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ABSTRACT

We describe two types of supersymmetric σ models: with field values in supercoset space and with superfields. The notion of Riemannian symmetric pair $(H,\frac{G}{H})$ is generalized to supergroups. Using the supercoset approach the superconformal-invariant model of composite U(n) Yang-Mills fields is introduced. In the framework of the superfield approach we present with some details two versions of the composite N = 1 supersymmetric Yang-Mills theory in four dimensions with U(n) and U(m) × U(n) local invariance. We argue that especially the superfield σ models can be used for the description of pre-QCD supersymmetric dynamics.

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I. INTRODUCTION

Let us recall the most interesting properties of bosonic σ models [1]:

a) The interaction is introduced in the geometric way by means of the mapping

$$\varphi^A(x) : x \in \mathcal{S} \to \varphi^A \in \mathcal{M} , \tag{1.1}$$

where $\mathcal{M}$ is a <u>non-linear</u> Riemannian manifold, with possible additional (complex or quaternionic) structure [2].

b) If we put $\mathcal{M} = \frac{G}{H}$, the σ models describe G symmetry <u>broken spontaneously</u> to its subgroup H. By coupling the $\frac{G}{H}$ σ field to the massless G-gauge fields, the $\frac{G}{H}$ valued components of the gauge potentials become massive. One can say that the σ fields in the coupled system with gauge fields play the role of <u>geometric Higgs fields</u> [3].

c) If the topology of $\mathcal{M}$ is non-trivial, the configurations of σ fields can be classified by the characteristic classes of $\mathcal{M}$. In particular, one obtains the instanton solutions for Euclidean two-dimensional models ($\mathcal{S} = R^2$ or $\mathcal{S} = R^2 \cup 0 = S^2$) if $\mathcal{M}$ is a complex manifold with Kähler structure [1-5] and one gets the instanton solutions in four Euclidean dimensions ($\mathcal{S} = R^4$ or $\mathcal{S} = R^4 \cup 0 = S^4$) if $\mathcal{M}$ is endowed with quaternionic structure [1,2,5].

d) In σ models one can introduce minimal gauge coupling with <u>composite</u> gauge field. The formula for the composite gauge fields can be obtained from the coupled gauge + σ field system if we abandon the free kinematic gauge term. Denoting the Lagrangian density for the σ model by $\mathcal{L}_\sigma(\varphi,\partial_\mu\varphi)$, we can have:

i) the theory with <u>elementary</u> gauge fields:

$$\mathcal{L} = \mathcal{L}_\sigma(\varphi,\nabla_\mu\varphi) + \frac{1}{4g_0^2}\mathcal{L}_{YM} \tag{1.2}$$

if $g_0 \neq 0$. In such a system one can observe the <u>geometric Higgs mechanism</u> (see b);

ii) coupled system with <u>composite</u> gauge fields is obtained by putting in (1.2) $g_0 \to \infty$. The fields A_μ ($\nabla_\mu = \partial_\mu - A_\mu$) are determined from the <u>algebraic</u> equation:

$$\varphi^A \frac{\partial}{\partial(\nabla_\mu\varphi^A)} \mathcal{L}_\sigma(\varphi,\nabla_\mu\varphi) = 0 . \tag{1.3}$$

The gauge potentials play the role of Lagrange multipliers.

In the description of composite $U(n)$ gauge fields there are distinguished the complex Grassmanian σ models, for which

$$\mathcal{M} = G_{n+m,n}(C) = \frac{U(n+m)}{U(m) \otimes U(n)} = \frac{C^{(n+m)\cdot n}}{GL(n;c)} \quad . \tag{1.4}$$

Due to the Narasimhan and Ramanan(NR)theorem[6] any connection (gauge potential) on differentiable principle U(m) bundle over compact base $\mathcal{S}$ can be defined (in a given co-ordinate map on $\mathcal{S}$) as

$$A_\mu^{ij} = \bar{\phi}^{iJ} \partial_\mu \phi^{Jj} \qquad \begin{array}{l} i,j - U(n) \text{ indices} \\ J,K = 1\ldots n+m \ (m \text{ sufficiently large}) \end{array} \tag{1.5}$$

where $\phi^{Ji} \in S_{n+m,n}(C)$ (complex Stiefel manifold), i.e.

$$S_{n+m,n}(C): \quad \bar{\Phi}^{iJ}\Phi^{Jj} = \delta^{ij} \qquad \Phi^{Ji} \text{ complex} \quad . \tag{1.6}$$

It can be shown that $S_{n+m,n}(C)$ describes the collection of n-dimensional subspaces in C^{n+m} containing the origin and represents a natural generalization of the complex projective plane CP(n+m-1).

One can write the following two U(n) invariant actions for $G_{n+m,n}(C)$ σ fields:

i) Bilinear in σ-field derivative:

$$\mathcal{L}_\sigma^{(2)} = \frac{1}{2}(\overline{\nabla_\mu\Phi})^{iJ}(\nabla_\mu\Phi)^{Ji} + \lambda^{ij}(\bar{\Phi}^{iJ}\Phi^{Jj} - \delta^{ij}) \ , \tag{1.7}$$

where

$$\nabla_\mu^{ij} = \partial_\mu\delta^{ij} - A_\mu^{ij} = \partial_\mu\delta^{ij} - \bar{\Phi}^{iJ}\partial_\mu\Phi^{Jj} \quad . \tag{1.7a}$$

The formula (1.7) describes the "ordinary" $G_{n+m,n}(C)$ σ model, which was considered by several authors in two dimensions (see, e.g. [7,8]) because the action (1.7) is conformal invariant only if D = 2.

ii) Four-linear in σ field derivative:

$$\mathcal{L}_\sigma^{(4)} = -\frac{1}{4}F_{\mu\nu}^{ij}F^{\mu\nu\, ij} + \lambda^{ij}(\bar{\Phi}^{iJ}\Phi^{Jj} - \delta^{ij}) \ , \tag{1.8}$$

where

$$F_{\mu\nu}^{ij} = [\nabla_\mu, \nabla_\nu]^{ij} = (\nabla_{[\mu}\Phi)^{iJ}(\nabla_{\nu]}\Phi)^{Jj} \quad . \tag{1.8a}$$

The formula (1.8) describes the generalized $G_{n+m,n}(C)$ σ model equivalent to the U(n) Yang-Mills theory with composite gauge fields. It is interesting to observe that because the canonical dimensionality of σ fields in any number of dimensions is equal to zero, the four-linear Lagrangian density (1.8) is conformal invariant only if $D = 4$.

