2)(1+t_-^2)} e^{-2i\gamma\rho} \, ,$$

$$A_\mu = -i\,\sigma_{\mu\nu} S^\nu \, ; \qquad (17)$$

$$S_\nu = \left(\frac{t_+}{1+t_+^2} y_\nu^+ + \frac{t_-}{1+t_-^2} y_\nu^-\right) ; \quad y_\mu^\pm = \left(1, \pm\frac{\vec{x}}{|\vec{x}|}\right),$$

$$\varphi = c\,a\,e^{i\gamma\rho} \, .$$

This situation is somewhat disappointing and may create some problems for the use of our solutions in the framework of physical Minkowski theories.

A possible way out is to notice that all quantities invariant under the transformations (11) are Minkowski real and it is possible that only those quantities represent physical observables.

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