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ON THE QUANTUM FIELD THEORY OF CHARGES AND MONOPOLES *

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ABSTRACT

A treatment of the interaction between charges and monopoles is presented, in terms of functional integration over closed paths. The Lorentz covariance is preserved in all the steps of the procedure and the symmetry between electric charges and magnetic poles in the interaction is clearly displayed. Some instances of application are discussed.

I. INTRODUCTION

The problem of magnetic monopoles experiences recurrent periods of interest in physics, mainly induced by theoretical considerations, in lack of any cogent experimental evidence of such a kind of particles. At present the interest is due to the classical solutions of the equations of the non-abelian gauge theories, where in some cases isolated magnetic poles appear [1]. From this point of view the magnetic poles do not come in as primitive entities and therefore their properties depend on the features of the theory from which they emerge. There are, however, common properties of this kind of hypothetical particles which can be seen as properties of point-like monopoles. Of course the relevant features are their electromagnetic interactions. As is well known, the interest for the monopoles was originally roused by the papers of Dirac [2] and received contributions by Cabibbo and Ferrari [3], Weinberg [4], Schwinger [5], Wu and Yang [6]. A systematic research on the dynamics of the monopoles, and their possible treatment in the general framework of quantum field theory has been presented in a series of papers by Zwanziger [7] and Zwanziger, Brandt and Neri [8]. More recently, an approach to this problem following the ideas of Yang [9] was also presented by Dao Xing Xia [10] *). There are different techniques in dealing with the problem of the quantum field theory of the monopole, but in any case one must, at a certain moment, face the problem of preserving the Lorentz invariance of the theory; in one way or another the Dirac quantization prescription [2] is crucial for implementing the Lorentz and rotational invariance. This condition is in principle not compatible with the perturbation expansion, where the S matrix (or the generating functional) must be treated as an analytical function of the coupling constants e and g. In this paper we present a treatment of the quantum charge-monopole interaction which is based on the formalism of the path integrals, which recently received a new interest, for different problems, by the work of Polyakov [11] of Migdal and Makeenko [12]. In this formalism the amplitudes containing closed loops of standard quantum field theory can be expressed by a sum over closed particle trajectories. This approach to the monopoles was basically introduced by Brandt, Neri and Zwanziger [8].

In our treatment, the electromagnetic field is considered as a derived quantity, either due to quantum particles or to external sources, which can also be used as a device for treating the case of real photons. The interactions among electrically or magnetically charged particles are then described in terms of retarded interactions and it is possible to express them in the path integration formalism in such a way as to introduce only the electromagnetic field and its dual, rather than the vector potentials. As a consequence we never introduce the so-called Dirac string *), which in some treatments is expressed in terms of an arbitrary vector n_μ, and we always maintain explicit Lorentz invariance. The Dirac quantization condition is of course necessary in order to construct this formalism and in particular to show that the charge-monopole interaction can indeed be written in two equivalent ways, according to whether we consider the charge or the monopole as the source of the electromagnetic field. We consider both cases where the ordinary charges and the monopoles are spinless bosons and spin one-half fermions.

As a final part we present some applications of the general formalism we have built up, estimating the possible effect of the presence of virtual monopoles in processes where, hopefully, a sort of perturbative treatment can give meaningful informations. Due essentially to the large mass ascribed to the monopole we find that whenever this kind of treatment is allowed the effect is expected to be very weak.

*) We thank Dr. S. C. Lim for an English translation of this paper.

II. PATH INTEGRAL FORMULATION OF QED FOR ORDINARY PARTICLES

We start by considering a situation where the external particles are only photons, in that case the charged particle field can be completely integrated over and we are left with an effective interaction among photons that can be represented by the determinants:

$$\left\{ \det \left[D^2 + \mu^2 \right] \right\}^{-1} \tag{1}$$

for the bosonic case and

$$\det \left[i\gamma \cdot D - \mu \right] \tag{1'}$$

for the fermionic case, where in both cases: $D_\mu = \partial_\mu + ieA_\mu$.

From now on, until new specification, the formulation will go on in four-dimensional euclidean space (with hermitian γ matrices), thus we write

*) It is also possible to avoid the introduction of the Dirac string by introducing different determinations of the vector potential in different regions, taking care that in the common domain of definitions they differ only by a gauge transformation, as is done in the Yang formulation, Refs. 9 and 10.

$$\left\{\det\left[-D^2+\mu^2\right]\right\}^{-1} = \Delta_B \tag{2}$$

for Bose fields and

$$\det\left[\gamma\cdot D-\mu\right] = \Delta_F \tag{2'}$$

for Fermi fields.

We start with the Bose charged field (Eq.(2))

$$\left\{\det\left[-D^2+\mu^2\right]\right\}^{-1} = \exp\left[-\operatorname{tr}\ln\left(-D^2+\mu^2\right)\right] = \sum_\ell \frac{1}{\ell!}\left\{\operatorname{tr}\left[-\ln\left(-D^2+\mu^2\right)\right]\right\}^\ell . \tag{3}$$

We then write

$$\operatorname{tr}\left[-\ln\left(-D^2+\mu^2\right)\right] = \operatorname{tr}\int_0^\infty \frac{dT}{T}\left\{e^{-(-D^2+\mu^2)T} - e^{-cT}\right\} + \ln c\ ,$$

c is an arbitrary constant, here introduced to ensure the convergence of the integral representation when $T\to 0$; dropping the c-dependent term will introduce an infinite constant. Disregarding this constant we get

$$\operatorname{tr}\left[-\ln\left(-D^2+\mu^2\right)\right] = \operatorname{tr}\int_0^\infty e^{-\mu^2 T}\frac{dT}{T}\, e^{D^2 T} . \tag{4}$$

The last factor looks like a finite-time evolution operator, where D^2 plays the rôle of a Hamiltonian (in four-dimensional space). According to the Feynman formalism it can be represented through a path integral

$$\langle \nu_1 | e^{D^2 T} | \nu_2 \rangle = \mathcal{N}\int_{\substack{q(0)=q_{\nu_1}\\ q(T)=q_{\nu_2}}} \mathcal{D}q\, e^{-\int_0^T\left[\frac{1}{4}\dot q^2(s)+ie\dot q A(q)\right]ds} .$$