The main aim of our lecture is to show how to introduce the supersymmetric generalization of the Lagrangian (1.8). [4] Using the superfield formalism one can show that the "bosonic" σ fields ψ^{Ji} should be supplemented by a supersymmetric partner ψ^{Ji}, which plays in supersymmetric composite SU(3) Yang-Mills theory the role of the quark-field variable. In such a way,by introducing supersymmetric dynamics with composite gauge fields,one can resolve the long-standing difficulty with supersymmetric generalization of QCD - that gluons and quarks cannot belong to the same linear supermultiplet. Using the technique of σ models - a natural framework for the composite gauge fields - one introduces in supersymmetry QCD only one elementary quark supermultiplet containing quarks and "quarkinos"; the gluon and gluino fields are composite.

Let us recall [2] that there are in principle three ways of supersymmetric generalization of the mapping (1.1);

a) supercoset approach: one supersymmetrizes the manifold $\mathcal{M}$ of field values; in particular by replacing the coset space of a Lie group G by a supercoset space of a Lie supergroup $\tilde{G}$;

b) Superfield approach: one supersymmetrizes the co-ordinate manifold $\mathcal{S}$, replacing it by a superspace (usually a flat superspace);

c) Fully supersymmetrized approach: both $\mathcal{S}$ and $\mathcal{M}$ are becoming the supermanifolds.

In Sec.II we shall discuss the σ fields with their values in supercoset manifold. For physical reasons we shall consider the models without spin-statistics difficulty which can be avoided only if the supergroup $\tilde{G}$ unifies the geometric and internal bosonic symmetries [5]. As the basic geometric structure for these models we introduce the supersymmetric generalization of the Riemannian pair $(H, \frac{G}{H})$(see. e.g. [13]) to the case of a supergroup $\tilde{G}$. It appears that in the supercoset approach one can also introduce supersymmetric composite Yang-Mills theory, with fundamental variables (quarks?)

described by supertwistors [27,28]. The main part of our presentation is contained however in Sec.III, where we consider the σ models described by superfields. After a brief resume of the theory of supersymmetric σ models with bilinear Lagrangians (see e.g. [14-22]) we discuss more explicitly the above-mentioned supersymmetric version of the generalized four-linear σ model, which describes subcanonical pre-QCD dynamics. In Sec.IV we discuss physical aspects of the presented scheme and recall major unsolved problems.

II. SUPERSYMMETRIC σ MODELS: SUPERCOSET APPROACH

a) "Physical" supergroups

In order to avoid the violation of spin-statistics theorem for the fields forming the linear representation of a supergroup G, one should consider the "physical" supergroups with the following bosonic sectors:

spin groups (space-time symmetry)	$\times$	internal symmetry (flavour, colour generations) .	(2.1)

In four dimensions, there are only four possible spin groups in Minkowski and four in Euclidean space:

i) Minkowski space-time

1) Lorentz spin $\quad SL(2,C) \simeq Sp(2;C) \simeq O(3,1)$
2) Anti-de-Sitter spin $\quad Sp(4;R) \simeq O(3,2)$
3) de-Sitter spin $\quad Sp(1,1;H) \simeq O(4,1)$
4) Conformal spin $\quad SU(2,2) \simeq O(4,2)$

ii) Euclidean space ($d = 4$)

1) Euclidean spin $\quad SU(2) \times SU(2) \simeq O(4)$
2) Anti-de-Sitter spin $\quad Sp(1,1;H) \simeq O(4,1)$
3) de-Sitter spin $\quad Sp(2;H) \simeq O(5)$
4) Conformal spin $\quad SL(2;H) \simeq O(5,1)$.

There are unique formulae for the superextensions of the above listed spin groups:

i) Minkowski space — Internal symmetry

1) $Sp(2;C) \to OSp(N;2;C)$ $\quad O(N;C)$
2) $Sp(4;R) \to OSp(N;4)$ $\quad O(N)$
3) $Sp(1,1;H) \to UU_\alpha(1,1;N;H)$ $\quad O(N;H)$
4) $SU(2,2) \to SU(2,2;N)$ $\quad U(N)$

ii) Euclidean space ($d = 4$)

1) $SU(2) \times SU(2) \to SU(2;N) \times SU(2;N)$ $\quad U(N) \times U(N)$
2) $Sp(1,1;H) \to UU_\alpha(1,1;N;H)$ $\quad O(N;H)$
3) $Sp(2;H) \to UU_\alpha(2;N;H)$ $\quad O(N;H)$
4) $SL(2;H) \to SL(2;N;H)$ $\quad GL(N;H)$.

Because $O(N;C)$, $O(N;H)$ and $G:(N;H)$ are non-compact, the only good candidates for the description of internal symmetries are

- graded de-Sitter groups $\quad OSp(N;4)$
- graded conformal groups $\quad SU(2,2;N)$
- graded Euclidean group $\quad SU(2;N) \times SU(2;N)$

We would like to mention here that the classification of matrix supergroups as describing linear frames in real and complex superspaces can be found in [23] (see also [24]); for the supergroups of isometries in quaternionic superspaces see [25].

b) Supersymmetric generalization of Riemann symmetric pair $(H,\frac{G}{H})$

The symmetric space formed from a connected Lie group H is a triple (G,H,n), where

- H is a closed subgroup of G,
- $\hat{n}$ is an involutive automorphism of G leaving the subgroup H invariant.

