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ABSTRACT

An idea of L-R asymmetry is proposed for the grand unification schemes. The idea provides an intrinsic mechanism to obtain standard model charges of fermions in the case of more than one weak gauge boson. It is elaborated within a scheme based on the partial symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ where the coupling constants g_L and g_R corresponding to the chiral SU(2) factors are assumed to be different from each other. Then, the embedding of this structure within the simple symmetry SO(10) is shown. In both cases, a consistent description of vector particle masses is given. These two schemes are considered as primary models to realize the L-R asymmetry idea due to the lack of family unification. However, in a subsequent work, we will show that the SO(14) unification of the three families can be obtained within the framework of L-R asymmetry. All formulations are carried out with the aid of a mathematical method that we recently proposed for the Lie algebra representations of classical groups.

I. INTRODUCTION

The symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is now well-established [1] for the present energy level of our understanding about the fundamental forces in nature. It also gives a basic framework towards the unification of these forces. The second attempt in this direction is the embedding of this partial symmetry in a simple symmetry such as SU(5).[2] However, the generation (or family) number problem prevents us from accepting the fact that this minimal model is the ultimate for grand unification [3]. Therefore, several models are proposed in the literature to overcome this replication problem of generations [4]. Except for the semi-simple horizontal [5] and maximal symmetry [6] models, these can be grouped into two parts:

i) Reducible models of unitary symmetries [7];
ii) Orthogonal models of horizontal symmetries [8].

It will be pointed out in another work that generally an anomaly free reducible set of representations of unitary groups can be included in an appropriate representation of orthgonal groups. The typical manifestation of these orthogonal models is an intrinsic L-R symmetry encounteredin the weak interaction sectors. In fact, this L-R symmetry idea is already known as an alternative to the V-A asymmetry and the corresponding partial GUT is based on the symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$. [9] The main idea here is that the weak interactions which behave as V-A asymmetric at present energy levels [10] get essentially a vector-like character just like the strong and electromagnetic ones. This is expected at sufficiently high energies and is realized by a discrete symmetry [*] between the weak coupling constants governing V-A and V+A weak interactions. Phenomenologically, the crucial point for the L-R symmetric models is the existence of two neutral and also two charged gauge vectors mediating the weak interactions. Especially, from the point of view of the expectations from the next generation accelerators which will operate in the near future, the number of weak gauge bosons is really crucial. However, the symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_4$ should manifest itself at present, whatever the number of weak gauge bosons may be, which are expected to be seen in future. Consequently, the $SU(2)_L$ gauge bosons should have minimal masses among all massive gauge bosons when one considers effective interactions. This is provided in L-R symmetric models with a large asymmetry between the expectation values of the corresponding Higgs scalars.

*) To the author's knowledge in all symmetric models this discrete symmetry is accepted. There is only one exception which is shown in Ref.11a. However, the point of view in Ref.11a is completely different from this paper based on an unpublished work (Ref.11b).

Instead, we elaborate another idea. Here the idea of the existence of the right-handed interactions which are expected to be seen at high energies is maintained as in the L-R symmetric models, i.e. we assume two neutral and two charged bosons of the SU(2) type. But, the right-handed (weak) interactions are suggested as a different type of interaction in the sense that they are unified with the others only at some symmetry limit. Hence, no discrete symmetry between the left and the right weak coupling constants g_L and g_R is assumed. This does not invoke for any novelty to describe present charged current phenomenology [10]. On the other hand, the essential difference between the V-A asymmetric and L-R symmetric models is in the weak charge assignments of the right-handed fermions and this requires further investigations of the effective neutral currents for L-R asymmetric models.*) With the introduction of the L-R asymmetry, the chiral fermion charges **) are obtained intrinsically just as in the standard model and consequently there is no need for any additional phenomenological investigation if there is a consistent description of the vector masses. These are carried out by an appropriate mixing matrix ***) among the neutral vectors of the model, which are not weak interaction eigenstates. This mixing matrix is an orthogonal matrix expressed in terms of two angular parameters θ and φ which are defined as

$$\frac{1}{g_L} = \frac{1}{e}\sin\theta\sin\varphi \quad , \quad \frac{1}{g_R} = \frac{1}{e}\sin\theta\cos\varphi \quad .$$

With this specification, the neutral current of the standard model will be provided but in terms of an effective parameter $x \equiv \sin\theta \sin\varphi$ which is equivalent to the Weinberg angle in our model. An 3/8 value is obtained for this parameter at symmetry limit. A consistent description of vector masses is also realized with the aid of this mixing matrix.

On the other hand, it is known that the partial symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ can be embedded in the simple symmetry SO(10). [15] This can easily be seen from the isomorphism of

$$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R \sim SO(6) \otimes SO(4) \quad ,$$

*) The constraints of the present phenomenology on these models have recently been studied in Ref.12.

**) Recent results on this point are given in Ref.13.

***) We proposed this structure three years ago in an unpublished work (Ref.14).

where $SO(6) \otimes SO(4)$ is the maximal subgroup of maximal rank of the group SO(10). We will show that this embedding is also possible for the L-R asymmetric model. The masses of the fermions and vector bosons are considered also for this simple mode of symmetry. However, it is not expected to get a satisfactory mass spectrum for fermions within the models with family replication. For all of these points, we use a mathematical formalism which we developed recently [16] for the Lie algebra representations of classical groups.

