IC/81/147

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

A CLUSTER EXPANSION FOR BOUND THREE-ALPHA PARTICLES AS A THREE-BODY PROBLEM

Ahmed Osman

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1981 MIRAMARE-TRIESTE

IC/81/147

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A CLUSTER EXPANSION FOR BOUND THREE-ALPHA PARTICLES AS A THREE-BODY PROBLEM *

Ahmed Osman **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

A three-body model is proposed to study the nuclear bound states. The nucleus is described as a bound state of three clusters. A cluster expansion is introduced for the three cluster bound state problem. The present integral equations are treated by simple approximate solutions, which lead to effective potentials by using the present cluster expansion. The ^{12}C nucleus is described as a three-alpha particle bound state. The binding energy of ^{12}C is calculated numerically using the present cluster expansion as bound three-alpha clusters. The present three-body cluster expansion calculations are very near to the exact three-body calculations using separable potentials. The present theoretical calculations are in good agreement with the experimental measurements.

MIRAMARE - TRIESTE
August 1981

* To be submitted for publication.

** Permanent address: Physics Department, Faculty of Science, Cairo University, Cairo, Egypt.

I. INTRODUCTION

The cluster model [1] representing the bound states of nuclei had been found as a successful model in describing light nuclei which makes it as an important competative model with the shell model. In that respect, different studies had been introduced by many authors [1] for the many-particle scattering problem. Wildermuth [1] developed the resonating group theory in nuclear collisions to describe the elastic scattering of two bound clusters approximately. But it seems that the equations obtained from the resonating group approach for the N-body scattering process do not introduce an exact representation for the many-body scattering problem. Several authors [2] developed this approach formulating coupled integral equations for the N-body scattering problem and possibly could be subjected to approximate solutions. These integral equations had been approximated by Osborn and collaborators [2] to describe the three-body scattering theory following the cluster approach.

The time-independent N-body scattering theory had been used by some authors [3] in extending the Faddeev approach to the N-body problem. The obtained time-independent integral equations have kernels free from momentum space delta functions. These kernels being fully connected form operators which reserve the unique solutions for the integral equations. The use of this method needs the off-shell transition matrix elements of the different combination of different body problems as input, which in turn limit the practical computations to very light nuclei. This method with the suggested operators has been applied by us [4] for systems with few particles.

The three-body problem had been solved in an exact solution by Faddeev [5]. The integral equations obtained by Faddeev introduce an exact, correct and successful description of the scattering amplitude of the three-body problem. The Faddeev equations were generalized by many authors [5] for the multiparticle scattering problems constructing N-body integral equations. Osman [6] developed in the framework of the Faddeev formalism, a cluster expansion to describe the bound states of nuclei. In this cluster expansion introduced by Osman, the nuclei are considered to be composed of smaller clusters. With this description, Osman obtained a set of generalized Faddeev-equations which could be applied to the many cluster problem. The connectedness and solvability of the obtained integral equations are considered. In that work, we also investigated some approximations like the hard cluster model and the cluster model inner parameters. This model had been applied with numerical calculations of different binding energies of different light nuclei. The obtained calculated values of the binding energies of the different nuclei are in agreement with the experimentally measured values according to the cluster decompositions considered.

It has been shown that the alpha-particle model [7] of light nuclei is one of the successful models in extracting the nuclear properties of nuclei. The most interesting nucleus according to the alpha-particle model is the ^{12}C nucleus. On the basis of this model, the ^{12}C nucleus is represented as three finite structureless alpha particles. This representation of the ^{12}C nucleus as three-alpha particles had been studied theoretically by many authors using variational calculations [8], three-body model according to Faddeev calculations [9], the resonating group theory [10], the orthogonality condition model [11], the hyperspherical techniques [12] or by using a cluster decomposition with generalized Faddeev equations [13]. Most of these calculations had been done by using static and local alpha-alpha potentials which fit the alpha-alpha scattering phase shifts. Since the Pauli principle gives rise to a three-body interaction in the case that three composite particles come very close together, then, effective three-alpha potential is suggested [14] to be added to the alpha-alpha potential.

In the present work, a cluster expansion of the bound states of nuclei is proposed following a three-body model approach. The nucleus is considered as decomposed of N distinguishable particles interacting with each other by pairwise potentials. A complete basis in momentum space is required for the integral equations of the three-body problem. In the formulations of Faddeev [5] this basis is taken as plane waves of three free particles. But Karlsson and Zeiger [15] introduced the basis to be taken as one free particle adjointed to an interacting pair in a scattering state or in a bound state. The Karlsson-Zeiger equations differ from the Faddeev equations in that it easily allows the decomposition of the three-body Green's functions into a sum of propagators of which one contains only the two-cluster state and the other one for the orthogonal three-particle continuum state. With this decomposition the integral equations are decoupled to study either the continuum or the two-cluster contributions separately. Thus using a three-body scattering procedure, integral equations for the N-body system are formulated and can be used for the scattering and bound states. In the present work, we are interested in the case of ^{12}C nucleus. The ^{12}C nucleus is considered as composed of bound three-alpha particles. The alpha particles are taken as tightly bound entities, without internal structure. The present integral equations of the cluster expansion are applied to the ^{12}C nucleus as decomposed of three bound structureless alpha particles. Numerical calculations of the integral equations are carried out for the binding energy of the ^{12}C nucleus. The present obtained theoretically calculated value of the binding energy is compared with the experimental value.

In Sec.II the three-body integral equations of the cluster expansion are introduced. Numerical calculations and results are given in Sec.III. Sec.IV is devoted to discussion and conclusions.