Introducing the decomposition of the Lie algebra g of G as follows:

$$g = h \oplus k \quad , \tag{2.2}$$

the involutive automorphism $\hat{\eta}$ leaves the generators h invariant, and changes the sign of those belonging to k. The algebra of G is endowed with the following Z_2 grading:

$$[h,h]\subset h \qquad [k,k]\subset h$$

$$[h,k]\subset k \quad . \tag{2.3}$$

Denoting $\hat{\eta}(g) = \eta g \eta^{-1}$ one can also write

$$g + \hat{\eta}(g)\subset h \qquad g - \hat{\eta}(g)\subset k \quad . \tag{2.4}$$

An example of the involution η *describing the Riemannian symmetric* pair $(U(n) \times U(m), G_{n+m,n}(C))$ i.e. defining the complex Grassmanian is the metric

$$I_{n,m} = \begin{array}{cc} & \begin{array}{cc} n & m \end{array} \\ & \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}\right)\begin{array}{c} n \\ m \end{array} \end{array} \quad . \tag{2.5}$$

Choosing however for even $n+m = 2N$ the so-called involution of the second kind $(\eta^2 = -1)$:

$$J_N = \left(\begin{array}{c|c} 0 & I_N \\ \hline -I_N & 0 \end{array}\right) \tag{2.6}$$

one obtains the Riemann symmetric pair $(Sp(N), \frac{U(2N)}{Sp(N)})$, important for D = 5 supersymmetry.

If we deal with a supergroup $\widetilde{G}$, the bosonic sector is the product of <u>two</u> bosonic supergroups $G_1 \otimes G_2$. Let us write $G_1 \otimes G_2$ as a direct product of two Riemannian symmetric pairs $(H_1, \frac{G_1}{H_1}) \otimes (H_2, \frac{G_2}{H_2})$ (respective involution: $\eta = \eta_1 \otimes \eta_2$). For some choices of $\hat{\eta}_1, \hat{\eta}_2$ the superalgebra G shows the following Z_4-graded structure:

$$\begin{array}{cccc} L_0 & L_1 & L_2 & L_3 \\ h = h_1 \oplus h_2 & f_+ & k = k_1 \oplus k_2 & f_- \end{array} \quad , \tag{2.7}$$

where $L_0 \oplus L_2$ is a bosonic sector, $L_1 \oplus L_3$ fermionic and [6)]

$$\left[L_i , L_k \right]_{\pm} \subset L_{i+k} \qquad i,j,i+j\subset Z_4 \quad . \tag{2.8}$$

The decomposition of the fermionic sector $f = (f_+,f_-)$ is induced by the choice of the involutions η_1, η_2 in the bosonic sector $g = g_1 \oplus g_2$. We shall present the following two examples obtained for the "physical" supergroups $OSp(2N;4)$ and $SU(2,2;2N)$ when the Z_4-grading occurs:

i) Supercoset space $\left(SL(2,C) \times U(N), \frac{OSp(2N;4)}{SL(2,C) \times U(N)}\right)$.

The involution η is the following $(4+n+m) \times (4n+m)$ matrix:

$$\eta = \begin{array}{cc} & \begin{array}{cc} 4 & 2N \end{array} \\ & \left(\begin{array}{c|c} \gamma_5 & 0 \\ \hline 0 & I_N \end{array}\right)\begin{array}{c} 4 \\ 2N \end{array} \end{array} \quad , \tag{2.9}$$

where we use $(\gamma_\mu,\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu])$ as $sp(4)$ generators. Because

$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \qquad \gamma_5\sigma_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \quad ,$$

the involution $\eta_1 = \gamma_5$ describes the Riemannian symmetric pair $\left(SL(2,C), \frac{Sp(4)}{SL(2,C)}\right)$ and $\eta_2 = I_N$ describes $\left(U(N), \frac{O(2N)}{U(N)}\right)$. The fermionic off-diagonal matrix generators

$$q_\pm \quad f_\pm = \left(\begin{array}{c|c} 0 & q_\pm \\ \hline -q_\pm C & 0 \end{array}\right) \tag{2.10}$$

satisfy the conditions $f_\pm = \pm(\gamma_5\ I_N)f_\pm$.

ii) Supercoset space $\left(Sp(1,1;H) \times Sp(n), \frac{SU(2,2;2N)}{Sp(1,1;H) \times Sp(N)}\right)$.

The involution η has a form

$$\eta = \begin{array}{cc} & \begin{array}{cc} 4 & N \end{array} \\ & \left(\begin{array}{c|c} J_4 & 0 \\ \hline 0 & J_N \end{array}\right)\begin{array}{c} 4 \\ N \end{array} \end{array} \tag{2.11}$$

i.e. it is a product of two second kind involutions in the bosonic sector $SU(2,2) \times U(2N)$.

Using the four-linear grading (2.7) ("supersymmetric Riemann quadruple") one can introduce a new class of the supersymmetric σ models with the field values in the part of fermionic sector of a supergroup. These models generalize the purely fermionic σ models, considered in [26,27], with the following choice of the manifold $\mathcal{M}$:

real fermionic projective plane:	$\frac{OSp(N;4)}{O(N) \times Sp(4)}$	(real fermionic variables)

Complex fermionic projective plane:	$\dfrac{SU(2,2;N)}{SU(2,2)\times U(N)}$	(complex fermionic variables)

For these two choices, by introducing Cartan one-forms with fermionic parameters and considering invariant products of <u>four</u> one-forms, one can also obtain the Lagrangians four-linear in σ-field derivative.