II. THE L-R ASYMMETRY

We develop the L-R asymmetry idea on the partial symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$. As in all models based on this symmetry, the fermions sit in the direct product multiplets

$$(\psi)_{Ai} \doteq \begin{bmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{bmatrix} \qquad (2.1)$$

being in line with the charge operator

$$Q = \frac{1}{2}\left[\lambda_L^3(2) + \lambda_R^3(2)\right] \otimes 1_4 + \frac{1}{\sqrt{6}}\, 1_2 \otimes \lambda^{15}(4) \quad . \qquad (2.2)$$

Now and then the SU(4) indices are A,B,C,... = 1,2,3,4 and the SU(2) indices i,j,k,... = 1,2. We commonly use $\lambda^\alpha(N+1)$ for the SU(N+1) Lie algebra generators normalized as

$$Tr\left[\lambda^\alpha \lambda^\beta\right] = 2\,\delta_{\alpha\beta} \quad . \qquad (2.3)$$

Although it is well known, we give the formation of the model in some detail which will be useful for further applications. The invariant chiral derivatives for the multiplets (2.1) are

$$D\psi_{L,R} = \partial\psi_{L,R} - \frac{i}{\sqrt{2}}\, g\, \Sigma(N)\psi_{L,R} - \frac{i}{\sqrt{2}}\, g_{L,R}\, \Sigma(1)\psi_{L,R} \,, \qquad (2.4)$$

where the Lorentz indices are suppressed. In such expressions, we use as group operators of SU(N+1)

$$\Sigma(N)=\sum_{a=1}^{N} Z_a H^a+\sum_{\alpha\in\phi^+(N)}\left[W_\alpha E^\alpha+W_{-\alpha}E^{-\alpha}\right] . \tag{2.5}$$

In this expression, $\phi^+(N)$ is the positive root system of SU(N+1), the generators H^a are the elements of the Cartan subalgebra of SU(N+1) and the generators $E_{\mp\alpha}$ are the remaining generators of the SU(N+1) Lie algebra. We call these generators H^a and $E_{\mp\alpha}$ neutral and charged operators of the Lie algebra respectively. Corresponding group vectors Z_a and $W_{\mp\alpha}$ are also called in the same manner. These group vectors may not always be the interaction eigenstates. The restriction of the general expression (2.5) to the present model needs use of the fundamental representations because the transformation properties of the chiral multiplets such as

$$\Psi_L\sim\Psi(4,2_L,0) \quad , \quad \Psi_R\sim\Psi(4,0,2_R) \tag{2.6}$$

in an obvious notation for $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$. It should be stressed that this choice lies behind almost all family unification schemes. The fundamental representation weights μ_A of SU(4) are determined as

$$\begin{aligned}\mu_1&=(1 \quad 1 \quad 1)\\ \mu_2&=(-1 \quad 1 \quad 1)\\ \mu_3&=(0 \quad -2 \quad 1)\\ \mu_4&=(0 \quad 0 \quad -3) \quad .\end{aligned} \tag{2.7}$$

The notation

$$\mu_A^a\equiv(m_{A1},m_{A2},m_{A3})=\frac{1}{\sqrt{2}}m_{A1}O_{1a}+\frac{1}{\sqrt{6}}m_{A2}O_{2a}+\frac{1}{\sqrt{12}}m_{A3}O_{3a} \tag{2.8}$$

was formally used in expressions (2.7). If the parameters O_{ab} are assumed as the elements of an orthogonal matrix, it can easily be investigated that the weights (2.7) are the weights corresponding to the fundamental representation of SU(4). However, we give the systematics of these expressions for all classical groups in a subsequent paper. Now, the neutral generators H^a are specified as

$$H^a_{AB}=\mu_A^a\,\delta_{AB} \tag{2.9}$$

or equivalently

$$H^a=\frac{1}{\sqrt{2}}T^b O_{ba} \quad , \tag{2.10}$$

where

$$\begin{aligned}T^1&\equiv\lambda^3(4)=diag(1,-1,0,0)\\ T^2&\equiv\lambda^8(4)=\frac{1}{\sqrt{3}}diag(1,1,-2,0)\\ T^3&\equiv\lambda^{15}(4)=\frac{1}{\sqrt{6}}diag(1,1,1,-3) \quad .\end{aligned} \tag{2.11}$$

If we define the fields

$$G_a=O_{ab}Z_b \tag{2.12}$$

it is seen that colourless QCD interactions arise from the fields G_1 and G_2 with the coupling constant g. This is the reason for the labelling $SU(4)_C$. However, it is also seen that the remaining field G_3 would not serve us as the photon field because it can only give the e- and d-quark charges for the elements of the chiral fermion multiplets. Then, "weak mixings" among the neutral vectors remaining from the colourless gluon fields G_1, G_2 are compelling. On the other hand, the charged and coloured interactions of $SU(4)_C$ originate from the positive root system $\phi^+(3)$ defined in terms of the fundamental representation weights μ_A as

$$\phi^+(3)=\{\mu_A-\mu_B \; ; \; A<B=1,2,3,4.\} \quad . \tag{2.13}$$

It is now seen how the coloured QCD interactions arise from the roots

$$\alpha_1=\mu_1-\mu_2 \; , \; \alpha_2=\mu_2-\mu_3 \; , \; \alpha_1+\alpha_2=\mu_1-\mu_3$$

which are the roots of the subgroup $SU(3)_C$ of $SU(4)_C$. Hence, the coloured gluon fields are W_{α_1}, W_{α_2}, $W_{\alpha_1+\alpha_2}$. Let us note here the expressions

$$W_{-\alpha}=W_\alpha^\dagger \quad , \quad E_{-\alpha}=E_\alpha^T \quad , \tag{2.14}$$

where $\dagger$ denotes hermitian and T transpose conjugation. The correspondence with the conventional Gell-Mann basis λ^α is, for example,

$$E_{\alpha_1} W_{\alpha_1} + E_{-\alpha_1} W_{-\alpha_1} = \frac{1}{\sqrt{2}} \left[W_1 \lambda^1(4) + W_2 \lambda^2(4) \right] , \tag{2.15}$$

where $W_{\alpha_1} \equiv \frac{1}{\sqrt{2}} (W_1 - iW_2)$. We should say that the existence of such a correspondence for all classical groups is due to Serre's theorem [17]. Further it will be useful here to recall the relations

$$\left[E_\alpha(4) \right]_{AB} \neq 0 \quad \text{for which} \quad \alpha = \mu_A - \mu_B . \tag{2.16}$$