This is obtained from the Feynman formulation by just setting $m = \frac{1}{2}i$. The normalization is known from the value of the integral for $A \equiv 0$; since we have four space dimensions the result is

$$\langle \nu_1 | e^{\partial^2 T} | \nu_2 \rangle = \frac{1}{16\pi^2 T^2}\, e^{-(q_{\nu_1}-q_{\nu_2})^2/4T} \tag{5}$$

Since we must finally take the trace, we sum over all paths with the condition $q(0) = q(T)$ and write

$$\operatorname{tr}\left[-\ln\left(-D^2+\mu^2\right)\right] = \int_0^\infty e^{-\mu^2 T}\frac{dT}{T}\int_{q(0)=q(T)} e^{-\int_0^T\left[\frac{1}{4}\dot q^2(s)+ie\dot q A(q)\right]ds}\,\mathcal{D}q(s) \tag{6}$$

From now on the normalization factor is contained in $\mathcal{D}q$. The use of Eq.(3) now gives the expression for the determinant

$$\sum_\ell \frac{1}{\ell!}\int \prod_{m=1}^{\ell} e^{-\mu^2 T_m}\frac{dT_m}{T_m}\int \mathcal{D}q^{(m)}\, e^{-\int_0^{T_m}\left[\frac{1}{4}\dot q_m^2(s)+ie\dot q_m A(q_m)\right]ds} = \Delta_B \tag{7}$$

For the Fermi field (Eq.(2')) we could proceed in an analogous way. Alternatively, we can proceed in the following way: when we expand the logarithm and take the trace term by term we realize that only when an even number of γ matrices occurs the trace is non-zero. Therefore

$$\operatorname{tr}\ln\left[\gamma\cdot D-\mu\right] = \operatorname{tr}\ln\left[-\gamma\cdot D-\mu\right] = \frac{1}{2}\operatorname{tr}\ln\left[-(\gamma\cdot D)^2+\mu^2\right]$$

Now $(\gamma\cdot D)^2 = D^2 + i\frac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}$ so that

$$\operatorname{tr}\ln\left[\gamma\cdot D-\mu\right] = \frac{1}{2}\operatorname{tr}\ln\left[-D^2+\mu^2+i\frac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}\right] =$$

$$= -\frac{1}{2}\int_0^\infty e^{-\mu^2 T}\frac{dT}{T}\int_{q(0)=q(T)}\left[e^{-\int_0^T\left[\frac{1}{4}\dot q^2(s)+ie\dot q(s)A(q)\right]ds}\right. .$$

$$\left. \cdot\ \operatorname{tr}\,\mathbb{P}\, e^{-\frac{ie}{2}\int_0^T \sigma_{\mu\nu}F_{\mu\nu}\,ds}\right]\mathcal{D}q \tag{8}$$

where the path ordering refers to the σ matrices and the path-ordered term depends on s through q : $F_{\mu\nu} \equiv F_{\mu\nu}(q(s))$.

We now proceed to elaborate the formalism for spinless bosons by introducing the kinetic term for the photons. It sometimes becomes convenient to introduce the symbols

$$j_\mu^{(m)}(x) = e\int ds\, \dot q_\mu^{(m)}\,\delta(x-q) \quad ; \quad J_\mu = \sum_m j_\mu^{(m)} + J_\mu^{ext} .$$

We perform the relevant functional integration over the e.m. field, which appears in the expression

$$\int \mathcal{D}A \; e^{-\int \frac{1}{4} F_{\mu\nu}F_{\mu\nu}\,dx \; - i\int A_\mu J_\mu \,dx} ,$$

where in the Lorentz gauge $-\frac{1}{4}\iint F_{\mu\nu}F_{\mu\nu}\,dx = \frac{1}{2}\int A\,\Box A\,dx$. Accordingly, we can write:

$$\int \mathcal{D}A\; e^{-i\int A_\mu J_\mu(x)\,dx \; + \frac{1}{2}\int A_\mu(x)\,\Box A_\mu(x)\,dx} =$$

$$= e^{\frac{1}{2}\int J_\mu(x)\,D(x-y)\,J_\mu(y)\,dx\,dy} ,$$

where $\Box D(x-y) = \delta(x-y)$ and the normalization, with respect to the free case, is understood. The result can be expressed, defining

$$A_\mu^{(m)}(x) = i\int D(x-y)\, j_\mu^{(m)}(y)\,dy \quad \text{and} \quad A_\mu^{ext}(x) = i\int D(x-z)\, J_\mu^{ext}(z)\,dz ,$$

as

$$Z_E(J^{ext}) = \sum_\ell \frac{1}{\ell!}\int \prod_{m=1}^{\ell} e^{-\mu^2 T_m}\frac{dT_m}{T_m}\int \mathcal{D}q^{(m)}\Big[e^{-\int^{T_m}\frac{1}{4}\dot q_m^2(s)\,ds} \cdot$$

$$\cdot\, e^{-\frac{1}{2} ie\int \dot q_\mu^{(m)}(s)\sum_n A_\mu^{(n)}(q)\,ds}\; e^{-ie\int \dot q_\mu^{(m)} A_m^{ext}(q)\,ds}\Big] . \qquad (9)$$

This expression is the generating functional through which we can express the interaction of any number of photons with any number of closed boson loops (a term representing the interactions of the external currents among themselves has been omitted).

III. PATH INTEGRAL FORMULATION OF THE GENERAL CHARGE MONOPOLE INTERACTION

3.1 Introduction of the monopoles

We ~~now~~ write down a generalization of Eq.(9) for the case where we have not only charged loops but also loops covered by monopoles *). We parametrize the magnetic loops with $p(s)$ and define the magnetic current

$$h_\nu^{(m)}(x) = g\int ds\, \dot p_\nu^{(m)}(s)\,\delta(x-p) \qquad H_\nu = \sum_m h_\nu^{(m)} + H_\nu^{ext} .$$

If we had <u>only</u> monopoles we would have as a result $Z_M(H^{ext})$, an expression strictly analogous to $Z_E(J^{ext})$. Such an expression would contain the potentials

$$W_\nu^{(m)}(x) = i\int D(x-y)\, h_\nu^{(m)}(y)\,dy$$

and

$$W_\nu^{ext}(x) = i\int D(x-z)\, H_\nu^{ext}(z)\,dz .$$

It is important to make a remark on the effect of the Wick rotation: in Minkowski space we write the Gauss theorem $\mathrm{div}\,\vec{\mathcal{E}} = \rho_E$, i.e. $\partial^i F_{0i} = J_0$ and the rotation to R_4 leaves everything invariant. For magnetic poles $\mathrm{div}\,\vec{\mathcal{B}} = \rho_M$, i.e. $\partial^i \tilde F_{0i} = H_0$, but now the left-hand side of this equation contains only space components, while the right-hand side is a time component, so in the rotation an unbalanced i is produced. To take this fact into account we interpret the constant g, appearing in euclidean formulation, as pure imaginary.