Leaving the general discussion of possible action forms to a future publication, we shall consider here only as an example a purely fermionic four-linear σ model representing the "fermionic copy" of the model (1.8).

c) <u>Composite Yang-Mills theory in terms of purely fermionic fundamental fields</u>

In order to satisfy the spin-statistics theorem we choose as the fundamental co-ordinates the complex supertwistor fields [27,28], transforming linearly as the fundamental representation of graded conformal supergroups SU(2,2;n). The invariant SU(2,2;n) invariant inner supertwistor product has a form

$$\psi^*_\alpha C_{\alpha\beta}\psi_\beta + u^*_i u_i = \mathrm{inv} \qquad \begin{array}{l}\alpha = 1\cdots 4\\ i = 1\cdots n\end{array} \quad , \tag{2.12}$$

where $\psi^*_\alpha C_{\alpha\beta}\psi_\beta$ denotes U(2,2) invariant twistor norm; ψ_α anticommute and U commute. We choose to parametrize our purely fermionic U(n) invariant σ-fields values by means of n supertwistors with constraints and additional equivalence relations. First, we observe that n supertwistors describe the supercoset space $\dfrac{SU(2,2'n)}{SU(2,2)}$ if they are restricted by the following constraints equation ($\bar\psi = \psi^* C$; compare also with (1.6)):

$$\bar\psi_{\alpha i}\psi_{\alpha j} + u^*_{ik}u_{kj} = \delta_{ij} \quad . \tag{2.13}$$

We restrict the co-ordinates $(\psi_{\alpha i}, U_{ik})$ to the purely fermionic manifold

$$F\,G_{4,n}(C) = \frac{SU(2,2;n)}{SU(2,2)\times U(n)} \tag{2.14}$$

if we introduce the following equivalence classes:

$$(\psi_{\alpha j}, U_{ij}) \sim (\psi_{\alpha j}U_{jk},\ U_{ij}U_{jk}) \quad , \tag{2.15}$$

where $U \in U(n)$. If we consider $(\psi_{\alpha j}, U_{ij})$ as the σ fields with the values in $FG_{4,n}(C)$, the choice of the representative of the equivalence class (2.15) corresponds to the freedom of choice of the U(n) gauge parameters in the theory. In particular, one can choose a special gauge in which $U_{ij} = \delta_{ij}$ and there remain only fermionic degrees of freedom.

One can write down the following two Lagrangians:

i) bilinear (compare with (1.7))

$$\tilde{\mathcal{L}}^{(2)}_\sigma = \frac{1}{2}(\overline{\tilde\nabla_\mu\psi})\tilde\nabla^\mu\psi + \frac{1}{2}(\overline{\tilde\nabla_\mu u})\tilde\nabla^\mu u + \lambda(\bar\psi\psi + \bar u u - 1) , \tag{2.16}$$

where

$$\tilde\nabla_\mu^{ij} = \partial_\mu\delta^{ij} - \tilde A_\mu^{ij} \qquad \tilde A_\mu^{ij} = \bar\psi^i_\alpha\partial_\mu\psi^j_\alpha + \bar u^i\partial_\mu u^j \tag{2.17}$$

generalizing that considered in [27,29]. Such a model is invariant under the "external" global SU(2,2'n) group, local U(m) group and in D = 2 under conformal transformation in space-time.

ii) Four-linear (compare with (1.8))

$$\tilde{\mathcal{L}}^{(4)}_\sigma = -\frac{1}{4}\tilde F_{\mu\nu}\tilde F^{\mu\nu} + \lambda(\bar\psi\psi + \bar u u - 1) , \tag{2.18}$$

where

$$\tilde F_{\mu\nu}^{ij} = [\tilde\nabla_\mu, \tilde\nabla_\nu]^{ij} = (\overline{\tilde\nabla_{[\mu}\psi_\alpha})^i(\tilde\nabla_{\nu]}\psi_\alpha)^j + \overline{\tilde\nabla_{[\mu}u}^i\nabla_{\nu]}u^j . \tag{2.19}$$

This Lagrangian has SU(2,2;N) global and U(N) local invariances; besides it is invariant under conformal space-time transformations for D = 4.

If we introduce the composite connections by replacing the coset space by the supercoset space, we add <u>dimensionless</u> fermionic σ fields, which extend the NR formula (1.5) by a completely analogous fermionic term (see (2.17)). Such a property we consider as disadvantageous from the point of view of the physical applications: in conventional field theory the fermions have the dimensionality higher by $\frac{1}{2}$ "mass units", and adding the internal symmetry indices does not change the dimensionality. In the supercoset approach unfortunately, spin and internal degrees of freedom have to be treated due to the spin statistics theorem in a symmetric way. One can avoid such a symmetric description by considering superfields, which imply the geometric introduction of spin via extension of space-time to the superspace manifold.

III. SUPERSYMMETRIC GENERALIZED σ MODELS

a) General and historical remarks

In the superfield approach one describes the spin degrees of freedom by additional Grassmanian co-ordinates $\theta = \{\theta_\alpha\}$

$$x \in \mathcal{S} \longrightarrow (x,\theta) \in \tilde{\mathcal{S}} \; ; \qquad \mathcal{M} \text{ unchanged} \qquad . \tag{3.1}$$

The addition of Grassman variables can be treated also as an extension of the "internal" manifold $\mathcal{M}$ by the exterior product of the fundamental spinor representation T_α in

$$\mathcal{S} \text{ unchanged}; \quad \mathcal{M} \longrightarrow \mathcal{M} \otimes \bigoplus_{i=0}^{D} \wedge T_\alpha \quad . \tag{3.2}$$

Because the σ superfield can be described as the set of antisymmetric spinor fields

$$\varphi^A(x,\theta) \sim \bigoplus_{i=0}^{D} \varphi^A_{\alpha_1 \cdots \alpha_i} \quad , \tag{3.3}$$

the formulation of the σ model using (3.2) leads to the equivalent "component formulation" of the superfield σ model.