Let us mention that the group algebra determines the non-zero elements in (2.16) only to the extent of ∓ 1 and only the possibility of +1 is chosen in the conventional Gell-Mann basis. The charged interactions of $SU(4)_C$ will originate from the roots

$$\begin{aligned} \alpha_3 &= \mu_3 - \mu_4 = (0 \;\; -2 \;\; 4) \\ \alpha_2 + \alpha_3 &= \mu_2 - \mu_4 = (-1 \;\; 1 \;\; 4) \\ \alpha_1 + \alpha_2 + \alpha_3 &= \mu_1 - \mu_4 = (1 \;\; 1 \;\; 4) , \end{aligned} \tag{2.17}$$

where the corresponding vectors

$$X_1 \equiv W_{\alpha_1+\alpha_2+\alpha_3} , \; X_2 \equiv W_{\alpha_1+\alpha_2} , \; X_3 \equiv W_{\alpha_1}$$

are the leptoquarks which are coupled to the vector-like currents

$$\bar{\psi}_{a1} \gamma \nu + \bar{\psi}_{a2} \gamma e$$

with a coupling strength $g/\sqrt{2}$. Because the leptoquarks are only $SU(4)_C$ fields, i.e. they are unmixed, their charge values can be directly read off from the third components of the expressions (2.17) as 4/3.

Our notation for $SU(2)_L$ and $SU(2)_R$ interactions are, being in line with (2.5),

$$\Sigma(2_{L,R}) = Z_{L,R}\, T(2) + W_{L,R}\, E(2) + W^\dagger_{L,R}\, E^T(2) ,$$

where $T(2) = \frac{1}{\sqrt{2}} \lambda^3(2)$. As is known, the condition

$$\frac{g_R^2}{M_{W_R}^2} \ll \frac{g_L^2}{M_{W_L}^2} \tag{2.18}$$

must be satisfied at present energies for the suppression of the right-handed charged currents. Because of the existence of left-handed and also right-handed charges for neutral weak interactions, it will be more useful to call the interactions originating from g_L and g_R as "light" and "heavy" weak interactions, respectively. *) Then, the light-weak interactions will be those which manifest themselves at present energies. Now, we will show that the light weak interactions have completely the same weak charges as those of the standard model. Hence, together with the condition (2.18), all phenomenological requirements of the symmetry $SU(3)_C \otimes SU(2)_L \otimes SU(2)_Y$ shall be provided within the context of the symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$. Although the difference is not appreciated at present energy levels, the true specification of the symmetry is apparently vital on the way going to the family unification.

As was previously emphasized, the compelling mixings among the neutral group vectors G_3, Z_L and Z_R are defined as

$$\begin{aligned} \frac{3}{\sqrt{6}} g G_3 &\equiv \sum_{a=1}^{3} e\, C_{1a} A_a \\ g_L Z_L &\equiv \sum_{a=1}^{3} e\, C_{2a} A_a \\ g_R Z_R &\equiv \sum_{a=1}^{3} e\, C_{3a} A_a , \end{aligned} \tag{2.19}$$

where the three neutral fields A_a are the weak interaction eigenstates. The corresponding chiral charges of these physical fields which are the couplings to the left and also the right-handed neutral fermion currents will be

$$\begin{aligned} Q^a_{\nu L} &= \frac{e}{2}(-C_{1a} + C_{2a}) , \quad Q^a_{uL} = \frac{e}{2}\left(\frac{1}{3} C_{1a} + C_{2a}\right) \\ Q^a_{eL} &= \frac{e}{2}(-C_{1a} - C_{2a}) , \quad Q^a_{dL} = \frac{e}{2}\left(\frac{1}{3} C_{1a} - C_{2a}\right) \end{aligned} \tag{2.20}$$

*) However, recently, there have been some works [18] which state that the heavy weak interactions are not too heavy.

and the right-handed charges can be obtained by replacing C_{2a} with C_{3a}. The specification of our model for the mixing parameters C_{ab} is as in the following:

$$C_{11} = 1 \qquad C_{12} = -\frac{x}{\Lambda} \qquad C_{13} = -\frac{z}{\Lambda}$$
$$C_{21} = 1 \qquad C_{22} = \frac{\Lambda}{x} \qquad C_{23} = 0$$
$$C_{31} = 1 \qquad C_{32} = -\frac{x}{\Lambda} \qquad C_{33} = \frac{1}{z\Lambda} \quad , \qquad (2.21)$$

where the definitions in terms of two angular parameters θ and φ are:

$$x = \sin\theta\sin\varphi \;,\; z\cos\theta = \sin\theta\cos\varphi \;,\; \Lambda = \sqrt{1-x^2} \;. \qquad (2.22)$$

