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QCD SUM RULES FOR THE DECAY AMPLITUDES OF PSEUDOSCALAR MESONS *

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ABSTRACT

Bounds on the π and K meson decay amplitudes are obtained to a good accuracy from QCD sum rules of the Laplace transform type. A relation between f_π and the ρ meson coupling to the photon is given. Using the heavy quarks $q^2 = 0$ sum rule to two loops we find our best bounds: $f_D \lesssim (101 \pm 25)$ MeV and $f_F \lesssim (147 \pm 41.6)$ MeV to be compared to $f_\pi \approx 93.3$ MeV. We also derive a relation between the D and F meson masses and the charm quark mass . Our results are extended to the beautiful B mesons.

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I. INTRODUCTION

There has been recent progress in extending the applicability domain of quantum chromodynamics (QCD) to obtain predictions on low-energy parameters (hadron masses and coupling constants). The approach is based on sum rules obeyed by the spectral functions of a specific two-point function of current operators, as a consequence of general analytical properties. There exists a variety of QCD sum rules in the literature [1-3] depending on how these analyticity and positivity properties are exploited. Of particular interest for low-energy phenomenology are the sum rules of the Laplace transform type:

$$\mathcal{F}(M^2) = \frac{1}{\pi}\int_0^\infty dt\, e^{-t/M^2}\, \mathrm{Im}\,\Pi(t), \tag{1}$$

and of the $Q^2 = 0$ type ($Q^2 \equiv -q^2 > 0$):

$$\mathcal{M}^{(n)}(Q^2) = \frac{(-1)^n}{n!}\left(\frac{\partial}{\partial Q^2}\right)^n \Pi(Q^2)\Big|_{Q^2=0}, \tag{2}$$

proposed by SVZ and collaborators [1], respectively, for the light and heavy quark system. Here $\frac{1}{\pi}\mathrm{Im}\,\Pi(t)$ denotes a specific spectral function (e.g. the hadronic vacuum polarization measured in the $e^+e^- \to$ hadrons); $\mathcal{F}(M^2)$ and $\mathcal{M}^{(n)}(Q^2)$ are quantities which in principle can be computed asymptotically in QCD. It is clear that the sum rules (1) and (2) are much more selective on the low-energy behaviour of the spectral function (small t) than the right-hand side of the usual dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t+Q^2}\, \mathrm{Im}\,\Pi(t) + \text{"subtraction"} . \tag{3}$$

The purpose of this letter is to report on some results obtained by applying the sum rules (1) and (2) to the two-point function $\Pi^{\mu\nu}(q)$ associated to the axial vector current $A^\mu \equiv \bar\psi_i \gamma^\mu \gamma^5 \psi_j$ (ψ_i denotes quark field with a flavour i)

$$\begin{aligned}\Pi^{\mu\nu}(q) &\equiv i\int d^4x\, e^{iqx}\langle 0|T\, A^\mu(x)(A^\nu(0))^\dagger|0\rangle \\ &= -(g^{\mu\nu}q^2 - q^\mu q^\nu)\,\Pi^{(1)}_{ij}(q^2) + q^\mu q^\nu\, \Pi^{(0)}_{ij}(q^2),\end{aligned} \tag{4}$$

and to the two-point functions

$$\psi_5(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\,\partial_\mu A^\mu(x)\,(\partial_\nu A^\nu(0))^\dagger|0\rangle, \tag{5}$$

associated to the divergence $\partial_\mu A^\mu(x) \equiv (m_i + m_j)\,\bar\psi_i \gamma_5 \psi_j$ of the axial vector current. In fact, $\psi_5(q^2)$ and $\Pi^{(0)}_{ij}(q^2)$ are related by the current algebra identity [1] via

$$(q^2)^2\, \Pi^{(0)}_{ij}(q^2) = \psi_5(q^2) - \psi_5(0) , \tag{6}$$

where

$$\psi_5(0) = -(m_i+m_j)\Big\{ \langle 0|\, \bar\psi_i \psi_i + \bar\psi_j \psi_j \,|0\rangle - m_i^3 \log\frac{m_i^2}{\nu^2} - m_j^3 \log \frac{m_j^2}{\nu^2} \Big\} + \mathcal{O}(m_i^4, \alpha_s) . \tag{7}$$

In the Nambu-Goldstone realization of chiral symmetry, the quantity $\langle 0|\bar\psi_i\psi_i|0\rangle$ is not zero, so care must be taken in using sum rules of the types (1) and (2).