In order to write a generating functional allowing also for charge monopole interaction, we look back, for a while, to the charge charge interaction. The interaction of the m^{th} charge with the n^{th} one is written as

*) The monopoles are treated as pointlike just as the ordinary charged particles.

$$Q_{m,n} = -ie\int \dot q_\mu^{(m)}(s)\, A_\mu^{(n)}(q(s))\, ds =$$

$$= -ie\int_{\Gamma_m} dq_\mu^{(m)} A_\mu^{(n)}(q) = -\frac{ie}{2}\int_\Sigma d\sigma_{\mu\nu}^{(m)} F_{\mu\nu}^{(n)} .$$

Having used the Stokes' theorem and defined $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ we then obtain

$$Q_{m,n} = e^2\int_\Sigma d\sigma_{\mu\nu}^{(m)}\partial_\mu\int_{\Gamma_n} D(x-q')\, dq_\nu'^{(n)} = e^2\int_{\Sigma_m}\int_{\Sigma_n} d\sigma_{\mu\nu}\, d\sigma'_{\nu\lambda}\, \partial_\mu\partial_\lambda D(x-x'), \quad (10)$$

where $x \in \Sigma_m$, $x' \in \Sigma_n$ and Σ_m, Σ_n are two surfaces having Γ_m and Γ_n as boundaries.

Let us now assume that the particle circulating in the loop Γ_m carries a magnetic pole instead of a charge. The common wisdom about monopoles is that they interact with the dual of the field strength in the same way as the charge interacts with the field strength.

So we write the interaction as

$$I_{m,n} = -\frac{ig}{2}\int_{T_m} d\tau_{\mu\nu}^{(m)} \tilde F_{\mu\nu}^{(n)} =$$

$$= \frac{eg}{2}\int_{T_m}\int_{\Sigma_n'} d\tau_{\mu\nu}\, d\sigma'_{\lambda\rho}\, \epsilon_{\mu\nu\kappa\lambda}\partial_\kappa\partial_\rho D(x-x') , \quad (11)$$

where $x \in T_m$, $x' \in \Sigma_n'$ and T_m is the surface having the trajectory of the monopole as boundary, Σ_n' is the same for the charge.

3.2 Discussion of the features of the charge monopole interaction

Having written the interaction, we must now face two problems: to show that the exponential of the interaction term J_{mn} depends on the closed paths and not on the surfaces and to verify that the interaction is symmetric between electric charges and magnetic poles as will be seen below. This second requirement means that we can also write the interaction in the form

$$I_{m,n} = \frac{ie}{2}\int_{\Sigma_n} d\sigma_{\mu\nu}^{(m)} \tilde M_{\mu\nu}^{(n)} ,$$

where, from now on,

$$M_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu .$$

We start showing that

$$\Delta I = -\frac{ie}{2}\int_{\Sigma_n} d\sigma_{\mu\nu}^{(m)} \tilde M_{\mu\nu}^{(n)} + \frac{ie}{2}\int_{\Sigma_n^*} d\sigma_{\mu\nu}^{(m)} \tilde M_{\mu\nu}^{(n)} = 2\pi n i \quad (12)$$

provided the Dirac quantization condition (which in euclidean space reads $eg = 2\pi i$) holds, where Σ and Σ^* have the same boundary.

We can write

$$\Delta I = \frac{ie}{3!}\int_{\mathcal V}\left[\partial_\lambda \tilde M_{\mu\nu} + \partial_\mu \tilde M_{\nu\lambda} + \partial_\nu \tilde M_{\lambda\mu}\right] d\mathcal V_{\lambda\mu\nu} ,$$

where $\mathcal V$ is the three-dimensional volume enclosed by Σ and Σ^*. The term in brackets is equal to $\epsilon_{\lambda\mu\nu\rho}\,\partial_\sigma M_{\sigma\rho} = i\epsilon_{\lambda\mu\nu\rho}\, h_\rho$ and calling Λ the path of the monopole

$$\Delta I = -\frac{eg}{3!}\int_{\mathcal V\otimes\Lambda}\epsilon_{\lambda\mu\nu\rho}\, d\mathcal V_{\lambda\mu\nu}\, dp_\rho\, \delta^{(4)}(x-p) = -eg\int_{\mathcal V\otimes\Lambda} d^4x\, \delta^{(4)}(x-p) = egn = 2\pi n i$$

where n is an integer.

In agreement with the Dirac equation condition we also assume that the electric and magnetic external currents have the following property: if we define

$$e_{ext} = \frac{1}{3!}\int J_\mu^{ext}\epsilon_{\mu\nu\rho\sigma}\, d\mathcal V_{\nu\rho\sigma}$$

and

$$g_{ext} = \frac{1}{3!}\int H_\mu^{ext} \epsilon_{\mu\nu\rho\sigma}\, d^\sigma \Upsilon_{\nu\rho\sigma}$$

then $e \cdot g_{ext} = e_{ext} \cdot g = 2\pi n i$. Since, in the complete generating functional, $I_{m,n}$ appears at the exponent, the change of Σ to Σ^* is irrelevant and therefore the exponential of the interaction term depends only on the path Λ.