The orthogonality of the mixings between the fields (G_3, Z_L, Z_R) and (A_1, A_2, A_3) are provided by the expression (2.21), since it is required by the invariance of the kinetic Lagrangian. Moreover, under the specification (2.21), the photon field is A_1 and the mediator of the light weak interactions is A_2. Here, the angular parameter φ specifies the disparity between the weak interactions as

$$tg\varphi \equiv \frac{g_R}{g_L} \qquad (2.23)$$

and for the description of the electromagnetic interactions, the expresssion

$$\frac{e^2}{g_L^2} = \frac{3}{8} \qquad (2.24)$$

should be satisfied at the symmetry limit where all coupling constants are equal. As is promised, the chiral charges (2.20) give the expressions

$$Q_{iL}^2 = \frac{e}{x\Lambda}\left(T_{3L}^i - x^2 Q^i\right)$$
$$Q_{iR}^2 = \frac{e}{x\Lambda}\left(-x^2 Q^i\right) \qquad (2.25)$$

if one uses the mixing parameters in (2.21). Here, the index i denotes the fermion species u, d, e, ν and T_{3L}^i, Q^i are their standard model

assignments. Hence, the charges in (2.25) are just those of the standard model but in terms of the effective parameter x instead of the Weinberg angle. Then, we can effectively take

$$x = \sin\theta_W$$

and (2.24) gives its symmetry limit value as 3/8. In conclusion, the simplifications of the model for the neutral weak interactions are given by the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}}\left[L_\mu L^\mu + \left(\frac{x M_2}{z M_3}\right)^2 R_\mu R^\mu\right] \qquad (2.26)$$

where

$$L_\mu \equiv J_{\mu L}^3 - x^2 J_\mu^{c.m.}$$
$$R_\mu \equiv J_{\mu R}^3 - z^2 J_\mu^{c.m.} + z^2 J_\mu^3 \;. \qquad (2.27)$$

However, the consistency of the description (2.26) is studied in the next section which describes vector masses.

III. DESCRIPTION OF THE MASSES

We have shown in the previous section that the present phenomenology which displays a V-A asymmetric character is obtained also within the context of the L-R asymmetry. However, this is intimately related with the true description of the vector masses. For this, we will take generally a Higgs multiplet $\varphi(4, 2_L, 2_R)$ and we will study its various motivations *). The corresponding invariant derivative now is

$$D\varphi = \partial\varphi - ig\Sigma(4)\varphi - ig_L\Sigma(2_L)\varphi - ig_R\Sigma(2_R)\varphi \qquad (3.1)$$

and the contributions to the charged vector masses are

$$(A+B+C+D)\,g^2 \longrightarrow X_a^{+} X_a$$
$$(A+B+C+D)\,g_{L,R}^2 \longrightarrow W_{L,R}^{+} W_{L,R} \;, \qquad (3.2)$$

---

*) The structure of the scalar multiplets in L-R symmetric models and some renormalization group analysis are given in Ref.19.

where the following definitions are valid for the vacuum expectation values (vev's) of Higgs fields φ_{Aij}

$$
\begin{aligned}
A &\equiv |\langle Re\, \varphi_{411} \rangle|^2 \\
B &\equiv |\langle Re\, \varphi_{412} \rangle|^2 \\
C &\equiv |\langle Re\, \varphi_{421} \rangle|^2 \\
D &\equiv |\langle Re\, \varphi_{422} \rangle|^2 \quad .
\end{aligned}
\tag{3.3}
$$

It can easily be seen that under the expressions (2.7) for the fundamental representation weights μ_A only the weight

$$\mu_4 = (0 \quad 0 \quad -3) \tag{3.4}$$

can contribute to the masses if one wants to preserve the symmetry $SU(3)_C$. Hence only the vev's in (3.3) can be a priori non-zero. Although they are seen to be apparent for SU(4), the specifications of the weights which can give masses to the particles as in the expression (3.4) is important especially for the unifying schemes of the groups with higher rank. We will turn to this point in the next section for SO(10). On the other hand, the mass matrix of the neutral vectors is expressed as

$$
\begin{aligned}
m_{ab} = {} & e(A+B+C+D)[C_{1a}C_{1b}+C_{2a}C_{2b}+C_{3a}C_{3b}] \\
& + e(A-B-C+D)[C_{2a}C_{3b}+C_{3a}C_{2b}] \\
& + e(-A-B+C+D)[C_{1a}C_{2b}+C_{2a}C_{1b}] \\
& + e(-A+B-C+D)[C_{1a}C_{3b}+C_{3a}C_{1b}] \quad .
\end{aligned}
\tag{3.5}
$$

Now, with the aid of the general expressions (3.2) and (3.5), we can study what is the useful choice of Higgs multiplets giving rise to a true description of the vector masses. First, the fact that the photon field has a zero mass rules out the existence of the multiplet $\varphi(4,2_L,2_R)$. The following contributions are thus expected for the neutral vector masees:

	$\varphi(0 2_L 2_R)$	$\varphi(4 2_L 0)$	$\varphi(4 0 2_R)$
M_{11}	$A_1 = D_1 = 0$ $B_1, C_1 \neq 0$	$C_2 = D_2 = 0$ $A_2, B_2 \neq 0$	$B_3 = D_3 = 0$ $A_3, C_3 \neq 0$
M_{22}	$(B_1+C_1)/x^2\Lambda^2$	$(A_2+B_2)/x^2\Lambda^2$	0
M_{33}	$(B_1+C_1)/z^2\Lambda^2$	$(A_2+B_2)\Lambda^2/z^2$	$(A_3+C_3)(1+z^2)^2/z^2\Lambda^2$
M_{12}	0	0	0
M_{13}	0	0	0
M_{23}	$-(B_1+C_1)/xz\Lambda^2$	$(A_2+B_2)z/x\Lambda^2$	0

(3.6)

We see from this table that all three of the multiplets $\varphi(0,2_L,2_R)$, $\varphi(4,2_L,0)$ and $\varphi(4,0,2_R)$ should be chosen for a consistent description of the vector masses. With this choice, the vector masses will be obtained as

$$
\begin{aligned}
m_a^2 &= (v_2+v_3)\,g^2 \\
m_{W_L}^2 &= (v_1+v_2)\,g_L^2 \\
m_{W_R}^2 &= (v_1+v_3)\,g_R^2 \\
m_{A_2}^2 &= (v_1+v_2)\,e^2/x^2\Lambda^2 \\
m_{A_3}^2 &= [v_1+\Lambda^4 v_2+(1+z^2)^2 v_3]\,e^2/z^2\Lambda^2 \quad ,
\end{aligned}
\tag{3.7}
$$

where $v_1 = z^2 v_2$. The following definitions are used in these expressions:

$$
\begin{aligned}
v_1 &\equiv |\langle \varphi(0 2_L 2_R) \rangle|^2 = B_1 \text{ or } C_1 \\
v_2 &\equiv |\langle \varphi(4 2_L 0) \rangle|^2 = A_2 \text{ or } B_2 \\
v_3 &\equiv |\langle \varphi(4 0 2_R) \rangle|^2 = A_3 \text{ or } C_3 \quad .
\end{aligned}
\tag{3.8}
$$

