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WITH SPIN DEPENDENT NN INTERACTION

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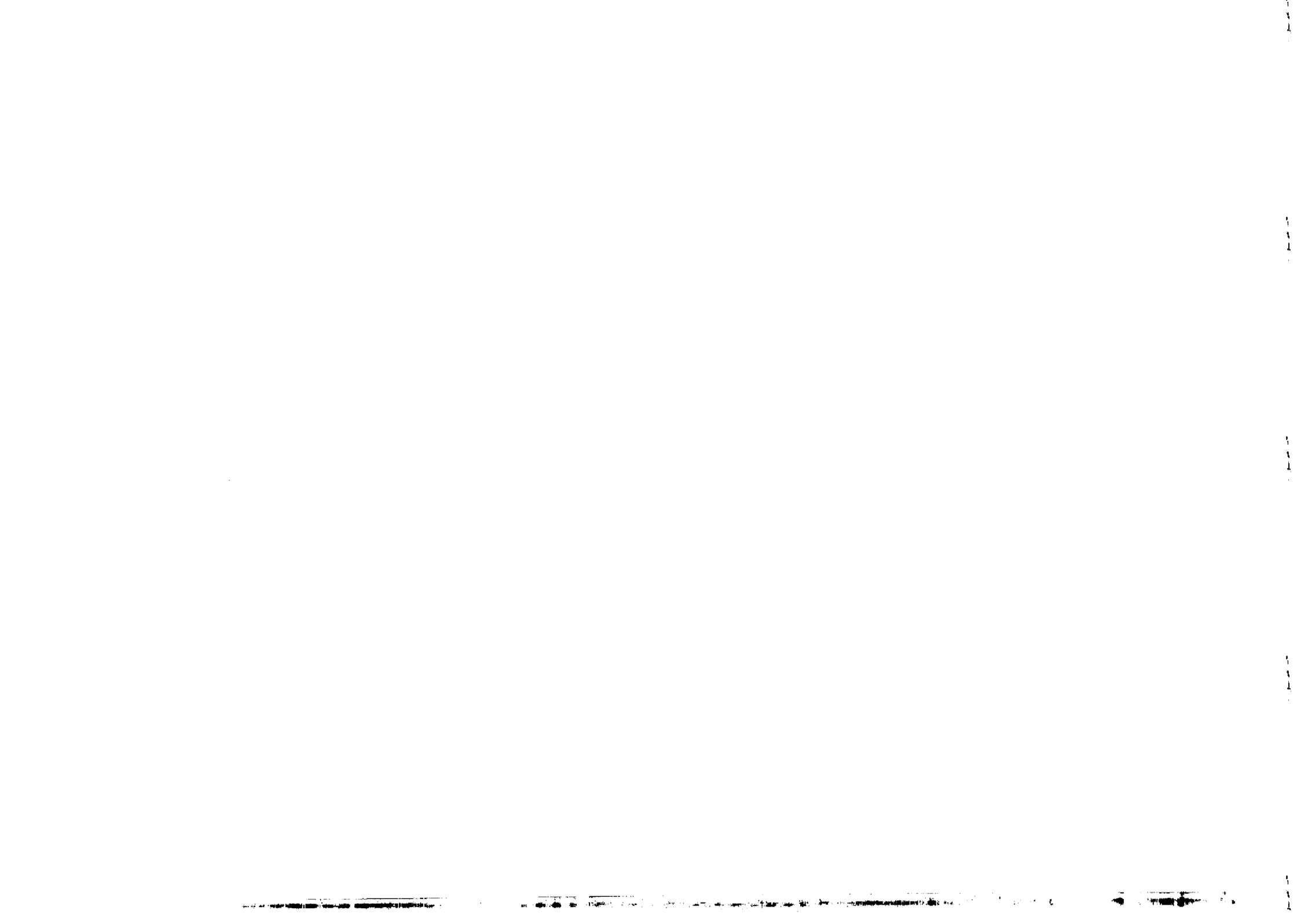