II. BOUNDS ON f_P ($P \equiv \pi, K, D, F$) FROM THE LAPLACE TRANSFORM SUM RULES

Formally, the Laplace transform sum rule is obtained by applying to both sides of Eq.(3) the operator

$$\mathcal{L} \equiv \lim_{\substack{Q^2 \to \infty \\ n \to \infty}} \; ; \frac{Q^2}{n} \equiv M^2 \quad \frac{(-1)^n}{(n-1)!} \left(\frac{\partial}{\partial Q^2}\right)^n . \tag{8}$$

The derivation of such a sum rule of SVZ [1b] has been discussed in Ref.5. In the phenomenological applications, the ρ meson coupling to the photon [1b,5], the light quark masses [5], the light quark vacuum condensate [6], the gluon component of the U(1) meson mass [7] have been bounded with a good accuracy, within the range of the sum rule scale $M \simeq M_\rho$ in the $\bar u d$ channel ($M \simeq M_\phi$ in the $\bar u s$ channel [6]) and with the value of Λ in the $\overline{MS}$ renormalization scheme [2] taken to be $\Lambda \simeq 70 \sim 210$ MeV from the sum rule analysis of the isovector part of the $e^+e^- \to$ hadrons data [9]. In the following we extend the applications of the Laplace transform sum rule in order to get information on the decay amplitude f_P of pseudoscalar mesons defined as

$$\langle 0|\, A^\mu(x)\, |P\rangle = f_P \sqrt{2}\, q^\mu \tag{9}$$

where $|P\rangle$ is the pseudoscalar state and q^μ its momentum. For the light quark systems, an appropriate sum rule which allows us to extract f_P, is the Laplace transform of $\mathrm{Im}\,\Pi^{(1+0)}_{ij}(t) \equiv \mathrm{Im}\,\Pi^{(1)}_{ij}(t) + \mathrm{Im}\,\Pi^{(0)}_{ij}(t)$.

It reads:

$$\frac{1}{\pi}\int_0^\infty dt\, e^{-t/M^2}\, \mathrm{Im}\,\Pi^{(1+0)}_{ij}(t) = \frac{M^2}{4\pi^2}\Big\{ 1 + \frac{\bar\alpha_s(M^2)}{\pi} + \left(\frac{\bar\alpha_s(M^2)}{\pi}\right)^2 \Big[F_3 - \frac{\beta_1}{2}\gamma_E - \frac{\beta_2}{\beta_1}\log\log\frac{M^2}{\Lambda^2}\Big] + \frac{4\pi^2}{M^4}\Big[m_i\langle\bar\psi_i\psi_i\rangle + m_j\langle\bar\psi_j\psi_j\rangle\Big] + \frac{\pi}{3}\frac{1}{M^4}\langle\alpha_s G^2\rangle + 6\,\frac{\bar m_i \bar m_j}{M^2} + \mathcal{O}\left(\frac{\bar\alpha_s}{\pi}\right)^3 + \mathcal{O}\left(\frac{1}{M^2}\bar\alpha_s , \frac{1}{M^4}\bar\alpha_s\right) + \mathcal{O}\left(\frac{1}{M^6}\right)\Big\} , \tag{10}$$

where $\gamma_E = 0.5772$ is the Euler constant, $\bar\alpha_s/\pi = 1/-\beta_1 \log M/\Lambda$ is the running QCD coupling; $\bar m_i = \dfrac{\hat m_i}{(\log M/\Lambda)^{\gamma_1/-\beta_1}}\left\{1 - \gamma_1\dfrac{\beta_2}{\beta_1^3}\,\dfrac{\log\log M^2/\Lambda^2}{\log M/\Lambda} + \dfrac{1}{\beta_1^2}\left(\gamma_2 - \gamma_1\dfrac{\beta_2}{\beta_1}\right)\dfrac{1}{\log M/\Lambda}\right\}$ is the running quark mass to two loops ($\hat m_i$ is the invariant mass); For $SU(3)_c \times SU(n)_F$, $F_3 = 1.986 - 0.115\, n_F$ is the three-loop calculation of $\Pi^{1+0}_{ij}(q^2)$ [10] [3], $\gamma_1 = 2$, $\beta_1 = -\frac{11}{2} + \frac{n_F}{3}$, $\beta_2 = -\frac{51}{4} + \frac{19}{12} n_F$, $\gamma_2 = \frac{101}{12} - \frac{5}{18} n_F$.