Next, we shall show that

$$e^{-\frac{ig}{2}\int_{T_m} d\tau_{\mu\nu}^{(m)} \tilde{F}_{\mu\nu}^{(m)}} = e^{\frac{ie}{2}\int_{\Sigma_m} d\sigma_{\mu\nu}^{(m)} \tilde{M}_{\mu\nu}^{(m)}} . \quad (13)$$

Let us put $d\tau_{\mu\nu}^{(m)} = dx_\mu \wedge dx_\nu$ and $d\sigma_{\mu\nu}^{(n)} = dy_\mu \wedge dy_\nu$ so that the second exponent is

$$B \equiv -\int_\Sigma eg\, \epsilon_{\mu\nu\rho\sigma}\, dy_\mu \wedge dy_\nu\, \partial_\rho \int_\Lambda D(y-p)\, dp_\sigma .$$

Calling $V_\sigma(\rho) = \epsilon_{\mu\nu\rho\sigma}\, dy_\mu \wedge dy_\nu\, \partial_\rho\, D(y-p)$ we rewrite

$$\int_\Lambda V_\sigma\, dp_\sigma = \frac{1}{2}\int_T \left(\partial_\tau V_\sigma - \partial_\sigma V_\tau\right) dx_\tau \wedge dx_\sigma$$

and

$$B = eg \int_\Sigma \int_T [dy_\mu \wedge dy_\nu][dx_\tau \wedge dx_\sigma]\, \epsilon_{\mu\nu\rho\sigma}\, \partial_\rho \partial_\tau D(y-x) .$$

Applying the same procedure, the first exponent A can be rewritten as:

$$A = eg \int_\Sigma \int_T [dy_\mu \wedge dy_\nu][dx_\tau \wedge dx_\sigma]\, \epsilon_{\tau\sigma\rho\nu}\, \partial_\mu \partial_\rho D(x-y) .$$

Now we use the identity

$$\epsilon_{\mu\nu\rho\sigma}\, dy_\mu \wedge dy_\nu\, dx_\tau \wedge dx_\sigma\, \partial_\tau \partial_\rho D +$$
$$+ \epsilon_{\tau\sigma\rho\nu}\, dy_\mu \wedge dy_\nu\, dx_\tau \wedge dx_\sigma\, \partial_\mu \partial_\rho D =$$
$$= \frac{1}{2}\epsilon_{\mu\nu\tau\sigma}\, dy_\mu \wedge dy_\nu\, dx_\tau \wedge dx_\sigma\, \Box D$$

which comes from the vector identity: $\epsilon_{\kappa\lambda\mu\nu}\,\delta_{\xi\alpha}$ + (cycl. perm in $\kappa\lambda\mu\nu\xi$) = 0. Therefore

$$B - A = \frac{1}{2} eg \int_{\Sigma \otimes T} \delta^{(4)}(y-x)\, \epsilon_{\mu\nu\tau\sigma}\, dy_\mu \wedge dy_\nu\, dx_\tau \wedge dx_\sigma .$$

Since $\frac{1}{4}\epsilon_{\mu\nu\tau\sigma}\, dy_\mu \wedge dy_\nu\, dx_\tau \wedge dx_\sigma$ is a four-dimensional volume element either $B-A = 0$ or $B-A = 4\pi i$ due to the Dirac quantization condition, and therefore Eq.(13) holds. Let us notice that if Γ and Λ have no common points then we can choose Σ and T, which are arbitrary surfaces having Γ and Λ as boundaries, without common points so that $B-A = 0$ (of course, we always make use implicitly of the Dirac quantization condition when we say that we can deform Σ or T). We can also formulate the charge monopole interaction in a way which is manifestly symmetric under the interchange $eg \to -eg$ by saying that it is

$$e^{\frac{1}{2}\left[-\frac{ig}{2}\int_{T_m} d\tau_{\mu\nu}^{(m)} \tilde{F}_{\mu\nu}^{(m)} + \frac{ie}{2}\int_{\Sigma_m} d\sigma_{\mu\nu}^{(m)} \tilde{M}_{\mu\nu}^{(m)}\right]} ,$$

with the advantage that it remains manifestly symmetric also when we expand in powers the exponential, as we do for instance in perturbation theory. The exponent represents the same charge monopole interaction as in Eq.(11) except for a contact term in the case where the trajectories cross. Later, when considering the case of spin 1/2, we shall see that there is a similar problem which cannot be avoided even before the expansion of the exponential. We can then choose to formulate the theory in the manifestly symmetric form, as written before, which implies an extra contact interaction with respect to Eq.(11) and its generalization in the case of spin 1/2. One can possibly introduce a regularization prescription such that the charge and monopole trajectories have no common points, so that the contact term never occurs. For instance, one can regularize by introducing a lattice: the charge trajectories are then defined on the links of the lattice, whereas the monopole trajectories are defined on the links of the dual lattice. This would be natural in the sense that in compact QED on a lattice the monopoles appear located on the sites of the dual lattice.

3.3 Expression and properties of the generating functional

At this point we can write the generating functional for the general case, in which we have both charges and monopoles and external currents of both kinds:

$$Z(J^{ext}, H^{ext}) = \sum_e \sum_r \frac{1}{e!\,r!} \left(\prod_1^e{}_m \int \mathcal{D}q^{(m)} \int e^{-\mu_Q^2 T_m} \frac{dT_m}{T_m} \right) \cdot$$
$$\cdot \left(\prod_1^r{}_m \int \mathcal{D}p^{(m)} \int e^{-\mu_P^2 T_m} \frac{dT_m}{T_m} \right) e^{-\sum_1^e{}_m Q_m - \sum_1^r{}_m P_m - I} , \tag{14}$$

where

$$Q = \int_0^T ds \left\{ \frac{1}{4} \dot q^2(s) + \frac{ie}{2} \dot q_\mu(s) \sum_1^e{}_j A_\mu^{(j)}(q) + ie \dot q_\mu(s) A_\mu^{ext} \right\} \tag{15}$$

$$P = \int_0^T ds \left\{ \frac{1}{4} \dot p^2(s) + \frac{ig}{2} \dot p_\mu(s) \sum_1^r{}_k W_\mu^{(k)}(p) + ig \dot p_\mu(s) W_\mu^{ext} \right\} \tag{15'}$$

$$I = \frac{ig}{4} \sum_1^e{}_m \sum_1^r{}_n \int_{T_m} d\tau_{\mu\nu}^{(n)} \tilde F_{\mu\nu}^{(m)} - \frac{ie}{4} \sum_1^r{}_n \sum_1^e{}_m \int_{\Sigma_m} d\sigma_{\mu\nu}^{(m)} \tilde M_{\mu\nu}^{(n)} +$$
$$+ \frac{ig}{2} \sum_1^r{}_m \int_{T_m} d\tau_{\mu\nu}^{(m)} \tilde F_{\mu\nu}^{ext} - \frac{ie}{2} \sum_1^e{}_m \int_{\Sigma_m} d\sigma_{\mu\nu}^{(m)} \tilde M_{\mu\nu}^{ext} . \tag{15''}$$