The charged current conditions $v_3 \gg v_2$ is adopted here and this leads us to the relation

$$m_{A_3}^2 \cong \left[\frac{\cos^2\theta}{\cos^2\theta_W} v_1 + \frac{\cos^2\theta_W}{\cos^2\theta} v_3 \right] g_R^2 \tag{3.9}$$

for the mass of the neutral heavy weak interaction mediator. On the other hand, it is seen that the well-known relation

$$m_{W_L}^2 = \cos^2\theta_W \, m_{A_2}^2 \tag{3.10}$$

is also obtained here. This is the relation which determines the ratio of the neutral and charged (light) weak interaction strength. Then, the preservation of this relation is a phenomenological constraint on the models. Let us emphasize that it is a consequence of doublet Higgses [20] in the standard model and it is also obtained here within the choice (3.6).

IV. SIMPLE EMBEDDING OF THE L-R ASYMMETRY

The partial symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ serves us as a primary step and it is natural to embed it in a simple symmetry within a framework of the unification of the fundamental forces. This was already realized within a SO(10) symmetry scheme [15] displaying the L-R symmetry. It is interesting to note that the same is also true for the L-R asymmetry and the chiral weak charges (2.20), which are essential for the realization of the L-R asymmetry, can be obtained only for SO(10) group. Hence, the same mixing matrix (2.21) will suffice also in this case. Let us immediately note that this works only for one family unification *).

Investigations of these points and also to acquire masses for vectors and fermions will be made in a formulation for orthogonal groups that we proposed recently [21]. In this formulation, the fundamental representation weights of SO(10) are expressed as

$$\begin{aligned} \mu_1 &= (1 \quad 1 \quad -2 \quad 0 \quad 0) \\ \mu_2 &= (-1 \quad 1 \quad -2 \quad 0 \quad 0) \\ \mu_3 &= (0 \quad -2 \quad -2 \quad 0 \quad 0) \\ \mu_4 &= (0 \quad 0 \quad 0 \quad 1 \quad 1) \\ \mu_5 &= (0 \quad 0 \quad 0 \quad -1 \quad 1) \end{aligned} \tag{4.1}$$

*) Within the framework of one family unification, there is another group having 16 dimensional representation, SO(12). But, it will easily be seen that the structure of the weak chiral charges cannot be obtained for this representation as in (2.20).

The remaining five weights of this representation having in essence ten weights are the opposites of the weights in (4.1). For SO(10), the definitions concerning (4.1) are, for a,b = 1,...,5

$$\begin{aligned} \mu_b^a &\equiv (m_{b1}, m_{b2}, m_{b3}, m_{b4}, m_{b5}) \\ &= \frac{1}{\sqrt{2}} m_{b1} O_{1a} + \frac{1}{\sqrt{6}} m_{b2} O_{2a} + \frac{1}{\sqrt{12}} m_{b3} O_{3a} + \frac{1}{\sqrt{2}} m_{b4} O_{4a} + \frac{1}{\sqrt{2}} m_{b5} O_{5a} \end{aligned} \tag{4.2}$$

As in (2.8), the validity of the expressions (4.1) can be investigated immediately by assuming that the parameters O_{ba} are the elements of an orthogonal matrix. In essence, the $SU(4) \otimes SU(2) \otimes SU(2)$ sub-structure of SO(10) is explicitly seen in these expressions.

On the other hand, the 16 chiral fields of a fermion family will be accommodated in the 16 dimensional spinor representation of SO(10). In the notation of Ref.21, this representation 16 consists of the module

$$\Pi(\lambda_5) = \left\{ \tfrac{1}{2}(\mu_{i_1} + \mu_{i_2} + \mu_{i_3} + \mu_{i_4} + \mu_{i_5}) \right\} \tag{4.3}$$

and also the sub-modules

$$\begin{aligned} \Pi_1 &= \left\{ \tfrac{1}{2}(\mu_{i_1} + \mu_{i_2} + \mu_{i_3} - \mu_{i_4} - \mu_{i_5}) \right\} \\ \Pi_2 &= \left\{ \tfrac{1}{2}(\mu_{i_1} - \mu_{i_2} - \mu_{i_3} - \mu_{i_4} - \mu_{i_5}) \right\} , \end{aligned} \tag{4.4}$$

where all indices i_k take the values from 1 to 5 while they are different from each other within the same weight. These modules have the dimensions

$$\begin{aligned} \dim \Pi &= 5!/(5-5)!\,5! \\ \dim \Pi_1 &= 5!/(5-5)!\,3!\,2! \\ \dim \Pi_2 &= 5!/(5-5)!\,1!\,4! \end{aligned}$$

It can easily be seen that these modules correspond to the representations 1, 10, 5 of the maximal subgroup SU(5) of SO(10) and hence give the decomposition

$$\underline{16} = \underline{1} + \underline{10} + \underline{5} \quad .$$

However a very convenient notation is

$$\frac{1}{2}(\epsilon_1\mu_1+\epsilon_2\mu_2+\epsilon_3\mu_3+\epsilon_4\mu_4+\epsilon_5\mu_5) \equiv (\epsilon_1\epsilon_2\epsilon_3\epsilon_4\epsilon_5) \quad , \tag{4.5}$$

where $\epsilon_i = \mp 1$ in accordance with the definitions of the modules in the expressions (4.3) and (4.4). It is clear that the quark weights between these weights must be in the form

$$\begin{bmatrix} + - - \epsilon\epsilon \\ - + - \epsilon\epsilon \\ - - + \epsilon\epsilon \end{bmatrix} \tag{4.6}$$