II. CLUSTER EXPANSION AND INTEGRAL EQUATIONS

In this section the cluster expansion is formulated. Integral equations for this cluster representation are to be developed. The integral equations will be deduced from a three-body problem approach. We follow in this cluster representation, the formulations of the Karlsson and Zeiger [15] equations. Firstly, let us write the different notations of the three-body system. The three particles forming the three-body system will be referred to by the indices i, j and k where each one has the numbers 1, 2 and 3 in a cyclic permutation. The mass of the particle i is denoted by m_i. The reduced mass of the pair j and k is given by

$$\mu_i = \frac{m_j m_k}{(m_j + m_k)} . \tag{1}$$

Then, the reduced mass of the bound state of the clusters (j + k) relative to the cluster i is

$$M_i = \frac{m_i (m_j + m_k)}{(m_i + m_j + m_k)} . \tag{2}$$

The momentum of the cluster i is $\underset{\sim}{p}_i$ and the relative momentum of the two clusters j and k is $\underset{\sim}{q}_i$. Then, in the three-body centre-of-mass system, the two independent momentum variables in the Jacobi representation are $\underset{\sim}{p}_i$ and $\underset{\sim}{q}_i$. The centre-of-mass Hamiltonian for the kinetic energy of the three-body system is given by

$$H_o = \frac{p_i^2}{2M_i} + \frac{q_i^2}{2\mu_i} = \frac{p_o^2}{2M_o} , \tag{3}$$

where

$$M_o^2 = \frac{m_i m_j m_k}{(m_i + m_j + m_k)} \tag{4}$$

and $\underset{\sim}{p}_0$ is a six-dimensional vector representing the pair of vectors $(\underset{\sim}{p}_i, \underset{\sim}{q}_i)$. If the particle i has initial momentum $\underset{\sim}{p}_i'$, then the amplitudes of the elastic and rearrangement scattering of the three-body system are given by $H_{ji}(p_j;p_i')$ where j = 1, 2 and 3. Also, the amplitudes of the break-up scattering process is given as a sum on three terms as summing over j the amplitude $E_{ji}(p_0;p_i')$. The six functions of H_{ji} and E_{ji} are expressed in integral forms as

$$H_{ji}(p_j;p_i') = V^{\alpha}_{ji}(p_j,p_i') - \sum_{k=1,2,3} 2M_k \int \frac{V^{\alpha}_{jk}(p_j,p_k'')\,H_{ki}(p_k'';p_i')}{p_k''^2 - k_k^2 - i\varepsilon_0}\,dp_k'' - \sum_{k=1,2,3} 2M_0 \int \frac{V^{\beta}_{jk}(p_j,p_0'')\,E_{ki}(p_0'';p_i')}{p_0''^2 - k_0^2 - i\varepsilon_0}\,dp_0'' , \tag{5}$$

$$E_{ji}(p_0;p_i') = V^{\gamma}_{ji}(p_0,p_i') - \sum_{k=1,2,3} 2M_k \int \frac{V^{\gamma}_{jk}(p_0,p_k'')\,H_{ki}(p_k'';p_i')}{p_k''^2 - k_k^2 - i\varepsilon_0}\,dp_k'' - \sum_{k=1,2,3} 2M_0 \int \frac{V^{\delta}_{jk}(p_0,p_0'')\,E_{ki}(p_0'';p_i')}{p_0''^2 - k_0^2 - i\varepsilon_0}\,dp_0'' . \tag{6}$$

Eqs.(5) and (6) are the Karlsson-Zweiger equations, and these equations form a set of coupled integral equations. The kernels of these equations depend on the simple energy-independent rearrangement potentials given by

$$V^{\alpha}_{ji}(p_j,p_i') = -(1-\delta_{ji})\,\phi_j(q_j^{(1)})\,\psi_i(q_i^{(2)}) \tag{7}$$

$$V^{\beta}_{ji}(p_j,p_0') = -(1-\delta_{ji})\,\phi_j(q_j^{(1)})\,\chi_i^{-}(q_i^{(2)},q_i') \tag{8}$$

$$V^{\gamma}_{ji}(p_0,p_i') = (1-\delta_{ji})\,t_j^{+}(q_j^{(1)},q_j)\,\psi_i(q_i^{(2)}) \tag{9}$$

$$V^{\delta}_{ji}(p_0,p_0') = (1-\delta_{ji})\,t_j^{+}(q_j^{(1)},q_j)\,\chi_i^{-}(q_i^{(2)},q_i') \tag{10}$$

If v_i is the two-body interaction and the binding energy is ε_i, then the functions appearing in Eqs.(7)-(10) are described easily. $\psi_i(q)$ is the bound state eigenfunction given as

$$\left(\frac{q_i^2}{2\mu_i} + v_i\right)\psi_i = -\varepsilon_i\,\psi_i . \tag{11}$$

From $\psi_i(q)$, the vertex function $\phi_i(q)$ can be constructed as

$$\phi_i(q) = \left(\frac{q^2}{2\mu_i} + \varepsilon_i\right)\psi_i(q) . \tag{12}$$

The scattering wave functions $\chi_i^{\pm}$ are the matrix elements in momentum space with the time limit and are given by

$$\chi_i^{\pm} = s - \lim_{t\to\mp\infty} e^{i\left(\frac{q_i^2}{2\mu_i}+v_i\right)t}\, e^{-i\frac{q_i^2}{2\mu_i}t} . \tag{13}$$

Then the half-on-shell t matrices are

$$t_i^{\pm}(q,q') = \langle q \,|\, v_i\,\chi_i^{\pm} \,|\, q' \rangle . \tag{14}$$

Decoupling these equations, a cluster expansion [2] of the three-body problem has to be constructed. Then Eyre and Osborn [2], using the transition operators and the formalism of Alt, Grassberger and Sandhas [16], obtained the equation

$$T(Z) = V'[-K_0(Z)V] - V'K(Z)\,T(Z) \tag{15}$$

shere V, V', $K_0(Z)$, K(Z) and T(Z) are 3 × 3 matrices which have operator elements $\delta_{ij}V_i$, $V_i(1-\delta_{ij})$, $\delta_{ij}G_0(Z)$, $\delta_{ij}G_i(Z)$ and $T_{ij}(Z)$. Z is complex and the Green's functions $G_0(Z)$ and $G_i(Z)$ are defined by $(H_0-Z)^{-1}$ and $(H_i-Z)^{-1}$. Then expanding $G_i(Z)$ into bound state (cluster) and continuum [2),6)] parts using projection operators [2], then K(Z) can be decomposed of the sum of cluster state $K^S(Z)$ and a continuum state $K^C(Z)$. Also for the case of T(Z), we have $T^S(Z)$ and $T^C(Z)$. Then the equations of the T(Z) matrices are obtained as

$$T^{S}(z) = V' - V' K^{S}(z) T^{S}(z) \quad (16)$$

$$T^{C}(z) = V' - V' K^{C}(z) T^{C}(z) \quad (17)$$

and then

$$T(z) = T^{S}(z)[-K_0(z)V] - T^{S}(z) K^{C}(z) T(z) \quad , (18)$$

$$T(z) = T^{C}(z)[-K_0(z)V] - T^{C}(z) K^{S}(z) T(z) \quad . (19)$$

These equations (18) and (19) are nearly the same as that obtained by us [6] and also contain the results obtained by Bollé [17]. Equations (18) and (19) are deduced by Eyre and Osborn [2].