According to the previous notations

$$A_\mu(x) = i \int D(x-y)\, j_\mu(y)\, dy$$
$$W_\mu(x) = i \int D(x-y)\, h_\mu(y)\, dy$$

and the external currents appear through

$$A_\mu^{ext}(x) = i \int D(x-y)\, J_\mu^{ext}(y)\, dy$$
$$W_\mu^{ext}(x) = i \int D(x-y)\, H_\mu^{ext}(y)\, dy .$$

As a consequence of our previous discussion we see that all the factors entering in the definition of Z depend on the paths (Γ, Λ) and not on the surfaces (Σ, T), so all the functional integrations are, in principle, unambigously defined.

Let us remark that also in Eqs.(15) and (15') we can express the interaction terms, by using the Stokes' theorem, through the field strengths, i.e. $F_{\mu\nu}$ and $M_{\mu\nu}$ respectively, so that we can say in the whole expression Eq.(14) only the field strengths appear. We notice the symmetry

$$Z(e, g; \mu_Q, \mu_P; J_\mu^{ext}, H_\mu^{ext}) =$$
$$= Z(-g, e; \mu_P, \mu_Q; -H_\mu^{ext}, J_\mu^{ext}) . \tag{16}$$

Remember that, in euclidean four space, g and H^{ext} are imaginary, while e and J^{ext} are real.

In order to get the corresponding expressions for Minkowski space one has to perform the formal substitutions in the above formulae, calling here g_E the magnetic coupling constant in euclidean space, which is imaginary, and g_M the same coupling in Minkowski space which is real:

$$\dot q_\mu \dot q_\mu \longrightarrow -i \dot q^\mu \dot q_\mu \qquad \dot p_\mu \dot p_\mu \longrightarrow -i \dot p^\mu \dot p_\mu$$
$$\dot q_\mu A_\mu \longrightarrow -\dot q^\mu A_\mu \qquad \dot p_\mu W_\mu \longrightarrow -\dot p^\mu W_\mu$$
$$ig_E \int d\tau_{\mu\nu} \tilde F_{\mu\nu} \longrightarrow ig_M \int d\tau_{\mu\nu} \tilde F^{\mu\nu} ,$$

where now

$$A_\mu(x) = -\int D(x-y)\, j_\mu(y)\, dy \quad ; \qquad j_\mu = e\int \dot q_\mu\, ds\, \delta(x-q)$$
$$W_\mu(x) = -\int D(x-y)\, h_\mu(y)\, dy \quad ; \qquad h_\mu = g_M \int \dot p_\mu\, ds\, \delta(x-p)$$

and g_M and H^{ext} become real quantities with $eg_M = 2\pi n$ and similarly for the external currents: formally $g_E \to |g_E| = g_M$. The symmetry property (Eq.(16)) for Z remains valid in the Minkowski space formulation.

The generating functional for the case in which some of the charges and some of the monopoles have spin 1/2 can be obtained by inserting extra factors in the integrand of Eq.(14). For instance, for a spin 1/2 charge-spin 1/2 charge interaction we have to insert the factor U_{QQ} (calling $q(s_1)$ and $q(s_2)$ the two trajectories and $\sigma^{(1)}_{\mu\nu}$, $\sigma^{(2)}_{\mu\nu}$ the spin matrices, which act in different spin spaces)

$$U_{QQ} = \left(\tfrac{1}{2}\right)^2 \mathrm{tr}_1\, \mathrm{tr}_2\, \mathbb{P}_{s_1} \mathbb{P}_{s_2} \exp\Big\{ e^2 \int_0^{T_1}\!\!\int_0^{T_2} ds_1\, ds_2 \Big(\sigma^{(1)}_{\mu\nu} \cdot$$
$$\cdot \partial_\mu D(q(s_1)-q(s_2))\, \dot q_\nu(s_2) - \dot q_\mu(s_1)\, \partial_\nu D(q(s_1)-q(s_2))\, \sigma^{(2)}_{\nu\mu} +$$
$$+ \sigma^{(1)}_{\mu\alpha}\, \partial_\mu \partial_\nu D(q(s_1)-q(s_2))\, \sigma^{(2)}_{\nu\alpha} \Big) \Big\}. \tag{17}$$

The symbol $\mathbb{P}_{s_1}$ is a path ordering prescription referring to the matrix $\sigma^{(1)}$. tr_1 acts on the spin space of $\sigma^{(1)}$. This factor represents the spin contribution as can be seen from Eq.(8), where also one finds the factor $(-\frac{1}{2})$ for every spin 1/2 loop. The interaction of a spin 1/2 charge with a spin 1/2 monopole requires the insertion of the factor U_{QP} in the functional integrand:

$$U_{QP} = \left(-\tfrac{1}{2}\right)^2 \mathrm{tr}_1\, \mathrm{tr}_2\, \mathbb{P}_{s_1} \mathbb{P}_{s_2} \exp\Big\{ eg \int_0^{T_1}\!\!\int_0^{T_2} ds_1\, ds_2 \Big(\tilde\sigma^{(1)}_{\alpha\beta} \cdot$$
$$\cdot \partial_\alpha D(q(s_1)-p(s_2))\, \dot p_\beta(s_2) + \dot q_\mu(s_1)\, \partial_\nu D(q(s_1)-p(s_2))\, \tilde\sigma^{(2)}_{\nu\mu} -$$
$$- \tfrac{1}{2} \tilde\sigma^{(1)}_{\alpha\beta}\, \partial_\alpha \partial_\mu D(q(s_1)-p(s_2))\, \sigma^{(2)}_{\mu\beta} + \tfrac{1}{2} \sigma^{(1)}_{\mu\beta}\, \partial_\mu \partial_\alpha D(q(s_1)-p(s_2))\, \tilde\sigma^{(2)}_{\alpha\beta} \Big) \Big\} \tag{18}$$

(by definition $\tilde\sigma_{\alpha\beta} = \frac{1}{2}\, \varepsilon_{\alpha\beta\mu\nu}\, \sigma_{\mu\nu} = \sigma_{\alpha\beta}\, \gamma_5$).