They indeed exist for $\epsilon = +1$ in the module π_1 and for $\epsilon = -1$ in the module π_2. Antiquark weights are similarly

$$\begin{bmatrix} - + + \bar{\epsilon}\bar{\epsilon}' \\ + - + \bar{\epsilon}\bar{\epsilon}' \\ + + - \bar{\epsilon}\bar{\epsilon}' \end{bmatrix} \tag{4.7}$$

and they can only be included in the module π_1 in view of the specification (4.4). The remaining four weights are lepton and antilepton weights. With the aid of (4.1), these weights will be expressed as

$$q_{1L} = \begin{bmatrix} + - - - - \\ - + - - - \\ - - + - - \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 & -1 \\ 0 & -2 & 1 & 0 & -1 \end{bmatrix}$$

$$q_{2L} = \begin{bmatrix} + - - + + \\ - + - + + \\ - - + - - \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 0 & 1 \end{bmatrix}$$

$$q^c_{3L} = \begin{bmatrix} - + + + - \\ + - + + - \\ + + - + - \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 \end{bmatrix}$$

$$q^c_{4L} = \begin{bmatrix} - + + - + \\ + - + - + \\ + + - - + \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & -1 & 0 \\ 1 & -1 & -1 & -1 & 0 \\ 0 & 2 & -1 & -1 & 0 \end{bmatrix}$$

$$\begin{aligned} \ell_1 &= (+++++) = (0 \quad 0 \quad -3 \quad 0 \quad 1) \\ \ell_2 &= (+++--) = (0 \quad 0 \quad -3 \quad 0 \quad -1) \\ \ell_3 &= (---+-) = (0 \quad 0 \quad 3 \quad 1 \quad 0) \\ \ell_4 &= (----+) = (0 \quad 0 \quad 3 \quad -1 \quad 0) \end{aligned} \tag{4.8}$$

Of course, we cannot define which are those of leptons and antileptons among the weights ℓ_1, ℓ_2, ℓ_3 and ℓ_4 unless we determine the charge operator. For this, we will generally express the chiral charges defined as the coupling strengths of the fermions to the neutral vectors of SO(10). Similarly with the definition (2.12), it is also seen here that the fields G_1 and G_2 are colourless gluons and they cannot contribute to the electromagnetic charge operator because otherwise the three particles with equal electromagnetic charges cannot be accommodated in the same triplet of SU(3). Being in complete accordance with the definitions (2.19), we make the following definitions for the remaining three neutral vectors of SO(10):

$$\begin{aligned} \frac{3}{\sqrt{6}} g G_3 &= \sum_{i=1}^{3} e C_{1i} A_i \\ g G_4 &= \sum_{i=1}^{3} e C_{3i} A_i \\ g G_5 &= \sum_{i=1}^{3} e C_{2i} A_i \end{aligned} \tag{4.9}$$

where g is the coupling constant corresponding to the simple symmetry SO(10) which is expected to survive at some energy level. With the aid of these expressions the following chiral charges are obtained:

$$
\begin{aligned}
Q^i_{1L} &= \frac{e}{2}\left[\frac{1}{3}C_{1i} - C_{2i}\right] \\
Q^i_{2L} &= \frac{e}{2}\left[\frac{1}{3}C_{1i} + C_{2i}\right] \\
Q^i_{3R} &= \frac{e}{2}\left[\frac{1}{3}C_{1i} - C_{3i}\right] \\
Q^i_{4R} &= \frac{e}{2}\left[\frac{1}{3}C_{1i} + C_{3i}\right]
\end{aligned} \tag{4.10}
$$

If we compare these charges with those of (2.20) we immediately see that the the weights q_1, q_3 are related to the d quarks and q_2, q_4 to u quarks. Then for the leptons, the choices

$$\ell_1 = \nu_L \,,\quad \ell_2 = e_L \,,\quad \ell_3 = e_L^c \,,\quad \ell_4 = \nu_L^c \tag{4.11}$$

will be valid. This terminates the phenomenological aspect of the model because the same specification (2.21) for the mixing parameters will provide the embedding of the L-R asymmetry within the simple symmetry SO(10). However, the investigation of the mass mechanism is worthwhile because all fermions of the one family sit in the same irreducible multiplet in this case.

V. MASS MECHANISMS

It is appropriate to begin with the well-known decomposition

$$\underline{16} \otimes \underline{16} = [\underline{10} \oplus \underline{126}]_S \oplus [\underline{120}]_A$$

since this is the only source which gives Dirac masses to the fermions. Let us note that the consequences which will be obtained from this relation will not qualitatively change whether the scalars are composite or elementary. However, only this decomposition does not suffice for our aim. We need the modular decompositions of the irreducible representations

$$V(\lambda_1) = \underline{10} \,,\quad V(2\lambda_5) = \underline{126} \,,\quad V(\lambda_3) = \underline{120} \tag{5.1}$$

in this decomposition. The corresponding principal dominant weights in these expressions are denoted inside the paranthesis and consequently their modular decompositions will be written as

$$
\begin{aligned}
V(\lambda_1) &= \Pi(\lambda_1) \\
V(2\lambda_5) &= \Pi(2\lambda_5) \oplus \Pi(\lambda_3) \oplus 3\,\Pi(\lambda_1) \\
V(\lambda_3) &= \Pi(\lambda_3) \oplus 4\,\Pi(\lambda_1) \,,
\end{aligned} \tag{5.2}
$$

where $\Pi(2\lambda_5)$ is the 16 dimensional module of the principal dominant weight $2\lambda_5$ and $\Pi(\lambda_3)$ is the 80 dimensional module of λ_3.