Once this cluster expansion is formulated by these decoupled integral equations, then the bound state of the three-body system can be considered. As introduced by Faddeev [5], the three-body system can be solved by the three coupled system of wave functions

$$|\Psi_i\rangle = -\sum_{j=1,2,3} G_i(z) V_i (1-\delta_{ij}) |\Psi_j\rangle \quad (20)$$

for Z = -b. b being the binding energy of the system. Keeping in mind that, in the three-body bound state, there is no incoming part, the bound state scattering amplitudes can be defined. So, for the case that Z = -b, k_k^2 is replaced by $2M_k(\varepsilon_k - b)$, k_0^2 is replaced by $-2M_0 b$ and $\varepsilon_0 = 0$ in equation (5). Then, the effective channel potential is

$$H^{C}_{ji}(\underset{\sim}{p}_j ; \underset{\sim}{p}'_i ; -b) = V^{\alpha}_{ji}(\underset{\sim}{p}_j, \underset{\sim}{p}'_i) - \sum_{k=1,2,3} 2M_0 \int \frac{V^{\beta}_{jk}(\underset{\sim}{p}_j, \underset{\sim}{p}''_0) E^{C}_{ki}(\underset{\sim}{p}''_0 ; \underset{\sim}{p}'_i ; -b)}{p''^2_0 + 2M_0 b} d\underset{\sim}{p}''_0 \quad . (21)$$

The bound state cluster amplitudes are given by the homogeneous integral equation

$$H_j(\underset{\sim}{p}_j) = -\sum_{k=1,2,3} 2M_k \int \frac{H^{C}_{jk}(\underset{\sim}{p}_j ; \underset{\sim}{p}''_k ; -b)\, H_k(\underset{\sim}{p}''_k)}{p''^2_k - 2M_k\varepsilon_k + 2M_k b} d\underset{\sim}{p}''_k \quad (22)$$

with the quadrature relation

$$E_j(\underset{\sim}{p}_0) = -\sum_{k=1,2,3} 2M_k \int \frac{E^{C}_{ji}(\underset{\sim}{p}_0 ; \underset{\sim}{p}''_k ; -b)\, H_k(\underset{\sim}{p}''_k)}{p''^2_k - 2M_k\varepsilon_k + 2M_k b} d\underset{\sim}{p}''_k \quad . (23)$$

In Eqs.(21)-(23), H^C_{ji} and E^C_{ji} refer to the two clusterlike and the quasibreakup amplitudes. From Eqs.(22) and (23), the wave functions $|\Psi_i\rangle$ given by Eq.(20) can be represented in a conventional wave representation as

$$\langle \underset{\sim}{p}_0 | \Psi_i \rangle = \frac{2M_i\, \psi_i(\underset{\sim}{q}_i)\, H_i(\underset{\sim}{p}_i)}{p_i^2 - 2M_i(\varepsilon_i - b)} + \int \frac{\chi^{-}_i(\underset{\sim}{q}_i, \underset{\sim}{q}''_i)\, E_i(\underset{\sim}{p}_i, \underset{\sim}{q}''_i)}{\frac{p_i^2}{2M_i} + \frac{q''^2_i}{2\mu_i} + b} d\underset{\sim}{q}''_i \quad . (24)$$

Eqs.(21)-(23) introduce the cluster expansion in the case of bound state of the three-body problem. From Eq.(24), the measure of the cluster form in the three-body bound state can be defined.

III. NUMERICAL CALCULATIONS AND RESULTS

In Sec.II a cluster expansion of the bound state of nuclei is introduced. The integral equations given for this cluster representation are deduced from the three-body problem. With these decoupled integral equations, the binding energy of the nucleus can be obtained according to any three cluster decomposition of the nucleus.

In the present work, we are interested in the ^{12}C nucleus. Since the alpha particle is a very tightly bound particle, so we use the alpha-cluster model in describing the ^{12}C nucleus. Thus, we consider the ^{12}C nucleus as composed of bound three alpha particles. Thus, the ^{12}C nucleus is represented by a cluster expansion of bound three alpha clusters. The two-body interactions are the alpha-alpha potentials. The alpha-alpha interaction is taken as a separable Yamaguchi potential [18] in momentum space

$$v(\underset{\sim}{q}, \underset{\sim}{q}') = \lambda \frac{1}{q^2+\beta^2} \frac{1}{q'^2+\beta^2} , \tag{25}$$

where the parameters λ and β are adjusted to fit the two-body (alpha-alpha) physical bound state.

Numerical calculations of the present T matrices are carried out after reducing these T matrices by their lowest poles. For the J = 0 problem, the obtained integral equations are in two continuous variables, which in turn are reduced to integral equations in one variable [19] by using separable potentials. Then conformal mappings and Gaussian quadratures are proceeded. In solving the integral equations, we use the method of Kopal [20]. The integrals are replaced by 36 point mesh. In the process of integration, we approximated the kernels by a finite 30 × 30 matrix by choosing finite mesh sizes. Increasing the number of steps in the integration for a 45 × 45 matrix did not change the results.