The leading non-perturbative effects are parametrized by the vacuum expectation values $\langle\bar\psi_i\psi_i\rangle$ and $\langle\alpha_s G^2\rangle$, where $G^2 \equiv G^{\mu\nu}G_{\mu\nu}$ is the square of the gluon field tensor. The renormalization group invariant (RGI) quantity $\langle m_i\bar\psi_i\psi_i\rangle$ can be determined using PCAC and the recent result in Ref.6, while we take $\alpha_s\langle G^2\rangle \simeq (0.044^{+0.014}_{-0.06})$ GeV^4 from recent results on charmonium data analysis [12]. In the $\bar u d$ channel the spectral function $\mathrm{Im}\,\Pi^{(1+0)}_{ij}(t)$ can be saturated by the π and A_1. We estimate the A_1 contribution to the spectral function from the $\tau \to \nu_\tau A_1$ data [13] and using a narrow width approximation

$$\mathrm{Im}\,\Pi_{A_1}(t) \simeq \frac{\pi M_{A_1}^2}{2\gamma_{A_1}^2}\,\delta(t - M_{A_1}^2) \simeq (7.3 \pm 2.2)\,10^{-2}\,\mathrm{GeV}^2\,\delta(t - M_{A_1}^2) . \tag{11}$$

The continuum contribution to the sum rule is estimated using the QCD model from the threshold $\sqrt{t_c} \simeq 1$ GeV. This is controlled by the weight factor e^{-2t_c/m^2} in Eq.(10) [6]. We use as well a recent result of Ref.14 based on the Laplace transform of the third Weinberg sum rules in order to estimate the product $\bar m_u \bar m_d$. That we take to be of the order of $\frac{4}{3}\pi^2 f_\pi^2\left(\frac{m_\pi^2}{M^2}\right)^2$.

Using the positivity of some eventual higher resonances, we get the bound

$$2f_\pi^2 \lesssim \left\{ \frac{M^2}{4\pi^2}\left(1-e^{-2t_c/M^2}\right)\left[1+\frac{\bar\alpha_s}{\pi}+\left(\frac{\bar\alpha_s}{\pi}\right)^2\left[F_3-\frac{\beta_1}{2}\gamma_E-\frac{\beta_2}{\beta_1}\log\log\frac{M^2}{\Lambda^2}\right]\right.\right.$$
$$\left.\left.+\frac{\pi}{3}\frac{1}{M^4}\langle\alpha_s G^2\rangle\right]-e^{-M_{A_1}^2/M^2}\frac{M_{A_1}^2}{2\gamma_{A_1}^2}\right\}\left(e^{-m_\pi^2/M^2}+\frac{m_\pi^2}{2M^2}\right)^{-1}. \tag{12a}$$

For $M \simeq M_\rho$, the π contribution is optimized, the A_1 contribution is about 18% of the π one and the QCD correction is about 18%. So[4)]

$$f_\pi \lesssim (91 \pm 5)\ \mathrm{MeV} \tag{12b}$$

which reproduces well the accurate data from $\pi \to \mu\nu$ decay, $f_\pi \simeq 93.28$ MeV. It is also more instructive to give the chiral limit ($m_\pi^2 = 0$) of Eq.(12). Then, we get[5)]

$$f_\pi \lesssim \frac{M_\rho}{2\pi\sqrt{2}}\left\{1+\frac{\bar\alpha_s}{\pi}+\frac{\pi}{3}\frac{1}{M_\rho^4}\langle\alpha_s G^2\rangle\right\}^{1/2} \simeq 97 \sim 100\ \mathrm{MeV} \tag{13}$$

for $\Lambda \simeq 70 \sim 210$ MeV. We can also identify the sum rule in Eq.(10) with the ρ sum rule discussed in Refs.1 and 5 at the same M^2. For $M^2 \simeq M_\rho^2$ the π and ρ meson dominance to each sum rule is fully justified. So, we get to leading order of chiral symmetry breaking[6)]

$$f_\pi \simeq \frac{M_\rho}{2\gamma_\rho}e^{-1/2}\left\{1-e^{1-M_{A_1}^2/M_\rho^2}\cdot\frac{1}{2}\frac{M_{A_1}^2}{M_\rho^2}\frac{\gamma_\rho^2}{\gamma_{A_1}^2}\right\} \simeq (92\pm7)\ \mathrm{MeV}, \tag{14}$$

where we have used the data $\frac{\pi M_\rho^2}{2\gamma_\rho^2} \simeq (0.129 \pm 0.02)\ \mathrm{GeV}^2$.[7)]