Fermion masses

Within the Cartan-Dynkin (root-weight) theory of the Lie algebra representations, it can easily be shown that the weight corresponding to a scalar field having a vev to give a Dirac mass for a fermion f is determined as

$$f_L + f_L^C \,, \tag{5.3}$$

where f_L and f_L^C are the corresponding weights of f within the fermion representation of the group in which the fermions and anti-fermions are accommodated with the same chirality, for example L. Then, we must choose the following scalar fields under the specification (4.7):

$$
\begin{aligned}
\nu_L + \nu_L^c &= (+++++) + (----+) = \mu_5 = u_L + u_L^c \\
e_L + e_L^c &= (+++--) + (---+-) = -\mu_5 = d_L + d_L^c \,.
\end{aligned} \tag{5.4}
$$

Hence, it is seen that there is no hope to obtain a satisfactory spectrum for the fermion masses. Needless to say, the same is true also at the partial unification level and it could be hoped that a true determination of the fermion masses come from a symmetry which collects all families within a representation. Let us emphasize here a point which is seen to be lacking in the literature: essentially only the modules and not the representations can give masses for particles. To be explicit, let us consider the representation $\underline{126}$ of SO(10) as an exemplary case. It is said that in the models including this representation the representation $\underline{126}$ gives masses to the fermions. In fact, the fermions gain their masses from the weights (5.4) of the module $\Pi(\lambda_1)$ which is included in the representation $\underline{126}$. This is explicit in the modular decompositions (5.2) and it is clear that posing the problem in such a manner will **extensively simplify** the discussions concerning fermion masses.

<u>Vector masses</u>

As in all other GUT's, this model is not able to describe the fermion masses. However, it must give a consistent description of vector masses to be phenomenologically acceptable. Then, as a first step, we must calculate the contributions from the modules $\pi(\lambda_1)$, $\pi(2\lambda_5)$ and $\pi(\lambda_3)$ to the vector masses. Let us remind the reader of the definition of the SO(10) root system, in order to classify these contributions. The positive roots of SO(10) are determined by the fundamental representation weights as

$$\Phi^+[SO(10)] = \{ \mu_i \mp \mu_j \; ; \; i < j = 1,..5. \} \tag{5.5}$$

We know that all charged interactions originate from these roots. For instance, the vector W_L corresponding to the charged $SU(2)_L$ vector is determined by

$$\nu_L - e_L = (+++++) - (+++--) = \mu_4 + \mu_5 = u_L - d_L \; ; \tag{5.6}$$

while the vector W_R corresponding to the charged $SU(2)_R$ by

$$e_L^c - \nu_L^c = (---+-) - (----+) = \mu_4 - \mu_5 = d_L^c - u_L^c \; . \tag{5.7}$$

These can easily be investigated by the statement (2.16). Since the weights $\mu_4 \mp \mu_5$ are included in $\Phi^+[SO(10)]$, then the vectors $W_{L,R}$ exist within the scheme SO(10). All other charged interactions can be determined in the same manner. For instance, the coloured QCD interactions will arise from the roots α_1, α_2 and $\alpha_1 + \alpha_2$ which form the positive root system of an SU(3) subgroup of SO(10). The fact that these vectors are massless impose stringent conditions on the weights which can give masses for the particles and hence these conditions will be determinative on gauge hierarchies of the model. In view of this fact, only the weights having the topology

$$(0 \quad 0 \quad \cdot \cdot \cdot) \tag{5.8}$$

can give masses for particles, when one considers all possible modules of SO(10). This is because the colourless gluons are not mixed with the other neutral vectors of SO(10). Let us recall that the weights in (5.4)

are already of this form. This topology forces only the weights which are in the form of

$$\mp N(\mu_1 + \mu_2 + \mu_3) + P[\mu_4, \mu_5] \quad , \tag{5.9}$$

where $P(\mu_4, \mu_5)$ denotes all possible combinations of the weights μ_4 and μ_5 and N = 0,1,2,.... Now, with the restriction to the modules in <u>16</u> ⊗ 16 decompositions, only the following Higgs fields can get non-zero vev's:

$$\begin{aligned}
\mp\mu_4 &= (0 \quad 0 \quad 0 \quad \mp 1 \quad \mp 1) \equiv \varphi_{\mp 1} \\
\mp\mu_5 &= (0 \quad 0 \quad 0 \quad \pm 1 \quad \mp 1) \equiv \varphi_{\mp 2} \\
\mp(\mu_1 + \mu_2 + \mu_3) &= (0 \quad 0 \quad \mp 6 \quad 0 \quad 0) \equiv \varphi_{\mp 3} \\
\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 &= (0 \quad 0 \quad -6 \quad 0 \quad 2) \equiv \varphi_4 \\
\mu_1 + \mu_2 + \mu_3 - \mu_4 - \mu_5 &= (0 \quad 0 \quad -6 \quad 0 \quad -2) \equiv \varphi_5 \\
-\mu_1 - \mu_2 - \mu_3 + \mu_4 - \mu_5 &= (0 \quad 0 \quad 6 \quad 2 \quad 0) \equiv \varphi_6 \\
-\mu_1 - \mu_2 - \mu_3 - \mu_4 + \mu_5 &= (0 \quad 0 \quad 6 \quad -2 \quad 0) \equiv \varphi_7
\end{aligned} \tag{5.10}$$

These can immediately be understood by Lemma 2 of Ref.21 . Thus, among the 256 Higgs fields we choose the 10 appropriate Higgs fields with the aid of this formulation. In fact, the same calculation is carried out easily for any higher representations.

