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INDUCED YUKAWA COUPLING AND FINITE MASS *

Y. Fujimoto
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We propose that the Yukawa couplings in the unified theories could be of induced nature. The idea is implemented in the gauge theory with either weak or horizontal $SU_L(2) \times SU_R(2)$ symmetry. A related subject of finite fermion mass is also discussed.

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I. INTRODUCTION

On the unified field theories it is a standard practice to introduce elementary scalars to induce the spontaneous breaking A-H-K (Anderson, Higgs and Kibble) mechanism and also to make fermions massive through Yukawa interaction. The resulting theory contains very many parameters, particularly due to various Yukawa couplings, and this fact motivates us to look for a simpler form of the unified theory with less number of parameters. To economize the theory there have been two approaches. One way 1) is to regard only fermions as elementary. In the other approach 2) fermions and gauge bosons are elementary fields. In both approaches the scalars are bound states of fermions and the scalar self-interaction and Yukawa interaction which are independently introduced in the standard unified theory are supposed to be some secondary products induced by the fundamental interaction, i.e. four fermion interaction in Ref.1 and gauge interaction in Ref.2. We could achieve in this manner a greatly simplified unified theory. However, the theoretical status of these approaches are in our opinion far from certain, the scheme being hampered by the difficulty in dealing with the strong coupling problem.

In the present paper first we take the view that the gauge bosons and fermions are the elementary particles, and then assume that scalars are created through gauge interaction as bound states of fermions and they interact with gauge bosons in the ordinary manner and form the standard potential so that spontaneous breaking takes place 3). By assuming we dissociate the underlying strong interaction from the theory. In this setting we calculate the induced Yukawa coupling and finite mass for the sub-set of fermions. Yukawa coupling can be induced through the one-loop diagram as is shown in Fig.1. Also closely related diagrams (Fig.2) can lead to finite mass. (The diagrams concerned are of the common origin and come from gauge boson scalar interaction, $A_\mu \phi^* A^\mu \phi$.) In this way we could achieve a unified theory with less parameters.

In the following we discuss the subjects separately, induced Yukawa coupling in Sec.II and finite mass in Sec.III. As an internal local symmetry we choose $SU_L(2) \times SU_R(2)$. It refers to either weak 4) or horizontal group 5).

II. INDUCED YUKAWA COUPLING

For illustration purpose we choose a model with local $SU_L(2) \times SU_R(2)$ symmetry. The model consists of two fermion doublets $\psi_L = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L$, $\psi_R = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_R$, singlet fermions χ_L, χ_R . ψ_2 and χ mix with each other (see Fig.1). In Fig.1 dotted lines denote mixings and ϕ_0, ϕ_1, ϕ_2 are associated scalars.

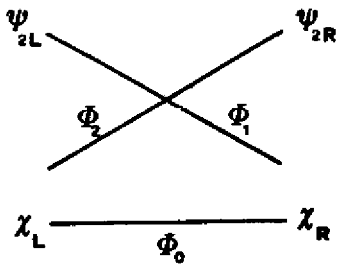

Fig.1

ϕ_0 is a singlet and ϕ_1 and ϕ_2 transform as (2,1), (1,2) with respect to $SU_L \times SU_R(2)$. Scalars are employed to cause mixings here but the origin of the mixing is not essential to our discussion. The important point is that there is no direct coupling between ψ_L and ψ_R and that ψ_2 has to be massive so that non-vanishing Yukawa coupling results. (The trick was first used by Georgi and Glashow [6] and later recounted by Barr and Zee [7] to calculate finite mass.) The gauge bosons W_L and W_R mix with each other through a scalar multiplet ϕ which does not have a direct Yukawa coupling with ψ_L and ψ_R.

In this setting we can calculate the Yukawa coupling (f) between ψ and ϕ induced through the diagram in Fig.2.

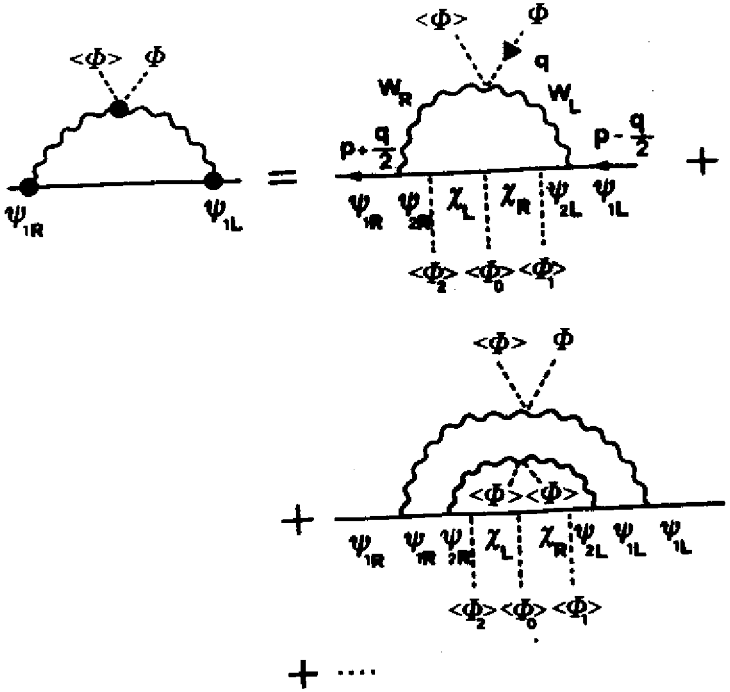

Fig.2

In Fig.2 $\langle\phi\rangle$ refers to VEV (vacuum expectation value).