The factor U_{QP} comes from the term $\sigma_{\mu\nu}\tilde F^{\mu\nu}$ in the charge-monopole interaction, in addition to the term appearing also for the bosons. This factor U_{QP} is manifestly symmetric for the charge monopole interchange and $eg \to -eg$. Since

$$\tilde\sigma^{(1)}_{\mu\beta}\, \partial_\mu \partial_\alpha D(q-p)\, \sigma^{(2)}_{\alpha\beta} + \sigma^{(1)}_{\mu\beta}\, \partial_\mu \partial_\alpha D(q-p)\, \tilde\sigma^{(2)}_{\alpha\beta} = \tilde\sigma^{(1)}_{\mu\beta}\, \sigma^{(2)}_{\mu\beta}\, \Box D(q-p)$$

it can also be written as $-\tilde\sigma^{(1)}_{\alpha\beta}\, \partial_\alpha\, \partial_\mu\, D(q-p)\, \sigma^{(2)}_{\mu\beta}$ plus a contact term, due to the fact that $\Box D(q-p) = \delta(q-p)$. We have already made a comment on this contact term and the possibility of introducing a regularization so that the charge and monopole trajectories never cross and the contact term gives no contribution.

IV. APPLICATION OF THE FORMALISM

4.1 General considerations about the perturbative expansion

The formulation of the charge monopole interaction that has been presented and appears in Eqs.(14), (15) and (18) is explicitly covariant. This result can be compared with some previous descriptions of the same problem where the string of the Dirac monopole introduces arbitrarily chosen directions. In our case we avoid the introduction of the string since in the path integral formalism, the interaction appears at the exponent and the Dirac quantization condition makes the ambiguities disappear.

We saw, more precisely, that we can express the generating functional in terms of the electromagnetic field strengths, rather than through the potentials. Then it should be possible to perform a perturbative expansion in such a way that only $F_{\mu\nu}$ and $M_{\mu\nu}$ appear. Actually this agrees with the standard QED result according to which a Feynman amplitude containing only closed charged loop and external photon lines is expressible in terms of the field strengths of the external lines. There are however two points that must be taken into account. The first is that the formalism is intrinsically ill-defined when the path of a monopole crosses (in four-dimensional space) the path of a charge. In our presentation we defined this situation with the prescription of symmetrizing the interaction term (see Eqs.(15") and (18)), this is a logically possible but not compelling choice. The second point is connected with the Dirac condition. As we repeatedly remarked, this condition ensures the definiteness of expressions like Eqs.(14) and (15") whenever expressions like $\frac{1}{2}ig \sum \int_{T_M} d\tau_{\mu\nu} \tilde{F}_{\mu\nu}$ appear at the exponent. On the other hand when the exponent is expanded in series (and the series cut at a certain definite term) the fact that a term like $\frac{1}{2}ig \sum \int_{T_M} d\tau_{\mu\nu} \tilde{F}_{\mu\nu}$ can acquire an addendum $2\pi ni$ gives rise to complications.

The now described difficulties never arise if we can imagine to divide the four space in regions, in some of them only the charges live, in the others only the monopoles. We can visualize the situation in a definite example: let us consider the vacuum polarization due to a virtual monopole that modifies the potential between two charges. In static situations we can enclose the trajectories of the charges, that are straight lines, within indefinite cylinders $\mathcal{C}_1$ of radius R. The rest of the space, $\mathcal{K}$, is available for the monopoles. In any point of the region $\mathcal{K}$ we have for the field of the external charges $\partial_\mu F_{\mu\nu} = \partial_\mu \tilde{F}_{\mu\nu} = 0$, so in that region we can follow the rule that to obtain the amplitude for monopole loops from the corresponding amplitude containing only charge loops, we transform*) F into $\tilde{F}$ and, of course (e, μ_Q) into (g, μ_P), $\mu_{Q,P}$ being the masses of the particles. Moreover it is evident that the possible contact term is excluded and since the surfaces T always lie inside $\mathcal{K}$ it cannot happen that a term like $g \sum \int_{T_M} d\tau_{\mu\nu} \tilde{F}_{\mu\nu}$ acquires an addendum of $2\pi ni$. The procedure we propose is a kind of regularization; in order to make it acceptable it must be shown that in a standard situation it reproduces, in the limit $R \to 0$, the standard results.

4.2 Vacuum polarization

Here we consider the vacuum polarization in more detail. By introducing the electromagnetic field strength due to an external charge describing the path $q(s)$:

$$F_{\mu\nu}(y;q) = ie \int \partial_\mu D(y-q)\, \dot{q}_\nu(s)\, ds - (\mu \leftrightarrow \nu) \quad ,$$

the perturbative term can be written in Minkowski space as

$$\int ds_1\, ds_2\, M_Q(q(s_1), q(s_2)) \quad .$$

In terms of the standard vacuum polarization tensor $e^2(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\, \Pi(z;\mu_Q^2)$, we have

*) Inside $\mathcal{C}_1$ we always have $\partial_\mu \tilde{F}_{\mu\nu} = 0$, while $\partial_\mu F_{\mu\nu} = 0$ does not hold in general, so the rule becomes ambiguous.

$$M_Q(q_1, q_2) = \frac{1}{2} e^2 \int F^{(1)}_{\mu\nu}(y_1; q_1)\, \Pi(y_1 - y_2; \mu_Q^2)\, F^{(2)\mu\nu}(y_2, q_2)\, dy_1 dy_2 . \tag{19}$$