In general we can put into superspace, besides the spin degrees of freedom, also some internal symmetries. Because the σ models are superconformal invariant one can split the internal symmetry group into the product $U(N) \times U(n+m)$, where the $U(N)$ internal symmetry is described by the extended superspace. Such a scheme would describe $N > 1$ supersymmetry. It is known that by assuming simple $N = 1$ supersymmetry in higher dimension, one can obtain the $N > 1$ supersymmetric theory (SST) by dimensional reduction; in particular

$N = 1$ SST in $D = 4 \sim N = 2$ SST in $D = 2$ [20] ;

$N = 1$ SST in $D = 6 \sim N = 2$ SST in $D = 4$ [19] .

In this lecture we shall discuss in some detail only the model with $N = 1$ supersymmetry in $D = 4$.

First superfield σ models [13,14] described $N = 1$ supersymmetry in $D = 2$, by means of the superfield

$$\varphi^A(x,\theta) = \varphi^A(x) + \theta^\alpha \psi^A_\alpha + \bar{\theta}\theta f^A \qquad \begin{matrix}\theta_\alpha \text{ real} \\ \alpha = 1,2\end{matrix} \quad , \tag{3.4}$$

where $\varphi^A \in \frac{O(3)}{O(2)} = S^2$ (supersymmetric $O(3)$ σ model); such a model can be formulated as a supersymmetric $CP(1)$ σ model if we use $N = 2$ superfields (complex superspace $(x,\bar{\theta},\theta)$) and impose the chirality conditions [16]. The formulation with $N = 2$ superfields in two dimensions has been generalized to any complex Kähler manifold-valued σ superfields $Z_i(x,\theta,\bar{\theta})$ by Zumino [20], who also found compact formula for the superfield action (see also [21]). In his formulation Zumino uses unconstrained σ superfields, with the parameters Z_i describing independent co-ordinate charts on $\mathcal{M}$. Using $N = 1$ unconstrained superfields, Freedman and Townsend have shown recently [30,31] that any bosonic σ model with field values in a Riemannian manifold $\mathcal{M}$ can be supersymmetrized for $D = 2$.

b) Composite $U(n)$ supersymmetric Yang-Mills theory in $D = 4$ as generalized σ model [8)]

The supersymmetric Lagrangian for $U(n)$ YM theory was first proposed by Ferrara, Zumino [32] and Salam, Strathdee [33]. It has the following form:

$$\mathcal{L} = \frac{1}{8} \operatorname{tr} \bar{D}_- D_+ \left[\bar{\Psi}_{--} \Psi_{++} \right] + \text{h.c.} \tag{3.5}$$

with the chiral spinor superfields $\Psi_{\pm\pm}$ defined as follows:

$$\Psi_{\alpha\pm\pm} = -\frac{i}{2\sqrt{2}} \bar{D}_\pm D_\mp \left[e^{\mp 2V} D_{\alpha\pm} \, e^{\pm 2V} \right] \; , \tag{3.6}$$

where $D^\pm \equiv D^\pm_\alpha = \frac{1}{2}(1 + i\gamma_5)\, D_\alpha$, $D_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{1}{2}(\not\partial\theta)_\alpha$ and $V \equiv V^{ij}$ is the real vector superfield with $U(n)$ adjoint indices:

$$V^{ij} = A^{ij} + \bar\theta\xi^{ij} + \frac{1}{4}(\bar\theta\theta)C^{ij} + \frac{1}{4}(\bar\theta\gamma_5\theta)\tilde{C}^{ij} + \\ + \frac{1}{4}\bar\theta i\gamma_\mu\gamma_5\theta V^{\mu ij} + \frac{1}{4}(\bar\theta\theta)\bar\theta\chi^{ij} + \frac{1}{32}(\bar\theta\theta)^2 D^{ij} \; . \tag{3.7}$$

If the superfield V transforms under the U(n) generalized gauge transformations [9]

$$(e^{-2V})' = e^{i\Lambda_+} e^{-2V} e^{-i\Lambda_-} \; , \tag{3.8}$$

where $\Lambda_\pm = \{\Lambda_\pm^{ij}\}$ are two chiral $n \times n$ matrix superfields, we obtain

$$\Psi'_{\alpha\pm\pm} = e^{i\Lambda_\pm}\Psi_{\alpha\pm\pm} e^{-i\Lambda_\pm} \tag{3.9}$$

and the Lagrangian (3.5) is invariant.

In order to obtain the <u>composite</u> gauge supermultiplet (3.7) we postulate

$$e^{-2V} = \Phi_+\Phi_- \; , \tag{3.10}$$

where $\Phi_+ = \{\Phi_+^{iJ}\}$ forms $n \times N$ rectangular matrix of left-chiral superfields ($N > n$) $\Phi_- = \{\Phi^{Ji}\}$ is $N \times n$ matrix of right-chiral superfields ($\varphi_- = (\varphi_+)^+ = \varphi$; $F_- = (F_+)^+ = F$)

$$\Phi_\pm = \varphi_\pm + \bar\theta\psi_\pm + \frac{1}{4}\bar\theta\theta F_\pm + \frac{i}{4}\bar\theta\gamma_5\theta F_\pm \mp \\ \mp \frac{1}{4}\bar\theta\gamma_\mu\gamma_5\theta\cdot\partial^\mu\varphi_\pm - \frac{i}{4}(\bar\theta\theta)\bar\theta\,\partial\!\!\!/\,\psi_\pm - \frac{1}{32}(\bar\theta\theta)^2\Box\varphi_\pm \tag{3.11}$$

transforming under the U(n) generalized gauge transformation as follows:

$$\Phi'_+ = e^{i\Lambda_+}\Phi_+ \qquad \Phi'_- = \Phi_- e^{-i\Lambda_-} \; . \tag{3.12}$$