In the case of the K meson, we have observed in Ref.6, that the natural scale of optimization of the Laplace transform sum rule is around M_ϕ in order to minimize the quark mass corrections. In this $\bar{u}s$ channel, Eq.(10) is now saturated by the K meson. Then, we deduce for $M \simeq M_\phi$:

$$f_K \lesssim (117 \pm 2)\ \mathrm{MeV} \tag{15}$$

to be compared to the data $f_K \simeq 1.16\, f_\pi$. Eqs.(12) and (15) are a further confirmation of the ability of the Laplace transform sum rules to predict the properties of a single resonance.

We extend the above analysis to the D and F mesons which saturate respectively the sum rule in the $\bar{c}d$ and $\bar{c}s$ channels. In Eq.(10) quark mass effect enters as a product of the light and the heavy ones, so we can choose the sum rule scale M around M_D in the dc channel where the mass effect is less than 20% of the leading QCD ones. In that case, we get to leading order

$$f_D \lesssim \frac{M_D}{2\pi}\sqrt{\frac{e}{2}} = 346.6\ \mathrm{MeV}, \tag{16}$$

while in the $\bar{c}s$ channel, we have to choose $M \simeq 2M_F$ in order to satisfy this 20% criterion. Clearly, we lose the optimization of the F contribution to the spectral function and the bound is expected to be bad. It will also be interesting to use sum rule involving <u>only</u> the pseudoscalar states. We can work with $\Pi_{ij}^{(0)}(q^2)$ (Eq.(4)) or $\psi_5(q^2)$ (Eq.(5)). However, as discussed in Ref.3b, the $\Pi_{ij}^{(0)}(q^2)$ sum rule involves leading non-perturbative effects (see Eqs.(6) and (7)) which tend to cancel the pole contribution to the sum rule [6]. Working with the Laplace transform of $\psi_5(q^2)$, we escape this difficulty [5]. In the $\bar{c}d$ channel, we saturate the $\psi_5(q^2)$ sum rule of Ref.5 by the D meson. Using the QCD model for the continuum and the positivity of higher resonance states, we get

$$f_D \lesssim \frac{\sqrt{3}}{4\pi}(\hat m_c+\hat m_d)\frac{1}{\left(\log M/\Lambda\right)^{\gamma_1/-\beta_1}}e^{M_D^2/2M^2}\left(\frac{M}{M_D}\right)^2\left(1-e^{-2t_c/M^2}\right)^{1/2}\cdot$$
$$\cdot\left\{1+\left(\frac{\bar\alpha_s}{\pi}\right)\left[\frac{11}{6}+\gamma_E+\frac{1}{-\beta_1}\left(\gamma_2-\gamma_1\frac{\beta_2}{\beta_1}\right)+\frac{\gamma_1\beta_2}{\beta_1^2}\log\log M^2/\Lambda^2\right]\right.$$
$$\left.-\frac{\bar m_c^2}{M^2}-\frac{4}{3}\pi^2\frac{1}{M^4}m_c\langle\bar\psi_d\psi_d\rangle+\frac{\pi}{6}\frac{1}{M^4}\alpha_s\langle G^2\rangle\right\}, \tag{17}$$

where we have neglected $\langle\bar\psi_c\psi_c\rangle$ and $\bar m_d^2/M^2$. The continuum threshold is $\sqrt{t_c} \simeq M_D + 2m_\pi$. We optimize the above inequality by demanding that the continuum contribution is around 10 ~ 36% of the leading QCD order and the quark mass correction is less than 50 ~ 20% in order to trust the series expansion in $\bar m_c^2/M^2$. Such conditions are satisfied for $M \simeq 2 \sim 3$ GeV to which corresponds the optimal bound[4)]

$$f_D \leq (240 \sim 450) \pm 80 \text{ MeV} , \tag{18}$$

where we have taken $\hat{m}_c \simeq (2.08 \pm 0.36)$ GeV [16]. We have also used PCAC and the recent result in Ref.5 in order to estimate the quantity $m_c \langle \bar{\psi}_d \psi_d \rangle$.
In the $\bar{c}s$ channel case we get