We now define two "cylinders" $\mathcal{C}_i$ within which we confine q_i; and we let $y_{1,2}$ vary over the rest of the space, $\mathcal{K}$. In that case we can define the effect of a monopole loop as

$$M_P(q_1, q_2) = \frac{1}{2} g^2 \int_{\mathcal{K}} \tilde{F}^{(1)}_{\mu\nu}(y_1; q_1)\, \Pi(y_1 - y_2; \mu_P^2)\, \tilde{F}^{(2)\mu\nu}(y_2, q_2)\, dy_1 dy_2 \tag{20}$$

where we can make the general observation that, since $\tilde{F}_1 \tilde{F}_2 = -F_1 F_2$, then the monopole contribution always has the sign opposite to the charge contribution. One can verify that the same prescription applied to the vacuum polarization due to the charge loops, Eq.(19), i.e. the prescription of restricting the integration over y_1 and y_2 to the region $\mathcal{K}$ outside the cylinders $\mathcal{C}_{1,2}$, gives back the standard result of QED when we let the radii of $\mathcal{C}_{1,2}$ go to zero *). The verification is easily done in the situation where the charges are fixed in space and so we calculate the radiative correction to the Coulomb potential. Using this fact, we can estimate the perturbative correction [13] due to monopoles of the static Coulomb potential to be $\frac{e}{4\pi r} \to \frac{e}{4\pi r}(1 + \eta_P)$ with:

$$\eta_P = \frac{-1}{16\sqrt{\pi}\,\alpha} \cdot \frac{e^{-2\mu_P r}}{(\mu_P r)^{3/2}}$$

for $r \gg \frac{1}{\mu_P}$ and using explicitly $eg = 2\pi$.

In order to have a quantitative estimate one can try a comparison of this correction with the standard electronic one η_Q and try to find, viz., the values of r at which $|\eta_P| \approx |\eta_Q|$. We cannot use this asymptotic expression for η_Q, but, better, the opposite approximation [13], holding for $r \ll \frac{1}{\mu_Q}$

*) If R were very small but non-zero we would obtain a correction to the result of the order of $(R\mu_Q)^2$.

$$\eta_Q \approx \frac{2\alpha}{3\pi}\left[-\ln \mu_Q r - \frac{5}{6} - \gamma\right].$$

In order to give definite numbers we can say that $|\eta_P| \approx |\eta_Q|$ for $r = \frac{1}{2\mu_P} F\left(\frac{2\mu_P}{\mu_Q}\right)$, where F is a function steadily decreasing from 4.8 to 3.4 when $\ln 2\mu_P/\mu_Q$ varies from 10 to 50, i.e. when μ_P varies from 5 GeV to 10^{18} GeV.

4.3 Light-light scattering

Another situation where we can compare the possible effects of the monopole with the effects of standard particles is the light-light scattering. Here, for an electron loop we have the matrix element

$$M_Q = e^4 \epsilon_{\mu_1}(k_1)\epsilon_{\mu_2}(k_2)\epsilon_{\mu_3}(k_3)\epsilon_{\mu_4}(k_4) I^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4;\mu_Q) \cdot \delta(k_1+k_2-k_3-k_4) \quad ,$$

where ϵ_{μ_i} are the polarization vectors of the external photons. As we have already discussed, the matrix element can be put into the form [13]

$$M_Q = e^4 f_{\mu_1\lambda_1}(k_1) f_{\mu_2\lambda_2}(k_2) f_{\mu_3\lambda_3}(k_3) f_{\mu_4\lambda_4}(k_4) \cdot J^{\mu_1\mu_2\mu_3\mu_4;\lambda_1\lambda_2\lambda_3\lambda_4}(k_1,k_2,k_3,k_4;\mu_Q)\,\delta(k_1+k_2-k_3-k_4)$$

with $f_{\mu\lambda} = k_\mu \epsilon_\lambda - k_\lambda \epsilon_\mu$.

Now if the effective interaction J represents a loop of monopoles we have to substitute $f_{\mu\lambda}$ with $\tilde{f}_{\mu\lambda}$, e^4 with g^4 and μ_Q with μ_P inside J. Performing these substitutions the tensorial expression of M in terms of the polarization vector changes, and indeed we know from Weinberg's observation [4] on the helicity states that the monopole behaves here in a different way in comparison with the case of the electron. However when the energy of the photons is small with respect to the mass circulating in the loop the scattering amplitude is expressed [13] only in terms of $(F_{\mu\nu}F_{\mu\nu})^2$ and $(F_{\mu\nu}\tilde{F}_{\mu\nu})^2$ and therefore for the monopole it has the same form as in the case of the electron loop. Quantitatively, the total cross-section [13] is expressed as $\sigma = C\omega^6$ when the CM energy ω goes to zero.

Let us write $C = C_e + C_I + \ldots$ where C_e is the pure electron contribution and C_I is the interference term between the contributions of the electron and of the monopole. We get for the ratio (in the case of a fermion monopole)

$$\frac{C_I}{C_e} = 2\left(\frac{\mu_e}{2\alpha\mu_P}\right)^4$$

i.e. a very small number for the conceivable values of the monopole mass μ_P.

4.4 Final considerations

In the examples that have been presented, the treatment was perturbative in g and the monopole-monopole interaction was not taken into account. This very strong interaction could give rise to tightly bound states where the peculiar features of the monopole are, possibly, difficult to recognize; moreover to deal with such situations we must commit ourselves with some definite description of the binding, which is beyond the scope of the present treatment. Here we have faced the problem: which is the contribution due to a definite kind of virtual particles to Green functions of external electrons or photons, assuming that these virtual particles interact with the external world as the monopoles are supposed to do, namely only through their coupling with the dual of the electromagnetic field.

APPENDIX

In order to make more direct the connection of the formalism described up till now with the standard perturbation expansion we calculate explicitly the second order vacuum polarization due to a loop of spinless bosons. We are thus led to consider the term

$$\Pi_{\mu\nu}(x,y) = \frac{\delta}{\delta A_\mu(x)}\frac{\delta}{\delta A_\nu(y)}\,\mathrm{tr}\left[-\ln\left(-D^2+\mu^2\right)\right] =$$

$$= -e^2\int e^{-\mu^2 T}\frac{dT}{T}\int_{q(0)=q(T)}\mathcal{D}q\int\delta(x-q(s'))\,\dot q_\mu(s')\,ds' \cdot$$

$$\cdot\int\delta(y-q(s''))\,\dot q_\nu(s'')\,ds'' \cdot e^{-\frac{1}{4}\int_0^T \dot q^2 ds}. \qquad \text{(A.1)}$$