We calculate in the Landau gauge the one-loop diagram in Fig.2. After resolving the mixing of W_K and W_R the first diagram in Fig.2 turns into Fig.3.

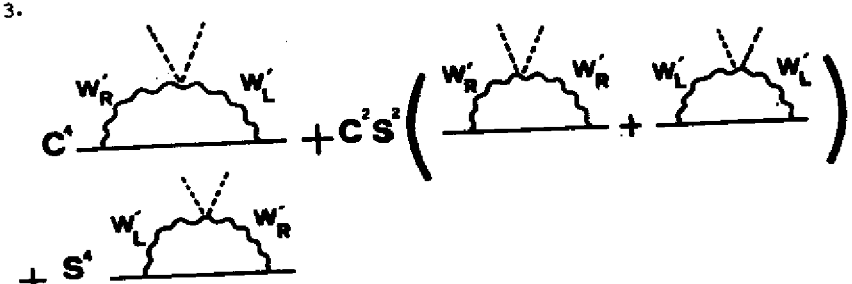

Fig.3

* prime denotes physical or diagonalized, ** $C = \cos\theta$, $S = \sin\theta$ θ denotes the mixing angle.

The result of calculation is expressed below in two limits:

1) $p, q \to 0$

$$f_{\bar{\psi}_1 \psi_1 \phi} = \frac{3g^4}{8\pi^2} m_{\psi_2} C' C'' \langle\phi\rangle \times$$

$$\times \left\{ \frac{C^4+S^4}{M_R^2-M_L^2} \left(\frac{M_R^2}{M_R^2-m_{\psi_2}^2} \log\frac{M_R^2}{m_{\psi_2}^2} - \frac{M_L^2}{M_L^2-m_{\psi_2}^2} \log\frac{M_L^2}{m_{\psi_2}^2} \right) \right.$$

$$+ C^2S^2 \left(\frac{\log M_R^2/m_{\psi_2}^2}{M_R^2-m_{\psi_2}^2} + \frac{\log M_L^2/m_{\psi_2}^2}{M_L^2-m_{\psi_2}^2} \right) -$$

$$\left. - (\psi_2 \leftrightarrow \chi) \right\} \qquad (1)$$

where $C' = \cos\theta'$, $C'' = \cos\theta''$, θ' and θ'' denote mixing angles associated with ψ_2 and χ. M_L and M_R are masses of W_L and W_R, respectively.

2) $P^2 = -\frac{q^2}{4} >> m_{\psi_2}^2$, M_L^2, M_R^2

$$f_{\bar{\psi}_1 \psi_1 \phi} \sim \text{const} \times \frac{\langle\phi\rangle m_{\psi_2}}{-g^2} + O\left(\frac{m_{\psi_2}^2}{g^2}, \frac{M_L^2}{g^4}, \frac{M_R^2}{g^4}, \frac{m_\chi^2}{g^4} \right) \qquad (2)$$

In both cases f is smaller than g^4, which could be viewed as a natural explanation of the smallness of the Yukawa coupling. It is interesting to note that the high energy limit in the latter case gives a vanishing Yukawa coupling.

To calculate $f_{\bar{\psi}_1 \psi_2 \phi}$ we only have to replace $W_{L,R}$ with $Z_{L,R}$ (the gauge boson associated with the third component of SU(2)) in Eqs.(1) and (2) (see Fig.4). The cross coupling $f_{\bar{\psi}_1 \psi_2 \phi}$ results from the diagram of Fig.5.

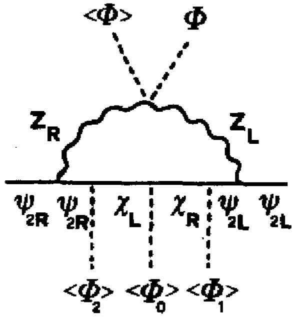

Fig.4

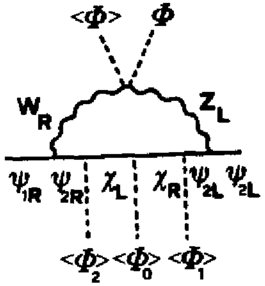

Fig.5

III. FINITE MASS

Another diagram which originates from $\phi^*\phi\, A_\mu A^\mu$ is the self-energy diagram of fermions (Fig.6)