$$f_F \leq f_D^{sup} \left(1 + \frac{\hat{m}_s}{\hat{m}_c}\right) \left(\frac{M_D}{M_F}\right)^2 \simeq 1.1 \, f_D^{sup} \tag{19}$$

for $\hat{m}_s \simeq 500$ MeV and $\hat{m}_c \simeq 2$ GeV, where f_D^{sup} is the upper bound in Eq.(18). Notice that the above bounds (Eqs.(18) and (19)) are insensitive to the value of $\Lambda \simeq 70 \sim 210$ MeV.

III. THE $q^2 = 0$ SUM RULE FOR THE HEAVY PSEUDOSCALAR MESONS

An alternative way to get bounds on f_D, f_F,... is to work with the quantity [8]

$$\varphi_{(1)} = \frac{1}{2!} \frac{\partial^2 \psi_5}{(\partial Q^2)^2} \bigg|_{Q^2=0} = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{dt}{t^3} \, \mathrm{Im}\, \psi_5(t) . \tag{20}$$

It relies on the fact that the perturbative expression of $\varphi_{(1)}$ in terms of the heavy quark mass is expected to exist provided that the heavy quark mass m_i^2 is bigger than the QCD scale Λ. To lowest order of QCD

$$\varphi_{(1)} = \frac{1}{4\pi^2} \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \text{ for } \begin{pmatrix} m_i^2 = m_j^2 \gg Q^2 \\ m_i^2 \gg Q^2 , \; m_j^2 = 0 \end{pmatrix} \tag{21}$$

which shows that $\varphi_{(1)}$ is a pure number, so it is blind of the external renormalization. Such an observation is helpful for the extraction of the $\bar{\alpha}_S/\pi$ contribution. This can be done working with the general result of $\psi_5(q^2)$ in the space-like region [17] or of $\mathrm{Im}\, \psi_5(t)$ in the time-like region [18]. We find it convenient using the last result. Taking only into account terms which are independent of the external renormalization [9], we find to two loops and including the leading non-perturbative effects [18]

$$\varphi_{(1)} \bigg|_{m_i \gg m_j} = \frac{1}{8\pi^2} \Bigg\{ 1 + \frac{\hat{m}_j}{\bar{m}_i} + \left(\frac{\bar{\alpha}_s}{\pi}\right) \frac{1}{15} \left(\frac{247}{9} - 2\pi^2\right) + \frac{16\pi^2}{\bar{m}_i^4} \hat{m}_i \langle 0 | \bar{\psi}_i \psi_i | 0 \rangle$$
$$- \frac{8\pi}{3} \frac{1}{\bar{m}_i^4} \alpha_s \langle 0 | G^2 | 0 \rangle + O\left(\left(\frac{m_j}{m_i}\right)^2 \log \frac{m_i}{m_j} , \; \bar{\alpha}_s^2 , \; \frac{1}{\bar{m}_i^6} \right) \Bigg\} , \tag{22}$$

where $\hat{m}_j$ is the invariant quark mass and $\bar{m}_i$ is the running mass evaluated at $Q^2 = \bar{m}_i^2$. In the following we shall neglect $\langle 0 | \bar{\psi}_i \psi_i | 0 \rangle$ for the heavy quarks (Wigner-Weyl realization of chiral symmetry) and we shall take $\langle \alpha_S G^2 \rangle \simeq 0.044$ GeV^4 [12]. As we can learn from Eq.(22), the quantity $\varphi_{(1)}$ is finite up to two loops and to the $1/\bar{m}_i^4$ terms, when $m_j^2 \to 0$. Such a result is encouraging and we expect $\varphi_{(1)}$ to be finite to higher orders, as required by the Kinoshita theorem [19]. We saturate the right-hand side of Eq.(20) by the D meson in the $\bar{c}d$ channel. Using the positivity of the continuum contribution to the spectral function, we get

$$| f_D | \leq \frac{M_D}{4\pi} \left\{ 1 + (0.06 - 0.11) \right\}^{1/2} \simeq 145 \text{ MeV} \tag{23}$$