We included here the Coulomb effects between the different alpha clusters which have been calculated using our previous formulation, Osman [9]. The present theoretically calculated value of the binding energy of the ^{12}C nucleus as composed of bound three alpha clusters is found to be 7.413 MeV. The experimental value of the binding energy of ^{12}C with respect to dissociation into three alpha particles (the $^{12}C(\gamma,3\alpha)$ threshold energy) is found by Ajzenberg-Selove and Lauritsen [21] as 7.274 MeV. The two values differ by a percentage of about 1.911. Thus, our present theoretically calculated value of the binding energy of ^{12}C nucleus is composed of bound three alpha clusters according to a cluster expansion is in good agreement with the experimental value of the binding energy.

IV. DISCUSSION AND CONCLUSIONS

In the present work, decoupled integral equations which introduce a cluster expansion of nuclei are given. These integral equations are manageable on a computer by using two-particle separable potentials. In this cluster representation, the nucleus can be described by smaller clusters. As a state of interest, is that of the nucleus ^{12}C as composed of bound three alpha clusters. The matrix elements of this cluster expansion have been solved

theoretically and numerically with the result of a calculated value of binding energy of ^{12}C in agreement with its experimental value as dissociation into three alpha clusters. This result indicates that such a cluster expansion for the three-body model successfully represents the elastic and rearrangement scattering as well as the bound state of light nuclei.

Thus we can conclude that, as long as the success of representing the ^{12}C as decomposed of three alpha particles, then the nucleus inside nuclei like to cluster themselves in the form of alpha clusters. Also, since the ^{12}C bound state problem is solved as bound three alpha clusters, this results in decreasing the number of interactions as dealing with three interacting alpha particles instead of dealing with twelve interacting nucleons. With this suppressions of the degrees of freedom, we can conclude also that the present cluster expansion can also lead to effective many-body forces.

ACKNOWLEDGMENTS

I am very grateful to Professors Abdus Salam and Paolo Budinich as well as to the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where most of this work was carried out. Thanks are also due to the Centro di Calcolo dell'Università di Trieste for the use of the facilities.

REFERENCES

1) K. Wildermuth and Y.C. Tang, A Unified Theory of the Nucleus (Vieweg, Braunschweig, 1977).

2) R. Raphael, P.C. Tandy and W. Tobocman, Phys. Rev. C14, 1355 (1976); J. Schwager, Ann. Phys. (N.Y.) 98, 14 (1976); D. Eyre and T.A. Osborn, Nucl. Phys. A299, 301 (1978); S.F.J. Wilk and T.A. Osborn, Ann. Phys. (N.Y.) 114, 410 (1978); D. Eyre and T.A. Osborn, Phys. Rev. C20, 869 (1979).

3) S. Weinberg, Phys. Rev. 133, B232 (1964); P. Grassberger and W. Sandhas, Nucl. Phys. B2, 181 (1967); O.A. Yakubovskii, Yad. Fiz. 5, 1312 (1967) [English translation: Sov. J. Nucl. Phys. 5, 937 (1967)]; V.F. Kharchenko and V.E. Kuzmichev, Nucl. Phys. A183, 606 (1972); I.H. Sloan, Phys. Rev. C6, 1945 (1972); Gy. Bencze, Nucl. Phys. A210, 568 (1973); E.F. Redish, Nucl. Phys. A225, 16 (1974); Gy. Bencze and E.F. Redish, Nucl. Phys. A238, 240 (1975); K.L. Kowalski, Phys. Rev. C16, 7 (1977); G. Cattapan and V. Vanzani, Phys. Rev. C19, 1168 (1979).

4) A. Osman, Phys. Letters 37B, 135 (1971); A. Osman, Atomkernenergie 22, 62 (1973); A. Osman, Particles and Nuclei (USA) 7, 29 (1974).

5) L.D. Faddeev, Žurn. Ėksp. Teor. Fiz. 39, 1459 (1960) [English translation: Sov. Phys. JETP 12, 1014 (1961)] ; L.D. Faddeev, Dokl. Akad. Nauk SSSR 138, 565 (1971) [English translation: Sov. Phys. Doklady 6, 384 (1961)] ; L.D. Faddeev, Dokl. Akad. Nauk SSSR 145, 301 (1962) [English translation: Sov. Phys. Doklady 7, 600 (1963)] ; L.D. Faddeev, Mathematical Aspects of the Three-Body Problems in the Quantum Scattering Theory, translated from Russian by the Israel Programme for Scientific Translations, Jerusalem (D. Davey, New York 1965); C. Lovelace, in Strong Interactions and High Energy Physics, Ed. R.G. Moorhouse (Oliver and Boyd Publishers, Edinburgh, Scotland, 1964); C. Lovelace, Phys. Rev. 135, B1225 (1964); L. Rosenberg, Phys. Rev. 140, B217 (1965).

6) A. Osman, Lettere al Nuovo Cimento 4, 817 (1970); A. Osman, Int. J. Theoret. Phys. (N.Y.) 16, 81 (1977).

7) E.V. Inopin and B.J. Tischenko, Žurn. Ėksp. Teor. Fiz. 38, 1160 (1960) [English translation: Sov. Phys. JETP 11, 840 (1960)] ; H. Morinaga, Phys. Letters 21, 78 (1966); D.M. Brink, Proceedings of the International School of Physics Enrico Fermi, Course XXXVI, Ed. C. Bloch (Academic Press, New York 1966), p.247; G.F. Bertsch and W. Bertozzi, Nucl. Phys. A165, 199 (1971); N. Takigawa and A. Arima, Nucl. Phys. A168, 593 (1971); A. Arima, H. Horiuchi, K. Kubodera and N. Takigawa, Adv. Nucl. Phys. 5, 345 (1972); Eds. M. Baranger and E. Vogt (Plenum Press, New York 1972).

8) P. Darriulat, Nucl. Phys. 76, 118 (1966); T.K. Lim, Nucl. Phys. A158, 385 (1970); J.V. Noble, Phys. Letters 31B, 253 (1970); R.M. Mendez-Moreno, M. Moreno and T.H. Seligman, Nucl. Phys. A221, 381 (1974); V.C. Aguilera-Navarro and O. Portilho, Ann. Phys. (N.Y.) 107, 126 (1977).