It can be shown that one gets from (3.10) the vector superfield V in the Wess-Zumino gauge

$$V = \frac{1}{4}\bar\theta i\gamma_\mu\gamma_5\theta V^\mu + \frac{1}{4}(\bar\theta\theta)\bar\theta\chi + \frac{1}{32}(\bar\theta\theta)^2 D \; , \tag{3.13}$$

if we impose on the components of the superfield (3.11) the following "σ-model-like" conditions:

$$\varphi^{+iJ}\varphi^{Jj} = \delta^{ij} \quad \varphi^{+iJ}\psi_-^{Jk} = 0 \quad \varphi^{+iJ}F^{Jj} = 0 \; . \tag{3.14}$$

The components (V^μ, χ, D) due to the relations (3.10) and (3.14) have the following composite form:

$$\begin{aligned} V_\mu &= i\varphi^+\partial_\mu\varphi + \frac{1}{2}\psi_+^T C\gamma_\mu\psi_- = A_\mu + j_\mu \\ \chi_+ &= -i\partial^\mu\varphi^+\gamma_\mu\psi_- - \psi_+ F \\ \chi_- &= -i\gamma^\mu\psi_+\partial_\mu\varphi - F^+\psi_- \\ D &= -2(\nabla_\mu\varphi)^+\nabla^\mu\varphi + i\psi_+^T C(\overleftarrow{\nabla\!\!\!/} - \overrightarrow{\nabla\!\!\!/})\psi_- + \\ &\qquad + 2j_\mu j^\mu - 2F^+F \; , \end{aligned} \tag{3.15}$$

where

$$(\nabla_\mu\varphi)^+ = (\partial_\mu - iA_\mu)\varphi^+ \qquad \nabla_\mu\varphi = \varphi(\overleftarrow{\partial}_\mu + iA_\mu) \tag{3.16}$$

and the Lagrangian (3.5) has in the gauge (3.14) the form

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}(F_{\mu\nu} + \nabla_{[\mu}j_{\nu]} - i[j_\mu, j_\nu])^2 \\ &\quad - \frac{i}{2}\chi_+^T C(\nabla\!\!\!/\,\chi_- - i\gamma^\mu[j_\mu, \chi_-]) + \frac{1}{2}D^2 + \\ &\quad + \lambda(\varphi^+\varphi - 1) + \eta(\varphi^+\psi) + (\psi^+\varphi)\eta^+ + \mu(\varphi^+F) + \\ &\qquad + (F^+\varphi)\mu^+ \; , \end{aligned} \tag{3.17}$$

where

$$F_{\mu\nu} = \partial_{[\mu}A_{\nu]} - i[A_\mu, A_\nu] \tag{3.18}$$

and the fields A_μ, j_μ, $\chi_\pm$ and D are given by the formula (3.15).

The formula (3.17) provides a supersymmetric generalization of the $G_{N,n}(C)$ σ model, described by the Lagrangian (1.8). Indeed, if we put in (3.17) the spinor fields $\psi^{Ji} \equiv 0$, one obtains the following Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + 2\left[(\nabla_\mu\varphi)^+(\nabla^\mu\varphi) + F^+F\right]^2 + \lambda(\varphi^+\varphi - 1) + \mu(\varphi^+F) + (F^+\varphi)\mu^+ . \qquad (3.19)$$

Substituting in (3.19) the algebraic equation for the auxiliary superfields F^{Ji}, one obtains the Lagrangian (1.8).

The Lagrangian (3.5) with the substitution (3.10) is invariant under:

a) the transformations of the superconformal group (parameters ϵ_1, ϵ_2; we use $\epsilon = \epsilon_1 + x_\mu\gamma^\mu\epsilon_2$)

$$\delta\varphi_\pm = \bar{\varepsilon}_\mp \psi_\pm \qquad \delta\psi_\pm = F_\pm\varepsilon_\pm - i\not{\partial}\varphi_\pm\varepsilon_\mp$$
$$\delta F_\pm = -i\bar{\varepsilon}_\pm \not{\partial}\psi_\pm + \frac{i}{2}\partial_\mu\bar{\varepsilon}_\pm\gamma^\mu\psi_\pm \; ; \qquad (3.20)$$

b) $U(n)$ generalized gauge transformations (parameters $\Lambda_\pm = (a_\pm, \xi_\pm, f_\pm)$; $a_- = (a_+)^+ = a$, $f_- = (f_+)^+ = f$)

$$\delta\varphi = -i\varphi a \qquad \delta\varphi^+ = i a^+\varphi$$
$$\delta\psi_- = -i\psi_- a - i\varphi\xi_- \qquad \delta\psi_+ = i a^+\psi_+ + i\xi_+\varphi^+$$
$$\delta F = -iFa + i\psi_-^T C\xi_- - i\varphi f$$
$$\delta F^+ = i a^+F^+ + i\xi_+^T C\psi_+ + i f^+\varphi^+ . \qquad (3.21)$$