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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MEDIUM ENERGY INELASTIC PROTON-NUCLEUS SCATTERING

WITH SPIN DEPENDENT NN INTERACTION *

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and

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ABSTRACT

The previously proposed effective profile expansion method for the Glauber multiple scattering model calculation has been extended to the case of proton-nucleus inelastic scattering with spin dependent NN interaction. Using the method which turns out to be computationally simple and of relatively wider applicability, a study of sensitivity of proton-nucleus inelastic scattering calculation to the sometimes neglected momentum transfer dependence of the NN scattering amplitude has been made. We find that the calculated polarization is particularly sensitive in this respect.

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I. INTRODUCTION

Extensive studies (e.g. Refs.1-6) over the past several years have shown that Glauber multiple scattering theory ⁷⁾ is a good tool for interpreting intermediate- and high-energy proton-nucleus scattering experiments in terms of the basic NN amplitude. Initially, in the absence of adequate information on the NN amplitude, it was hoped that perhaps at the incident proton energy $E \approx 1$ GeV the spin effects are small and so in most of the applications of Glauber theory to intermediate energy p-nucleus scattering, this effect was ignored. This made the theoretical problem considerably simple. However, growing realization of the importance of the spin effects and need for interpreting p-nucleus polarization experiments, prompted many authors to incorporate spin components of the NN amplitude into Glauber model calculations. For the elastic scattering a recent example is the work of Osland and Glauber ⁸⁾ where references to other earlier works may be found. In the case of inelastic scattering which is the subject of present study only a few attempts have been made in the past ^{9),10)}. A fairly recent good example is the work of Faldt and Osland ¹¹⁾. However, the formalism proposed by these authors is of limited applicability. This is mainly because they consider only the vibrational nuclei, assume zero range for the NN interaction and do not suggest how the correlation corrections could be incorporated in theory. Moreover, the z-ordering problem in the evaluation of the Glauber amplitude does not seem to be very clearly treated.

In this paper we present a new approach for evaluating the Glauber amplitude for p-nucleus inelastic scattering with spin dependent NN interaction. The approach is based on our recently proposed ¹²⁾ effective profile expansion technique and it proceeds without invoking any specific model for the target nucleus. It does not assume zero range for the NN force and it could be applied from moderately light to heavy nuclei. The fact that our formalism is not based on any specific model for the target nucleus makes it of special interest to those who are interested in extracting information on target transition densities from intermediate energy hadron-nucleus inelastic scattering experiments.

After developing a suitable formalism we next study sensitivity of p-nucleus inelastic scattering calculation to the previously neglected momentum transfer dependence of the basic NN amplitude. Our calculation shows that accounting for this dependence is quite important for a realistic analysis of the inelastic scattering experiments.

II. FORMALISM

In Glauber theory ⁷⁾ the inelastic scattering amplitude F_{f0} describing the excitation of a target nucleus from the ground state ψ_0 to some excited state ψ_f is given by

$$F_{f0}(\vec{q}) = \frac{i\mathbf{k}}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} \left[\delta_{f0} - (\psi_f | S(\vec{b}) | \psi_0) \right] d^2b, \quad (1a)$$

$$S(\vec{b}) = \mathcal{Z} \prod_{j=1}^A [1 - \Gamma(\vec{b} - \vec{s}_j)] \quad (1b)$$

where \mathbf{k} is the incident hadron momentum, \vec{q} the momentum transfer, \vec{b} the impact parameter, \vec{s}_j the projections of the target nucleon co-ordinates on the plane perpendicular to \vec{k} , and Γ is the profile function related to the elementary NN amplitude $f(q)$ as given below

$$\Gamma(\vec{b}) = \frac{1}{2\pi i k} \int e^{-i\vec{q}\cdot\vec{b}} f(q) d^2q. \quad (2)$$

Further, the symbol \mathcal{Z} in Eq.(1b) denotes the z-ordering operator. The need for such an operator arises for a spin dependent NN amplitude because, in such a case, different Γ_j 's do not commute. This operator implies that while evaluating the matrix elements of S the profile operator Γ_j are to be arranged from right to left in order of increasing z co-ordinates for the nucleons to which they correspond. It is because of this z-ordering that evaluation of the Glauber amplitude becomes more complicated when spin dependent elementary amplitudes are applied.