9) D.R. Harrington, Phys. Rev. 147, 685 (1966); J.R. Fulco and D.Y. Wong, Phys. Rev. 172, 1062 (1968); H. Hebach and P. Henneberg, Z. Phys. 216, 204 (1968); A. Osman, Phys. Rev. C4, 302 (1971); C.S. Han, D.S. Chuu and D.L. Lin, Nucl. Phys. A212, 241 (1973).

10) S.A. Afzal, A.A.Z. Ahmad and S. Ali, Rev. Mod. Phys. 41, 247 (1969); H.H. Hackenbroich, T.H. Seligman and W. Zahn, Nucl. Phys. A259, 445 (1975).

11) S. Saito, Progr. Theoret. Phys. 41, 705 (1969); Yu.F. Smirnov, I.T. Obukhovsky, Yu.M. Tchuvilsky and V.G. Neudatchin, Nucl. Phys. A235, 289 (1974); H. Horiuchi, Progr. Theoret. Phys. 53, 47 (1975).

12) J.L. Visschers and R. van Wageningen, Phys. Letters 34B, 455 (1971); M. Vallieres, H.T. Coelho and T.K. Das, Nucl. Phys. A271, 95 (1976).

13) A. Osman, Int. J. Theoret. Phys. (N.Y.) 16, 81 (1977); A. Osman, Phys. Rev. C17, 341 (1978); A. Osman, American Institute of Phys. (N.Y.) 47, 562 (1978).

14) L.M. Delves, Adv. Nucl. Phys. 5, 1 (1972), Eds. M. Baranger and E. Vogt (Plenum Press, New York 1972); J. Zofka and M. Sotona, Czech. J. Phys. B24, 1250 (1974); E.W. Schmid and H. Ziegelmann, The Quantum Mechanical Three-Body Problem (Pergamon Press, Oxford 1974); O. Portilho and S.A. Coon, Z. Phys. A290, 93 (1979).

15) B.R. Karlsson and E.M. Zeiger, Phys. Rev. D11, 939 (1975).

16) E.O. Alt, P. Grassberger and W. Sandhas, Nucl. Phys. B2, 167 (1967).

17) D. Bollé, Phys. Rev. D13, 1809 (1976).

18) Y. Yamaguchi, Phys. Rev. 95, 1628 (1954); Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 95, 1635 (1954).

19) A. Ahmadzadeh and J.A. Tjon, Phys. Rev. 139, B1085 (1965); T.A. Osborn and H.P. Noyes, Phys. Rev. Letters 17, 215 (1966); T.A. Osborn, Stanford Linear Accelerator Report No.SLAC 79 (1967).

20) Z. Kopal, Numerical Analysis (John Wiley & Sons., Inc., New York 1955).

21) F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A114, 1 (1968).

IC/81/3 RAJ K. GUPTA, RAM RAJ and S.B. KHADKIKAR - Proximity potential and the surface energy part of the microscopic nucleus-nucleus interaction with Skyrme force.

IC/81/9 E.W. MIELKE - Outline of a non-linear, relativistic quantum mechanics of extended particles.

IC/81/10 L. FONDA and N. MANKOC-BORSTNIK - On quantum quadrupole radiation.

IC/81/20 N.S. CRAIGIE, V. ROBERTO and D. WHOULD - Gluon helicity distributions from hyperon productions in proton-proton collisions.

IC/81/21 QING CHENGRU and QIN DANHUA - A discussion of the relativistic equal-time equation.

IC/81/22 F. CRISCIANI, G.C. GHIRARDI, A. RIMINI and T. WEBER - Quantum limitations for spin measurements on systems of arbitrary spin.

IC/81/23 S.C. LIM - Stochastic quantization of Proca field.

IC/81/24 INT.REP.* W. MECKLENBURG - On the zero modes of the internal Dirac operator in Kaluza-Klein theories.

IC/81/25 INT.REP.* SUN HONG-ZHON and HAN QI-ZHI - On the irreducible representations of the simple Lie group II - The tensor basis for the infinitesimal generators of the exceptional groups.

IC/81/26 ZHANG YUAN-ZHONG - On the possibility for a fourth test of general relativity in Earth's gravitational field.

IC/81/27 INT.REP.* A.N. PANDEY, Y.KUMAR, U.P. VERMA and D.R. SINGH - Molecular properties of a few organic molecules.

IC/81/28 E. GAVA, R. JENGO and C. OMERO - On the instanton effect in the finite temperature Yang-Mills theory and in the nonlinear σ-model.

IC/81/30 INT.REP.* G. PASTORE, G . SENATORE and M.P. TOSI - Electric resistivity and structure of liquid alkali metals and alloys as electron-ion plasmas.

IC/81/31 L. MIZRACHI - Duality transformation of a spontaneously broken gauge field.

IC/81/32 ZHANG YUAN-ZHONG - The approximate solution with torsion for a charged particle in a gauge theory of gravitation.

IC/81/33 INT.REP.* W. MECKLENBURG - Massive photons and the equivalence principle.

IC/81/34 INT.REP.* K. TAHIR SHAH - Metric and topology on a non-standard real line and non-standard space-time.

IC/81/35 INT.REP.* H. PRAKASH and N. CHANDRA - Unpolarized state of light revisited.

IC/81/36 INT.REP.* A.N. SINGH and R.S. PRASAD - Molecular Rydberg transistions in C_2H_4, HCOOH and $HCONH_2$.

* Internal Reports: Limited Distribution.
THESE PREPRINTS ARE AVAILABLE FROM THE PUBLICATIONS OFFICE, ICTP, PO BOX 586, I-34100 TRIESTE, ITALY.

IC/81/37 INT.REP.* H. PRAKASH - Definition and density operator for unpolarized fermion state.

IC/81/38 H.R. DALAFI - Microscopic description of double back-bending in ^{168}Er.

IC/81/39 INT.REP.* B.G. SIDHARTH - Bound state wave functions of singular and transitional potentials.

IC/81/40 J.A. MAGPANTAY - On the non-existence of extended physical states in the Schwinger model.

IC/81/41 B.G. SIDHARTH - A recurrence relation for the phase-shifts of exponential and Yukawa potentials.

IC/81/42 INT.REP.* M.P. TOSI and N.H. MARCH - Liquid direct correlation function, singlet densities and the theory of freezing.