The gauge conditions (3.14) and the Lagrangian (3.17) are preserved only under

a) local $U(n)$ transformations

$$\delta\varphi = -i\varphi a \qquad \delta\psi_- = -i\psi_- a \qquad \delta F = -iFa \; ; \qquad (3.22)$$

b) The superposition of superconformal and field-dependent generalized gauge transformations, with the parameters $\Lambda_\pm$ chosen as follows:

$$a = 0 \qquad \xi_\pm = iV_\mu\gamma^\mu\epsilon_\pm \qquad f_\pm = \mp i\bar{\epsilon}_\pm\chi_\mp \qquad (3.23)$$

defining the global $N = 1$ supergauge transformations

$$\delta\varphi = \bar{\epsilon}_-\psi_+ \qquad \delta\varphi^+ = \bar{\epsilon}_+\psi_-$$
$$\delta\psi_+ = F\epsilon_+ - i\not{\partial}\varphi\epsilon_- - V_\mu\gamma^\mu\epsilon_-\varphi^+$$
$$\delta\psi_- = F^+\epsilon_- - i\not{\partial}\varphi^+\epsilon_+ + \varphi V_\mu\gamma^\mu\epsilon_+$$
$$\delta F = -i\left(\bar{\epsilon}_+\not{\partial} - \frac{1}{2}\partial_\mu\bar{\epsilon}_+\gamma^\mu\right)\psi_+ - V_\mu\epsilon_-^T C\gamma^\mu\psi_+ + \bar{\epsilon}_+\chi_-\varphi$$
$$\delta F^+ = -i\left(\bar{\epsilon}_-\not{\partial} - \frac{1}{2}\partial_\mu\epsilon_-\gamma^\mu\right)\psi_- - \psi_-^T C\gamma^\mu\epsilon_- V_\mu + \bar{\epsilon}_-\varphi\chi_+ . \qquad (3.24)$$

It can be checked that after substituting (3.24) in (3.15) one obtains the standard supergauge transformations in supersymmetric YM theory (see e.g. [37]).

c) Supersymmetric Yang-Mills theory with $U(n) \times U(m)$ local invariance as $U(m)$ covariant generalized supersymmetric σ model.

In the $G_{n+m,n}(C)$ σ model there is a natural possibility of introducing another local gauge group, simply by substituting in the formula (1.5) new covariant derivatives

$$A_\mu^{ij} \longrightarrow \mathcal{A}_\mu^{ij} = \bar{\Phi}^{iJ} D_\mu^{JK}\Phi^{Kj} , \qquad (3.25a)$$

$$F_{\mu\nu}^{ij} \to \mathcal{F}_{\mu\nu}^{ij} = \partial_{[\mu} A_{\nu]} - i [A_\mu, A_\nu] , \qquad (3.25b)$$

where

$$D_\mu^{JK} = \partial_\mu \delta^{JK} - B_\mu^{JK} . \qquad (3.26)$$

We assume that B^{JK} is a U(m) gauge field; in principle one can gauge the whole U(n+m) global group.

There are two possible ways of adding these new gauge field into the theory:

i) One considers B_μ^{JK} as elementary and to the modified action (1.8) which looks as follows:

$$\mathcal{L}_\sigma^{(4)}(B) = -\frac{1}{4} \mathcal{F}_{\mu\nu}^{ij} \mathcal{F}^{\mu\nu\, ij} \qquad (3.27)$$

we add the free action of B_μ gauge fields

$$\mathcal{L}_{YM}(B) = -\frac{1}{4f_0^2} G_{\mu\nu}^{JK} G^{\mu\nu\, JK} . \qquad (3.28)$$

ii) By letting $f_0 \to \infty$ one can consider B_μ^{JK} as composite.

We see that one obtains two sets of composite gauge fields: gauging U(n) and U(m) local symmetries. If $m \leq n$ these two sets of composite gauge fields would be independent; if $m = r \cdot n + k$ ($r = 1,2,\ldots, k < n$) one obtains independent sets of composite vector potentials if we gauge the following subgroup of U(m):

$$G = [U(m)]^r \otimes U(k) .$$

An interesting example is provided by $n = 3$, $m = 2$: in such a model besides SU(3) gluons one can introduce an independent set of composite U(2) gauge fields; for CP(n) model $G = [U(1)]^n$.

In order to introduce the supersymmetric extension of the formula (3.25a) we modify the compositeness condition (3.10) as follows:

$$\Phi_+ e^{-2W} \Phi_- = e^{-2V} , \qquad (3.29)$$

where $W = W^{ij}$ is a new gauge superfield with the matrix values in U(m) algebra. The bosonic Lagrangian (3.27)&(3.28) should be replaced by the following:

$$\mathcal{L} = \frac{1}{8} \operatorname{tr} \bar{D}_- D_+ [\tilde{\varphi}_{--} \tilde{\varphi}_{++}] + \frac{1}{8f_0^2} \operatorname{tr} \bar{D}_- D_+ [\Xi_{--} \Xi_{++}] + \text{h.c.} , \qquad (3.30)$$

where

$$\tilde{\varphi}_{\alpha\pm\pm} = -\frac{i}{2\sqrt{2}} \bar{D}_\pm D_\mp \left[(\Phi_+ e^{-2W} \Phi_-)^{\pm 1} \cdot D_{\alpha\pm} (\Phi_+ e^{-2W} \Phi_-)^{\mp 1} \qquad (3.31)$$

and

$$\Xi_{\alpha\pm\pm} = -\frac{i}{2\sqrt{2}} \bar{D}_\pm D_\mp \left[e^{\mp 2W} D_{\alpha\pm} e^{\pm 2W} \right] . \qquad (3.32)$$

In particular, one can choose the new gauge superfield in the Wess-Zumino gauge, i.e.

$$W = \frac{1}{4} \bar{\theta} i \gamma_\mu \gamma_5 \theta B^\mu + \frac{1}{4} \bar{\theta}\theta \cdot \bar{\theta}\lambda + \frac{1}{32} (\bar{\theta}\theta)^2 d . \qquad (3.33)$$

In such a case the gauge conditions (3.14) can be preserved; one obtains the following modified formulae (3.15) for the composite components of supersymmetric U(n) gauge fields:

$$\tilde{V}_\mu = i \varphi^+ D_\mu \varphi + \frac{1}{2} \bar{\psi}_+ \gamma_\mu \psi$$
$$\tilde{\chi}_+ = -i \varphi^+ \not{D} \psi_- - \psi_+ F + \varphi^+ \lambda_+ \varphi$$

$$\widetilde{\chi}_- = -i\gamma^\mu \psi_+ D_\mu \varphi - F^+\psi_- + \varphi^+\lambda_-\varphi$$

$$\widetilde{D} = -2(D_\mu\varphi)^+(D_\mu\varphi) + i\,\bar{\psi}_+(\overleftarrow{\not D} - \overrightarrow{\not D})\psi_- + 2 j_\mu j^\mu -$$

$$-2|F|^2 + \varphi^+ d\varphi - 2\varphi^+\lambda_+\psi_- - 2\bar{\psi}_+\lambda_-\psi_- \quad (3.34)$$

We have assumed here that the supermultiplet (3.28) is elementary. The composite fields b_μ, λ, d are obtained by taking the limit $f_0 \to \infty$ in the formula (3.30).