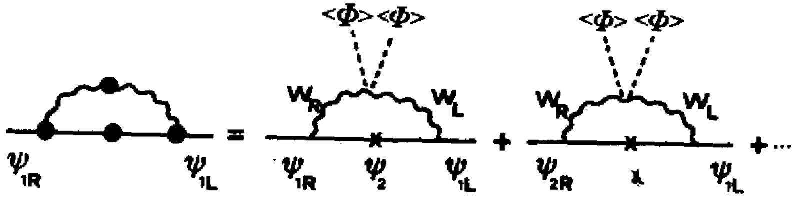

Fig.6

Calculating Fig.6 at one-loop level we obtain

$$m_{\psi_1} = \frac{3m_{\psi_2}}{16\pi^2} g^2 c'c^{e}cs\left\{\frac{M_R^2}{M_R^2 - m_{\psi_2}^2}\log\frac{M_R^2}{m_{\psi_2}^2} - \frac{M_L^2}{M_L^2 - m_{\psi_2}^2}\log\frac{M_L^2}{m_{\psi_2}^2} - (\psi_2 \leftrightarrow \chi)\right\}. \quad (3)$$

As the result shows the finite mass m_{ψ_1} is of the order of, or less than αm_{ψ_2}. If there exist large mixings then $m_{\psi_1} \sim \alpha m_{\psi_2}$, which may be the case for $SU_L^H(2) \times SU_R^H(2)$. Small mixing case is relevant for $SU_L(2) \times SU_R(2)$ since W_R is expected to be superheavy. In such a case we may regard ψ_1 and ψ_2 as neutrino and electron, and thus neutrino can acquire a very small Dirac mass. If the electron (but not the neutrino) obtains mass "dynamically" due to the electromagnetic interaction we may do without χ and the neutrino mass is given by

$$m_\nu = \frac{3m_e}{8\pi^2} g^2 cs\left\{\frac{M_R^2}{M_R^2 - m_e^2}\log\frac{M_R^2}{m_e^2} - (R \leftrightarrow L)\right\} \quad (4)$$

CS is expected to be reasonably smaller than 10^{-5} when grand unification is taken into account 8). And then the resulting neutrino mass is smaller than the experimental upper limit.

Lastly, together with the authors of Ref.6 we note that finite mass is not calculable if there is a direct Yukawa coupling between ψ_L and ψ_R.

IV. COMMENTS

We discussed the possibility of calculating the finite mass and Yukawa coupling. The analysis can be applied, with modifications, to the models based on various other groups. In case $U_Y(1)$ is added to the weak group to make it realistic the result changes slightly due to the extra mixings among neutral gauge bosons.

It may look disturbing to introduce a new fermion like χ. Actually, it is not if we regard it as a reflection of the presence of superheavy fermions which naturally appear in the grand unified theories beyond SU(5). 9) The philosophy that the structure of superheavy particles be reflected on that of light particles has been shared by many people.

Lastly, we disregarded in this paper the possible contribution from scalars. The reason is; our initial view was that all Yukawa couplings are dynamically induced. If so ~~their~~ contributions should be relatively negligible.

NOTE ADDED IN PROOF

After completion of the work I learned from Professor J. Strathdee that J.C. Pati, J. Strathdee and Abdus Salam also had the idea of induced Yukawa coupling in their recent work 10).

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REFERENCES

1) Y. Nambu and G. Jona Lasinio, Phys. Rev. 122 (1961) 345;
Y. Eguchi and H. Sugawara, Phys. Rev. D10 (1974) 4257;
H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. D15 (1977) 480.

2) L. Susskind and S. Dimopoulos, Nucl. Phys. B155 (1979) 237;
S. Weinberg, Phys. Rev. D19 (1979) 1277.

3) Abdus Salam and J. Strathdee, Phys. Rev. D18 (1978) 4713.

4) J. Pati and Abdus Salam, Phys. Rev. D8 (1973) 775.

5) T. Yamagida and Y. Maehara, Progr. Theor. Phys. 61 (1979) 1434;
F. Wilczek and A. Zee, Phys. Rev. Letters 42 (1979) 421.

6) H. Georgi and S.L. Glashow, Phys. Rev. D7 (1973) 2457.

7) S.M. Barr and A. Zee, Phys. Rev. D17 (1978) 1854.

8) Q. Shafi and C. Wetterlich, Phys. Letters B85 (1979) 52;
Y. Fujimoto, Phys. Letters B89 (1980) 347.

9) SU(7): P.H. Frampton, Phys. Letters B88 (1979) 299;
SU(8): Y. Fujimoto, Nucl. Phys. B182 (1981) 242;
J.E. Kim and H.S. Song, Phys. Rev. D22 (1980) 1753.
SU(9): P.H. Frampton, Phys. Letters 89B (1980) 352;
Y. Fujimoto and P. Sodano, Phys. Rev. D23 (1981) 1657;
SU(9) × SU(9): A. Davidson and K.C. Wali, Phys. Rev. D23 (1981) 477;
SU(11): H. Georgi, Nucl. Phys. B156 (1979) 126.

10) J.C. Pati, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/80/183.

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