It is useful to define $q(s) = q(0) + \bar q(s)$ with $\bar q(0) = \bar q(T) = 0$, so formally $\mathcal{D}q = dq_0\,\mathcal{D}\bar q$ (from now on, however, the bar over $\bar q$ will be omitted and the domain of the functional integration $\{q(0) = q(T)=0\}$ will be denoted by $\mathcal{Z}$). We take the Fourier transform of $\Pi_{\mu\nu}(x,y)$ and, integrating over dq_0, we get

$$\Pi_{\mu\nu}(k,k') = \int e^{ikx-ik'y}\,\Pi_{\mu\nu}(x,y) = -e^2(2\pi)^4\delta(k-k')\,\cdot$$

$$\cdot\int e^{-\mu^2 T}\frac{dT}{T}\int_{\mathcal{Z}}\mathcal{D}q\iint_0^T \dot q_\mu(s')\,\dot q_\nu(s'')\,e^{ik(q(s')-q(s''))}\,ds'ds''\;e^{-\frac{1}{4}\int_0^T\dot q^2 ds}. \qquad \text{(A.2)}$$

We verify explicitly gauge invariance $k_\mu\Pi_{\mu\nu} = \Pi_{\mu\nu}k'_\nu = 0$ owing to the boundary conditions for q ($k_\lambda\dot q_\lambda(s') \approx -i\frac{d}{ds'}$). So in standard form we can write

$$\Pi_{\mu\nu}(k,k') = -(2\pi)^4 e^2\,\delta^4(k-k')\left(\delta_{\mu\nu}A(k^2) - k_\mu k_\nu B(k^2)\right) \qquad \text{(A.3)}$$

with $A(k^2) = k^2 B(k^2)$.

As intermediate computational aid we introduce

$$\Phi[J] = \int_{\mathcal{Z}}\mathcal{D}q\;e^{-\frac{1}{4}\int_0^T\dot q^2 ds}\;e^{i\int_0^T\dot q(s)J(s)ds} = \int_{\mathcal{Z}^*}\mathcal{D}q^*\;e^{-\frac{1}{4}\int_0^T\dot q^{*2}ds}\;e^{-\int_0^T J^2(s)ds},$$

having written $q(s) = q^*(s) - 2i\int_0^s J(s')\,ds'$. Then $q^*(0) = 0$ but $q^*(T) = 2i\int_0^T J(s')\,ds' \equiv 2iQ^*$ and this condition defines $\mathcal{Z}^*$. With the normalization conditions for $\mathcal{D}q$ we obtain

$$\Phi[J] = e^{-\int_0^T J^2(s)ds}\;\frac{1}{16\pi^2 T^2}\;e^{Q^{*2}/T^2}.$$

In order to get $\Pi_{\mu\nu}$ we must now perform the functional derivatives with respect to $J_\mu(x)$ and then put $J_\mu(s) = k_\mu[\vartheta(s'-s) - \vartheta(s''-s)]$ so that $i\int\dot q(s)\,J(s)\,ds = ik\,[q(s') - q(s'')]$. In so doing we must remember that $\delta Q^*/\delta J(s') = 1$. We now give the form of

$$\left.\frac{\delta}{\delta J_\mu(s')}\frac{\delta}{\delta J_\nu(s')}\Phi[J]\right|_{J_\lambda = k_\lambda[\vartheta(s'-s)-\vartheta(s''-s)]}.$$

By taking into account that

$$\int_0^T J^2(s)ds = k^2|s'-s''|$$

and

$$Q^{*2}/T = k^2(s'-s'')/T, \quad \text{we obtain}$$

$$\frac{\delta}{\delta J_\mu}\frac{\delta}{\delta J_\nu}\Phi[J] = \frac{1}{16\pi^2T^2}\,e^{-k^2|s'-s''|}\,e^{k^2(s'-s'')/T}\,.$$

$$\cdot\left\{\delta_{\mu\nu}\left[-2\delta(s'-s'') + \frac{2}{T}\right] + 4k_\mu k_\nu\left[\vartheta(0)[1-\vartheta(0)] - \frac{|s'-s''|}{T} + \frac{(s'-s'')^2}{T^2}\right]\right\}.$$

This form individuates separately the contribution to $A(k^2)$ and to $B(k^2)$; in this last term there is an ambiguity, implied by the expression $\vartheta(0)\,[1-\vartheta(0)]$, that must be determined by means of the requirement of gauge invariance $k^2B^2 = A^2$. For practical purposes we can calculate $A(k^2)$, which is free from that ambiguity :

$$A(k^2) = \int \frac{dT}{T^2} e^{-\mu^2 T} \iint_0^1 du' du'' \frac{1}{8\pi^2} [1 - \delta(u'-u'')] e^{-k^2|u'-u''|T + k^2(u'-u'')^2 T} . \tag{A.4}$$

We see that $A(0) = 0$, as it must be, but $\partial A(k^2)/\partial k^2$ is not zero. So we must subtract this term; this operation corresponds to photon wave function renormalization *)

$$\bar{A}(k^2) = \int \frac{dT}{T^2} e^{-\mu^2 T} \iint_0^1 du' du'' \frac{1}{8\pi^2} [1 - \delta(u'-u'')] \sum_{2}^{\infty}{}_m \frac{1}{m!} (-k^2 T)^m \cdot$$

$$\cdot \left[|u'-u''|^2 - (u'-u'')^2 \right]^m = -\frac{1}{4\pi^2} k^2 \sum_{1}^{\infty}{}_n \left(-k^2/\mu^2\right)^n \frac{1}{n(n+1)} B(n+2, n+3) =$$

$$= -\frac{1}{8\pi^2} k^2 \sum_{1}^{\infty}{}_n \left(-k^2/4\mu^2\right)^n (2n-2)!! / (2n+3)!! . \tag{A.5}$$

Considering the same type of expansion for $B(k^2)$ and subtracting the divergent term corresponding formally to $B(0)$, we find an explicit determination of the ambiguous term compatible with gauge invariance, namely $\vartheta(0) = \frac{1}{2}$, and we can in conclusion write

$$\Pi_{\mu\nu}(k, k') = -(2\pi)^4 e^2 \delta(k-k') (\delta_{\mu\nu} - k_\mu k_\nu / k^2) \bar{A}(k^2) .$$

This final expression can, at this point, be compared and found to be consistent with the expression calculated in a standard way (see, e.g. Akhiezer-Berestetskij [14]).

*) The δ function never gives contribution to $\bar{A}$, its unique effect is to produce $A(0) = 0$.

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