In writing Eq.(1) we have not accounted for the Coulomb scattering for simplicity of presentation. However, in actual calculations it has been taken care of (of course in an appropriate way) as in earlier calculations ^{2),11)}. Also we treat all constituent particles of the target as identical.

It is fairly well known that the NN scattering amplitude in its most general form, obtained on the basis of symmetry consideration, may be expressed in terms of five complex amplitudes which are still operators in the i-spin space. Unfortunately, at present we lack precise information on all these amplitudes. It has however been shown by Osland and Glauber ⁸⁾ that of all the five amplitudes only those two which do not involve the target nucleon spin determine the scattering from spin zero nuclei. Therefore, we consider the following simplified form for the NN scattering amplitude:

$$f(q) = f_c(q) + f_s(q) \vec{q} \cdot (\vec{\sigma} \times \hat{k}), \quad (3)$$

where $\vec{\sigma}$ is the spin operator for projectile nucleon, \hat{k} is the unit vector along \vec{k} and $f_c(q)$ and $f_s(q)$ are the spin independent and spin dependent amplitudes, respectively.

To proceed further we define

$$\Gamma_0(\vec{b}) = \int \Gamma(\vec{b} - \vec{s}) \rho_0(\vec{r}) d\vec{r}, \quad (4a)$$

where $\rho_0(\vec{r})$ is the ground state density of the target:

$$\rho_0(\vec{r}_1) = \int |\psi_0(r_1 \dots r_A)|^2 \delta \left(\frac{\vec{r}_1 + \vec{r}_2 + \dots + \vec{r}_A}{A} \right) dr_2 \dots dr_3. \quad (4b)$$

From Eqs.(2) and (3) it follows that the quantity $\Gamma_0(\vec{b})$ may be expressed as

$$\Gamma_0(\vec{b}) = \Gamma_{0c}(\vec{b}) + i \vec{\sigma} \cdot \hat{n} \Gamma'_{0s}(\vec{b}), \quad (5a)$$

where

$$\hat{n} = \hat{k} \times \hat{b} \quad (5b)$$

$$\Gamma_{00}(b) = \frac{1}{ik} \int_0^\infty dq q J_0(qb) f_c(q) F_0(q) \quad (6a)$$

$$\Gamma'_{0s}(b) = \frac{1}{ik} \frac{d}{db} \int_0^\infty dq q J_0(qb) f_s(q) F_0(q) \quad (6b)$$

with $F_0(q)$ as the ground state form factor of the target:

$$F_0(q) = \int e^{i\vec{q}\cdot\vec{r}} \rho_0(\vec{r}) d\vec{r}. \quad (7)$$

Now we apply the effective profile expansion approach for evaluating the S-matrix element of interest. For this we define the effective profile function as

$$\gamma_j(\vec{b}) = \Gamma_0(\vec{b}) - \Gamma(\vec{b} - \vec{s}_j). \quad (8)$$

Using the fact that $\Gamma_0(b)$ commutes with itself the S-matrix given by Eq.(1a) may now be expanded as

$$S(\vec{b}) = (1 - \Gamma_0)^A + \sum_{j=1}^A (1 - \Gamma_0)^{j-1} \gamma_j (1 - \Gamma_0)^{A-j} + \mathcal{Z} \sum_{j < k} (1 - \Gamma_0)^{j-1} \gamma_j (1 - \Gamma_0)^{k-j-1} \gamma_k (1 - \Gamma_0)^{A-k} + \dots \quad (9)$$