With these specifications, the asymmetry between the masses of the vectors W_L and W_R can be seen out of the relations

$$\begin{aligned}
\mu_4 - \mu_5 &= \varphi_6 - \varphi_{-3} = \varphi_{-3} - \varphi_7 = \varphi_1 - \varphi_2 = \varphi_{-2} - \varphi_{-1} \\
\mu_4 + \mu_5 &= \varphi_4 - \varphi_3 = \varphi_3 - \varphi_5 = \varphi_1 - \varphi_{-2} = \varphi_2 - \varphi_{-1} \; .
\end{aligned} \tag{5.11}$$

It is clear that these ensure the condition (2.18). In fact, it is seen from the right-hand side of Eqs.(5.11)that the real module $\pi(\lambda_1)$ cannot contribute to the condition (2.18). This is a general behaviour concerning the masses of the vectors mediating L-handed and R-handed types of the same interaction. Only the complex modules can create a mass difference between two such particles. Consequently, only the existence of the scalar multiplet <u>126</u> is sufficient for the condition (2.18), i.e. this is a minimal choice of Higgses for a SO(10) theory. Moreover, similarly with the expressions (5.11) it will be seen that all charged particles included in SO(10)/SU(3) gain masses from this representation 126.

Now, to the masses of the neutral vectors. These are also calculated with the aid of the explicit forms (5.10) If we write the expressions (4.2) in the form of

$$\mu_A{}^a = \frac{1}{\sqrt{2}} t_A{}^b O_{ba} ,$$

where A denotes the weights in (5.10) the mass matrix for the unphysical fields in (4.8) will be

$$g^2 \sum_A |\langle \varphi_A \rangle|^2 t_A^a t_A^b \rho_A .$$

The coefficients ρ_A in this expression are the multiplicity factors of the modules of the representation 126, which are seen in the modular decompositions (5.2). In terms of the mixings defined in (4.8), this last expression will give the following mass matrix for the physical neutral fields A_1, A_2 and A_3:

$$\begin{aligned} \mathcal{M}_{ij} = & 3v_2[C_{2i}C_{2j} + C_{3i}C_{3j} - C_{2i}C_{3j} - C_{3i}C_{2j}]e^2 \\ & + 4v_4[C_{1i}C_{1j} + C_{2i}C_{2j} - C_{1i}C_{2j} - C_{2i}C_{1j}]e^2 \\ & + 4v_7[C_{1i}C_{1j} + C_{3i}C_{3j} - C_{1i}C_{3j} - C_{3i}C_{1j}]e^2 , \end{aligned} \tag{5.12}$$

where only the vacuum expectation values

$$v_2 = |\langle \varphi_2 \rangle|^2 + |\langle \varphi_{-2} \rangle|^2$$

$$v_{4,7} = |\langle \varphi_{4,7} \rangle|^2$$

survive here because the photon field has zero mass. The diagonalization of this matrix is explicit only under the condition

$$4v_4 z^2 = 3v_2$$

and this gives a consistent description of the vector masses.

It is seen that all three vev's v_2, v_4, v_7 can be included in the representation 126, although there could be other choices including the

the representations 10 and 120. However, it can be seen that the representation 120 cannot contribute to the fermion masses if one uses the possibility to choose the Clebsch-Gordan coefficients connecting the representation 120 to the direct product $16 \otimes 16$ to be antisymmetric, It also cannot contribute to the vector masses. Hence, there is a possibility with only one Higgs multiplet 126. In this case, the fermions and the mediators of the (light) weak interactions gain their masses from the same source in favour of the relation (5.11), while the heavy weak interaction mediators from another source in accordance with (2.18) which imposes here the condition $v_7 >> v_4$. However, the investigation of the relation (3.10) *) deserves further study in this simple case. In our opinion, this can have a meaning only within a family unification scheme and will soon be shown in a subsequent work.

VI. CONCLUSION

We show that the L-R asymmetry idea is useful for grand unification schemes and it can be formulated within the symmetry $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$. The idea states that the V-A asymmetry can be established with the two different sets of SU(2) vector bosons on condition that the corresponding weak interactions unify only at some symmetry limit. That is, contrary to the strong and electromagnetic interactions, the weak interactions cannot get a vector-like character before this limit. Moreover, in order to establish such a symmetry limit, the embedding of the model in a simple SO(10) scheme is carried out and it is shown that to some extent this simple description is consistent with only one Higgs multiplet 126.

This is not a unique reason to propose the L-R asymmetry idea. As is well-known, the family unification problem is the basic one for GUT's. It can even be said that the fermion mass spectrum problem needs the family unification. Hence, a model which cannot solve the family replication cannot be the ultimate for grand unification. We will show in a subsequent paper that this family unification problem can be solved with the aid of the L-R asymmetry idea. To briefly summarize this last point, let us focus our attention on some models [8] which attack the problem. Because of the surviving L-R asymmetry in these orthogonal models, there are excess fermions as being conjugate families having right-handed weak interactions symmetrically with left-handed weak interactions of the families. The similar excess fermions also exist in the context of maximal symmetries [23]. These are mirror fermions which come out

*) This point is studied for SU(5) unification in, for example, Ref.22.

just like the conjugate families. Let us remark here on the group theoretical role played by the ABJ anomalies [24]. The mirror fermions are needed in maximal symmetries because there are ABJ anomalies. Alternatively, there are no ABJ anomalies in orthogonal models however the conjugate families automatically exist.

On the other hand, the L-R asymmetry idea breaks the existence of conjugate families in orthogonal models and a model based on the known representation $\underline{64}$ of SO(14) leading to a 3+1 family unification will soon be given in a subsequent paper. In conclusion, we consider this paper as a necessary introduction for such a possibility.

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