IC/81/43 INT.REP.* M.P. TOSI and N.H. MARCH - Theory of freezing of alkali halides and binary alloys.

IC/81/44 INT.REP.* M.P. TOSI and N.H. MARCH - Freezing of ionic melts into super ionic phase.

IC/81/45 INT.REP.* B.G. SIDHARTH - Large ℓ behaviour of the phase-shifts.

IC/81/46 G. CAMPAGNOLI and E. TOSATTI - Self-consistent electronic structure of a model stage-1 graphite acceptor intercalate.

IC/81/47 INT.REP.* E. MAHDAVI-HEZAVAH - Renormalization effects in the SU(16) maximally gauged theory.

IC/81/48 S.C. LIM - Stochastic massless fields I. Integer spin.

IC/81/49 G. ALBERI and Z. BAJZER - Off-shell dominance in rescattering effects for antiproton deuteron annihilation.

IC/81/50 INT.REP.* K. TAHIR SHAH - Self-organisation through random input by biological and machine systems - the pre-cognition subsystem.

IC/81/51 INT.REP.* P. P. SRIVASTAVA - Instanton and Meron solutions for generalized CP^{n-1} model.

IC/81/52 INT.REP.* G. PASTORE, G. SENATORE and M.P. TOSI - Short-range correlations in multi-component plasmas.

IC/81/53 L. FONDA - General theory of the ionization of an atom by an electrostatic field.

IC/81/54 R. d'AURIA, P. FRÉ and T. REGGE - Group manifold approach to gravity and supergravity theories.

IC/81/55 L. MASPERI and C. OMERO - Variational approach for the N-state spin and gauge Potts model.

IC/81/56 S. NARISON - QCD sum rules for the light quarks vacuum condensate.

IC/81/57 N.S. CRAIGIE - Spin physics at short distances as a means of studying QCD.

IC/81/58 E. SOKATCHEV - Irreducibility conditions for extended superfields.

IC/81/59 J. LUKIERSKI and L. RYTEL - Geometric origin of central charges.

IC/81/60 G. ALDAZABAL, D. BOYANOVSKY, VERA L.V. BALTAR, L. MASPERI and C. OMERO Simple renormalization groups and mean-field methods for Z(N) spin models.

IC/81/61 ABDUS SALAM - Gauge interactions, elementarity and superunification.

IC/81/62 INT.REP.* J. MAGPANTAY, C. MUKKU and W.A. SAYED - Temperature dependence of critical magnetic fields for the Abelian Higgs model.

IC/81/63 RAJ K. GUPTA, NEELAM MALHOTRA and R. AROUMOUGAME - Role of proximity forces in fragmentation potentials for central and oriental heavy-ion collisions.

IC/81/64 Ch. OBCEMEA, P.FROELICH and E.J. BRÄNDAS - Generalized virial relations and the theory of subdynamics.

IC/81/65 INT.REP.* W. MECKLENBURG - Attempts at a geometrical understanding of Higgs fields.

IC/81/66 N. KUMAR and A.M. JAYANNAVAR - Frequency and temperature dependent mobility of a charged carrier on randomly interrupted strand.

IC/81/67 N.S. CRAIGIE - On the hadronic problem in QCD.

IC/81/68 INT.REP.* M.P. DAS - Electron structure calculations for heavy atoms: A local density approach.

IC/81/69 INT.REP.* M.H.A. HASSAN and I.A. ELTAYEB - On the topographic coupling at the core mantle interface.

IC/81/70 R. HOJMAN and A. SMAILAGIC - Exact solutions for strong gravity in generalized metrics.

IC/81/71 INT.REP.* B.K. AGARWAL - Electromagnetic form factor for a composite proton.

IC/81/72 J. BLANK, M. HAVLICEK, M. BEDNAR and W. LASSNER - Canonical representations of the Lie superalgebra Osp(1,4).

IC/81/73 INT.REP.* F.F. GRINSTEIN - Padé approximants and the calculation of spectral functions of solids.

IC/81/74 INT.REP.* I.A. ELTAYEB - Hydromagnetic stability of the Kennedy-Higgins model of the Earth's core.

IC/81/75 S. RANDJBAR-DAEMI - A recursive formula for the evaluation of $\langle\psi T_{\mu\nu}(x)\psi\rangle$ and its application in the semiclassical theory of gravity.

IC/81/76 INT.REP.* P. BALLONE, G. SENATORE and M.P. TOSI - Coexistence of vapour-like and liquid-like phases for the classical plasma model.

IC/81/77 INT.REP.* K.C. MATHUR, G.P. GUPTA and R.S. PUNDIR - Reduction of the Glauber amplitude for electron impact rotational excitation of quadrupolar molecular ions.

IC/81/78 INT.REP.* N. KUMAR and A.M. JAYANNAVAR - A note on the effective medium theory of randomly resistive-reactive medium.

IC/81/79 INT.REP.* S.N. PANDEY - Some aspects of gravitational waves in an isotropic background Universe.

IC/81/80 E. GAVA and R. JENGO - Perturbative evaluation of the thermal Wilson loop.

IC/81/81 INT.REP.* ABDUS SALAM and J. STRATHDEE - On the Olive-Montenon conjecture.

IC/81/82 INT.REP.* M.P. TOSI - Liquid alkali metals and alkali-based alloys as electron-ion plasmas.

IC/81/83 INT.REP.* M.A. El-ASHRY - On the theory of parametric excitation of electron plasma.

IC/81/84 INT.REP.* P. BALLONE, G. SENATORE and M.P. TOSI - On the surface properties of a semi-infinite classical plasma model with permeable boundary.

IC/81/85 G. CALUCCI, R. JENGO and M.T. VALLON - On the quantum field theory of charges and monopoles.

IC/81/87 V.K. DOBREV, A.Ch. GANCHEV and O.I. YORDANOV - Conformal operators from spinor fields - I: Symmetric tensor case.

IC/81/88 V.K. DOBREV - Stiefel-Skyrme-Higgs models, their classical static solutions and Yang-Mills-Higgs monopoles.

IC/81/89 Trieste International Symposium on Core Level Excitations in Atoms, Molecules and Solids, 22-26 June 1981 (Extended abstracts).