where we have used $M_D \simeq 1.87$ GeV, $m_d/m_c \simeq 0 \simeq \langle 0 | \bar{\psi}_c \psi_c | 0 \rangle$. The first correction in Eq.(23) comes from the $\bar{\alpha}_S$ term and the second one from the $\langle 0 | \alpha_S \, G^{\mu\nu}_{(a)} \, G^{(a)}_{\mu\nu} | 0 \rangle$ operator. In the $\bar{c}s$ channel we get

$$| f_F | \leq | f_D |^{sup} \left(\frac{M_F}{M_D}\right) \left(1 + \frac{\hat{m}_s}{\hat{m}_c}\right)^{1/2} , \tag{24}$$

where $(f_D)^{sup}$ is the upper bound in Eq.(5). For $\hat{m}_s \simeq (300 \sim 500)$ MeV, Eq.(24) gives

$$| f_F | \leq (169.5 \sim 176.7) \text{ MeV} . \tag{25}$$

The results in Eqs.(23) and (25) are stronger than in Eqs.(18) and (19), due to the fact that the Laplace sum rule scale M has to be chosen big enough so as to minimize the quark mass corrections. We can again improve the result in Eqs.(23) and (25) working with higher n^{th} derivatives of $\psi_5(q^2)$ due to the increasing contribution of resonances with n. The QCD expression of the moments in the case $\bar{m}_i \gg m_j$ is given in Ref.18 and appears to depend crucially on the quark mass value m_i^2 as well as on the way how it is renormalized [9]. Using directly the result in Ref.18, the QCD corrections to the moments are individually important. For large n, the moments behave as:

$$\mathcal{L}_{(n)} \equiv \frac{(-1)^{n+1}}{(n+1)!}\left(\frac{\partial}{\partial Q^2}\right)^{n+1}\psi_5\Big|_{Q^2=0} \simeq \frac{3}{4\pi^2}\frac{1}{(\overline{m}_i^2)^{n-1}}\frac{1}{n^2}\Big\{1 + n\frac{\hat{m}_j}{\hat{m}_i} +$$

$$\frac{4}{3}\left(\frac{\bar{\alpha}_s}{\pi}\right)\left[\frac{17}{4}+\frac{\pi^2}{3}+\frac{3}{2}\log n - 3n\log 2\right] - \frac{2}{3}\pi\left(\frac{n}{\overline{m}_i}\right)^4\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$+ \frac{2}{3}\pi^2\left(\frac{n}{\overline{m}_i}\right)^4\langle 0|m_i\overline{\psi}_i\psi_i|0\rangle + \mathcal{O}\left(\frac{1}{n}\right)\Big\}, \tag{26}$$

so care must be taken when working with higher moments. Already in the case of $\psi_5'''(0)$ each QCD correction to the moments is of the order of 60% but fortunately they tend to cancel out

$$\mathcal{L}_{(2)} \equiv \frac{(-1)^3}{3!}\left(\frac{\partial}{\partial Q^2}\right)^3\psi_5(Q^2)\Big|_{Q^2=0} = \frac{1}{64\pi^2}\frac{1}{\overline{m}_c^2}\Big\{1 + 0.57 - 0.66\Big\}, \tag{27}$$

where the first correction comes from the $\bar{\alpha}_S$ term, the second one from the $\langle 0|\alpha_S G^2|0\rangle$ term. The $\mathcal{L}_{(2)}$ sum rule will give the bound in the $\bar{c}d$ channel [4])

$$f_D \le \frac{M_D}{4\pi}\frac{1}{2}\frac{M_D}{\overline{m}_c}\Big\{1 + \text{QCD corrections}\Big\} \simeq (101 \pm 25)\,\text{MeV}. \tag{28}$$

Clearly, we get stronger bound than in Eq.(23) but, unfortunately, the uncertainty in the derivation of the bound has also increased, and so, it becomes useless to go to moments with higher n. In the $\bar{c}s$ channel the strange quark correction to $\mathcal{L}_{(2)}$ is of the order of 50%. In that case, we get, using a similar analysis as for the D meson

$$f_F \le (147.5 \pm 42.6)\,\text{MeV}\ . \tag{29}$$

A further use of the higher moments can be obtained from the ratio

$$R_n = \frac{\mathcal{L}_{(n)}}{\mathcal{L}_{(n+1)}}\ , \tag{30}$$

where the QCD corrections to R_n become moderate, as well as the leading quark mass dependence. For large n, the $\bar{\alpha}_S/\pi$ correction goes like "constant" + $\mathcal{O}(\frac{1}{n})$ and the $(1/\overline{m}_i^4)$ contribution behaves like n^3. Using, for example, the low moments R_1 and the fact that the continuum contribution to R_1 is positive [7]), we get in the $\bar{c}d$ channel