If the first gauge group U(n) describes for n = 3 the colour SU(3) group (or "grand unification" group SU(5) if n = 5), one can try the second group U(m) as describing electroweak gauge fields (or the extension of SU(5) describing technicolour SU(3) group). At the present stage, however, we found it difficult to obtain properly the chiralities of the members of famous SU(5) fundamental families.

IV. DISCUSSION

Both supersymmetric versions of generalized σ models, presented in Sec.II.d and Sec.III.b and III.c, share the following properties:

- U(n) gauge fields are composite;
- quark fields are elementary in fundamental U(n) × U(m) linear representation;
- the Lagrangian does not contain a separate bilinear free quark part, what might be interpreted as a dynamical manifestation of the confinement property.

The fundamental variables in superfield $G_{n+m,n}(C)$ σ models (Sec.III.b) are

- scalar n·m independent Goldstone fields, transforming linearly under U(n) × U(m) and non-linearly under the transformations from $\frac{U(n+m)}{U(n) \otimes U(m)}$, with the dimensionality $d(\varphi) = 0$;
- fermionic n·m independent quark variables, with dimensionality $d(\psi) = \frac{1}{2}$.

We see that the fundamental fields are subcanonical. The scalar fields naively should describe the theory with infra-red divergencies, but we suspect

that similarly like in two-dimensional bosonic σ models we obtain the four-dimensional theory with spontaneous mass generation [10]. It should also be observed that the subcanonical dimension of the quark fields matches perfectly with canonical dimensionalities of the composite meson and baryon fields

$$d(\psi) = \frac{1}{2} \begin{cases} \nearrow d(\bar{\psi}\psi) = 1 \text{ (mesons)} \\ \searrow d(\psi\psi\psi) = \frac{3}{2} \text{ (baryons)} \end{cases} \quad . \qquad (4.1)$$

We would like to recall here that $d(\psi) = \frac{1}{2}$ quark fields were proposed first by Heisenberg (see, e.g. [41,42]); we see now that the supersymmetry requirements allow us to construct consistently a two-level theory, with local fields on both levels: subcanonical fundamental "Urfeld" level and canonical composite level.

The use of $G_{n+m,n}(C)$-valued superfields has several advantages; in particular one can introduce the composite superfields which are colour singlets and for n = 3 correspond to the supersymmetric generalization of the formulae $\bar{\psi}\psi$ and $\psi\psi\psi$ for composite meson and baryon fields

$$\text{mesons:} \quad (e^{-2M})^{JK} = \Phi_-^{Ji}\,\Phi_+^{iK} \quad , \qquad (4.2a)$$

$$\text{baryons:} \quad (B_\pm)^{JKL} = \epsilon_{ijk}\,\Phi_\mp^{iJ}\,\Phi_\mp^{jK}\,\Phi_\mp^{kL} \quad . \qquad (4.2b)$$

It s an interesting task now to see how the equations for fundamental field variables determine the dynamics of composite superfields M and $B_\pm$.

Finally it should be stressed that the four-linear models presented here are classical and globally supersymmetric with simple (N = 1) conformal supersymmetry.

One can develop the ideas presented here at least in three directions:

a) The most urgent is to understand even the simplest quantized four-linear model (without supersymmetry) and in particular to show that it is renormalizable (what should follow from the absence of dimensional coupling constants). We would like to report here that some preliminary results in this direction in the framework of $\frac{1}{N}$ expansion technique (N = n+m) has been recently obtained [43];

b) One can look for the generalizations with $N > 1$, local coupling to supergravity and the relation with σ model structures found, e.g. in $N = 8$ supergravity [44].

c) In order to apply to hadronic systems one should understand how to introduce mass splitting mechanisms and break spontaneously exact supersymmetry.

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FOOTNOTES

1) For more details see e.g. [1,2,3].

2) For the first general discussion of complex structures in σ models see [4]; for quaternionic structure see [1,5].

3) Such a point of view on the mass generation mechanism for gauge fields was advocated by A.A. Slavnov.

4) Such a generalization was obtained by the author with B. Milewski[9]. I learned recently that the results presented in Sec.III.b were obtained independently by Y. Bars and M. Günaydin [10].

5) This difficulty has its counterpart in spin statistics problems in the supersymmetric SU(2;1) version of the Salam-Weinberg theory [11-13].

6) We denote by $[\cdot\,;\cdot]_\pm$ commutator or anticommutator, in accordance with conventional Z_2 graded superalgebra rules.

7) For a more geometric formulation of the SSYM theory see [34-36].

8) The content of Secs.III.b and c has been obtained in collaboration with B. Milewski (see [9]).

9) In [9] we use rather unconventional terminology: we call generalized gauge transformations the supergauge transformations, and supergauge transformations are called simply supersymmetry transformations.

10) See in particular [38], also [39,40]. The author would like to thank Dr. G. Parisi for suggesting a possibility of cancellation of infrared divergencies.

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IC/80/17 INT.REP.* NAMIK K. PAK: Introduction to instantons in Yang-Mills theory. (Part I).