IC/81/90 INT.REP.* A. TAWANSI, A.F. BASHA, M.M. MORSI and S.El-KONSOL - Effects of gamma-radiation and rare earth additives on the insulation of sodium silicate glasses.

IC/81/91 INT.REP.* A. TAWANSI, A.F. BASHA, M.M. MORSI and S. El-KONSOL - Investigation of the electrical conductivity of γ-irradiated sodium silicate glasses containing multivalence Cu ions.

FIGURE CAPTIONS

Fig. 1. The differential cross-sections of the nuclear stripping reaction $^{28}Si(d, p)^{29}Si$ of incident deuteron energy 18.0 MeV, leaving the residual nucleus ^{29}Si in its ground state. The solid curve is our present three-body calculations with Coulomb forces included. The dashed-dotted curve is the three-body calculations without Coulomb forces. The dashed curve is the DWBA calculations. The experimental data are taken from reference (17).

Fig.2. The differential cross-sections of the nuclear stripping reaction $^{40}Ca\ (d,p)^{41}Ca$ of incident deuteron energy 7.0 MeV, leaving the residual nucleus ^{41}Ca in its ground state. The solid curve is our present three-body calculationswith Coulomb forces included. The dashed-dotted curve is the three-body calculationswithout Coulomb forces. The dashed curve is the DWBA calculations. The experimental data are taken from reference (18).

Fig.3. The angular distributions of deuteron elastic scattering on ^{28}Si at deuteron incident energy of 18.0 MeV. The solid curve is our present three-body calculations including Coulomb forces. The dashed- dotted curve is the three-body calculations without Coulomb forces.

Fig.4. The angular distributions of deuteron elastic scattering on ^{40}Ca at deuteron incident energy of 7.0 MeV. The solid curve is our present three-body calculations including Coulomb forces. The dashed-dotted curve is the three-body calculations without Coulomb forces.