$$M_D \le 2\,\overline{m}_c\,(Q^2=\overline{m}_c^2)\Big\{1 - 0.5 + 0.55\Big\}^{1/2} \simeq (2.7 \pm 0.6)\,\text{GeV}\ , \tag{31}$$

where the corrections in { } come respectively from the $\bar{\alpha}_S$ term and from the gluon condensate term. In the $\bar{c}s$ channel, we deduce

$$M_F \le (2.2 \pm 0.5)\,\text{GeV}\ , \tag{32}$$

where the quark mass corrections to R_1 tends to decrease the upper value of the bound.

For completeness, we extend our analysis to the beautiful B mesons. Using $M_B \simeq 5.2$ GeV [20] and the invariant b-quark mass $\hat{m}_b \simeq 6.5 \sim 8.2$ GeV [16] i.e. $\overline{m}_b(m_b^2) \simeq 3.7 \sim 4.5$ GeV, we get from the $\mathcal{L}_{(2)}$ sum rule in the $\bar{b}u$ or $\bar{b}s$ channel

$$f_B \le (346.6 \sim 284.3)\,\text{MeV} \tag{33}$$

while in the $\bar{b}c$ channel

$$f_B \le (385 \sim 316)\,\text{MeV}\ . \tag{34}$$

The R_1 sum rule in Eq.(30) gives in the $\bar{b}u$ or $\bar{b}s$ channel the constraint

$$M_B \le (6 \sim 7.4) \pm 0.5\ \text{GeV} \tag{35}$$

while in the $\bar{b}c$ channel we get [11])

$$M_B \le (4 \sim 4.9) \pm 0.8\ \text{GeV} \tag{36}$$

IV. CONCLUSIONS

We have used QCD sum rules for the understanding of the fundamental decay amplitudes of pseudoscalar mesons which control the breaking of the chiral flavour symmetry. An experimental measurement of the decay amplitudes of the heavy pseudoscalar mesons is needed.

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FOOTNOTES

1) For a recent review on current algebra, see, e.g. Ref.4 and references therein.

2) For a recent review on dimensional regularization and renormalization, see, e.g. Ref.8.

3) We use the three-loop result of the two-point function associated to the vector current due to the fact that in the chiral limit ($\bar{m}_i = 0$) the $SU(n)_L \times SU(n)_R$ chiral symmetry is not spontaneously broken by gluon exchange to all orders of perturbation theory [11].

4) Our error is the quadratic sum of the error due to other sources (experiment) and of the estimated QCD error. We estimate the error due to QCD as the quadratic sum of the square of each individual QCD correction.

5) Notice that our f_π is $\frac{1}{\sqrt{2}}$ times the f_π of SVZ. We consider this result as an improvement of the result given by SVZ [1b]

6) Analogous result can also be obtained using the Laplace transform of the first Weinberg sum rule discussed in Ref.14. In the chiral limit, and using the positivity of the scalar contribution to the sum rule, the estimate in Eq.(14) could be replaced by a lower bound on f_π .

7) Recall that $\Gamma_{\rho \to e^+e^-} \simeq \frac{2}{3} \alpha^2 \pi \, M_\rho / 2\gamma_\rho^2$ (α being the QED fine structure constant).

8) Note that the quantity used by NRY [3a] depends crucially on the non-perturbative effects due to the Ward identity in Eq.(6).

9) One must notice that the moments of ψ_5 has ~~less~~ power of $\bar{m}_i^2$ than that of Ref.18. So, care must be taken for the corrections due to the mass renormalization.

10) We expect that the continuum contribution to $\mathcal{L}_{(n)}$ is bigger than to $\mathcal{L}_{(n+1)}$ because the latter is more weighted by low energy ($t = \bar{m}_c^2$).

11) After the completion of this work, we learn that an analysis of the B meson mass and coupling has also been done in Ref.[21] . Our bounds agree with their result coming from higher moments analysis. However, working with higher moments could be useless in the strange channel if the s-quark mass is higher than 150 MeV due to the important quark mass correction.

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