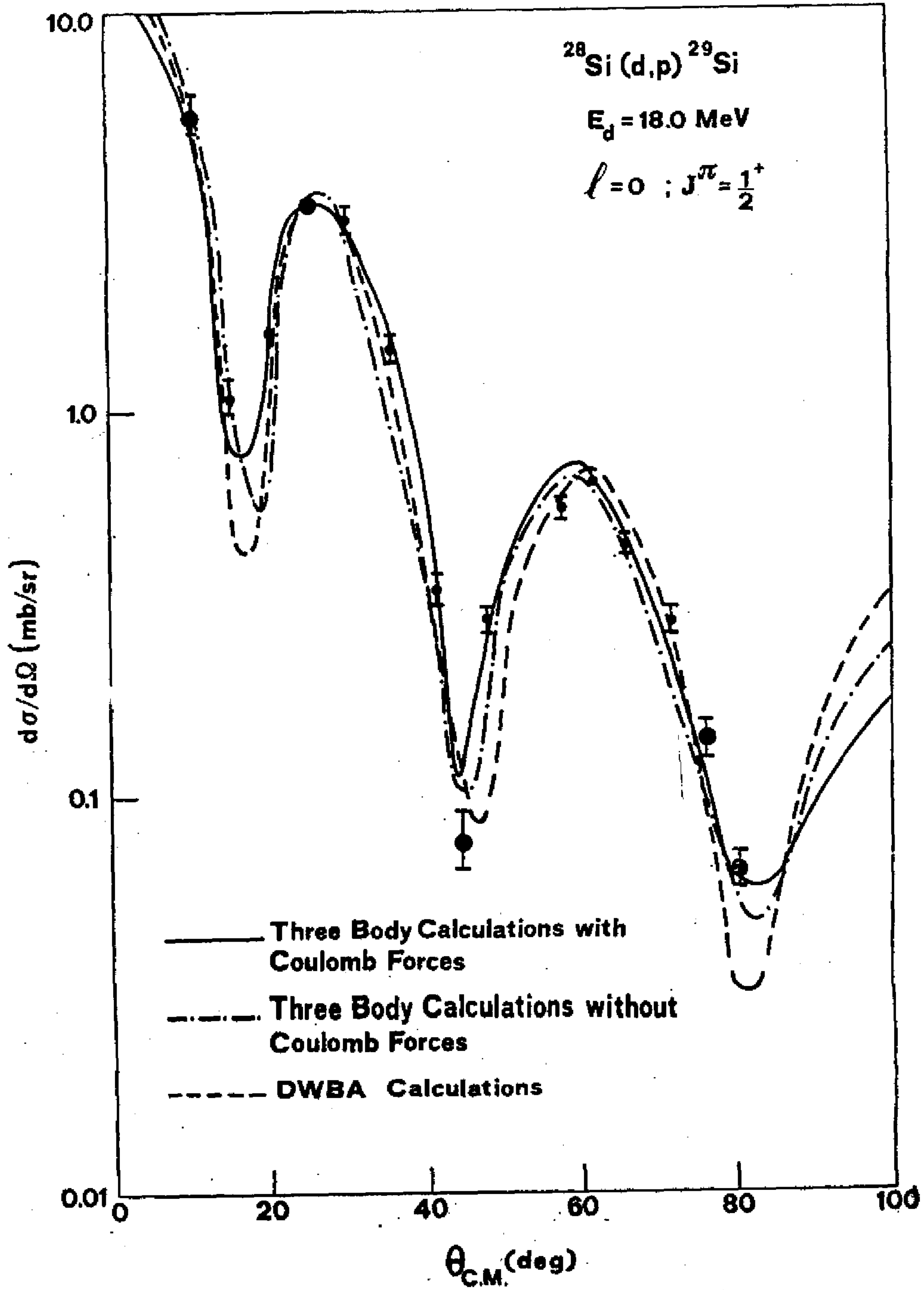
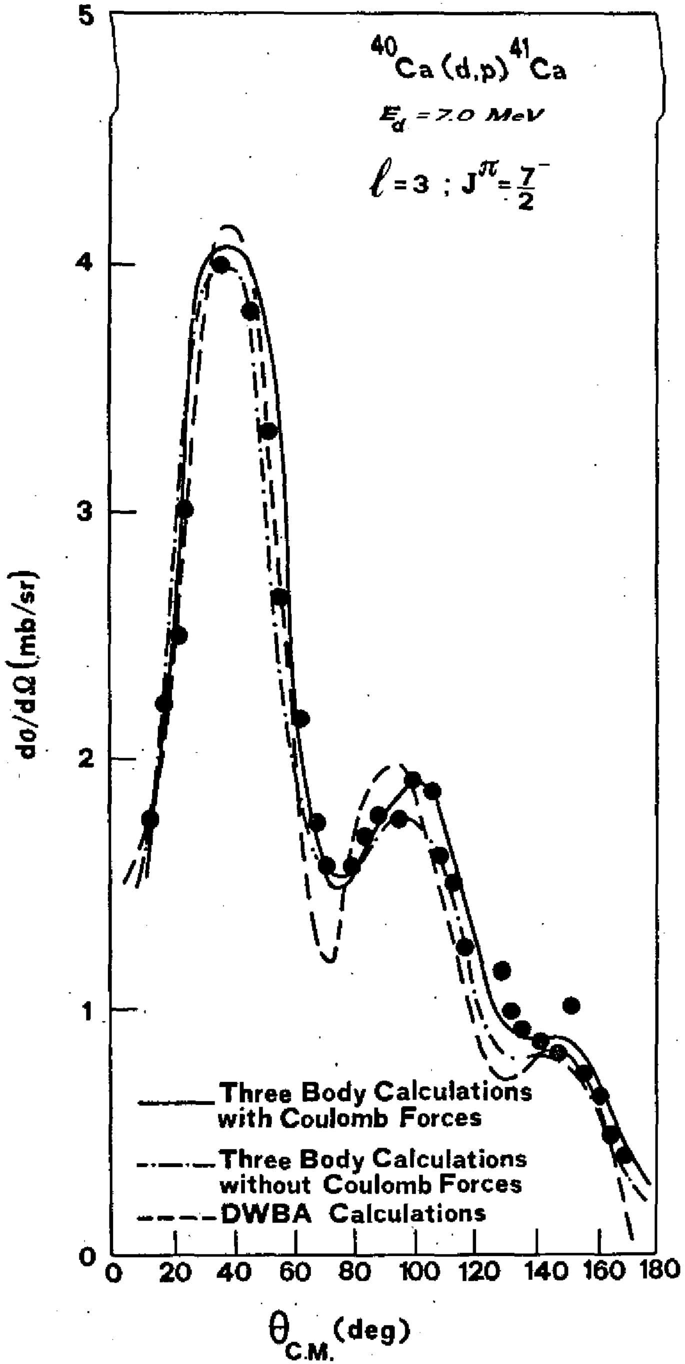
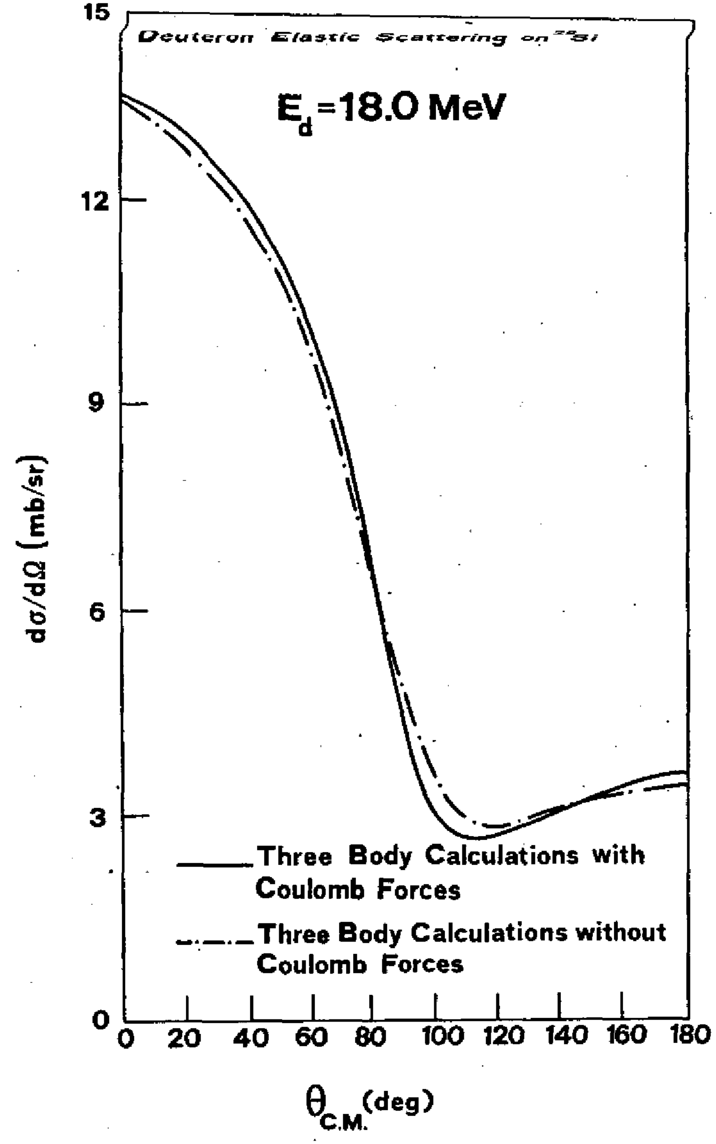


FIG.1



40Ca (d,p) 41Ca
Ed = 7.0 MeV
ℓ = 3 ; Jπ = 7/2−
dσ/dΩ (mb/sr)
5
4
3
2
1
0
0 20 40 60 80 100 120 140 160 180
θC.M.(deg)
Three Body Calculations with Coulomb Forces
Three Body Calculations without Coulomb Forces
DWBA Calculations

FIG. 2



Deuteron Elastic Scattering on 28Si
Ed = 18.0 MeV
dσ/dΩ (mb/sr)
15
12
9
6
3
0
0 20 40 60 80 100 120 140 160 180
θC.M.(deg)
Three Body Calculations with Coulomb Forces
Three Body Calculations without Coulomb Forces

FIG. 3