The above expansion may now be applied for evaluating the S-matrix elements for the elastic and inelastic scattering cases. Neglecting terms involving two or more γ 's they are

$$S_{00}(\vec{b}) = (1 - \Gamma_0(b))^A \quad (10)$$

and

$$S_{f0}(\vec{b}) = - \sum_{j=1}^A (1 - \Gamma_0)^{j-1} (\psi_f | \Gamma_j | \psi_0) (1 - \Gamma_0)^{A-j} \quad (11)$$

The expression (10) for $S_{00}(b)$ is essentially the same as obtained by Auger and Lombard³⁾ and more recently by Osland and Glauber⁸⁾ (note that we are not distinguishing between target protons and neutrons). The expression for the inelastic scattering case is, however, different from that derived by Faldt and Osland¹¹⁾. But before discussing it in detail it must be pointed out that one of the advantages of the present approach is the absence of the z-ordering problem in the evaluation of the first two terms in the expansion (9) which are the main contributors to the elastic and the inelastic scattering, respectively. The z-ordering is of course present in other terms of the expansion which are of higher order in γ and hence depend upon the two-body and other higher order correlations¹²⁾. Experience tells us such terms do not significantly contribute to the scattering for medium and heavy nuclei upto moderate momentum transfers, and therefore will not be considered here. As already hinted at, another advantage of the present approach is that it does not invoke any specific model for the target nucleus unlike the work of Faldt and Osland¹¹⁾ who base their formulation on the vibrational model for nuclei. In consequence the inelastic scattering amplitude evaluated on the basis of Eq.(11) depends on the intrinsic transition density of the target (as will become clear later) and not on a transition density appropriate to a specific model. Thus the present approach is more suited for determination of nuclear transition densities from the inelastic scattering which is one of the aims of such medium energy experiments.

Coming to the evaluation of $S_{f0}(\vec{b})$ as given by Eq.(11) we note that $\Gamma_f(\vec{b}) \equiv (\psi_f | \Gamma | \psi_0)$ may be written as

$$\Gamma_f(\vec{b}) = \frac{1}{2\pi i k} \int e^{-i\vec{q}\cdot\vec{b}} f(q) F_{f0}(\vec{q}) d^2q \quad (12)$$

where $F_{f0}(\vec{q})$ is the inelastic form factor related to the transition density $\rho_{f0}(\vec{r})$ as

$$F_{f0}(\vec{q}) = \int e^{i\vec{q}\cdot\vec{r}} \rho_{f0}(\vec{r}) d\vec{r} \quad (13)$$

Considering transition to a state of angular momentum L and writing

$$\rho_{f0}(\vec{r}) = \rho_L(r) Y_{LM}^*(\Omega) \quad (14)$$

where Y_{LM} is the spherical harmonic and $\rho_L(r)$ the radial transition density, the transition form factor may be expressed as

$$F_{f0}(\vec{q}) = i^L F_L(q) Y_{LM}^*(\hat{q}) \quad (15)$$

with

$$F_L(q) = 4\pi \int_0^\infty dr [r^2 J_L(qr) \rho_L(r)] \quad (16)$$

Since in Glauber theory \vec{q} is normal to \vec{k} , therefore, if the latter is chosen as the quantization axis, we have

$$Y_{LM}^*(\hat{q}) = i^{L+M} B_{LM} e^{-iM\phi_q} \quad \begin{array}{l} L+M \text{ even} \\ = 0 \quad L+M \text{ odd} \end{array} \quad (17)$$

where

$$B_{LM} = \frac{(2L+1)^{1/2}}{4\pi} \frac{[(L-M)! (L+M)!]^{1/2}}{(L-M)!! (L+M)!!} \quad (18)$$

Substituting Eqs.(17) and (15) in Eq.(12) we obtain

$$\Gamma_f(\vec{b}) = \left[g_{LM}^c(b) + i \vec{\sigma} \cdot \hat{n} g_{LM}^s(b) - \vec{\sigma} \cdot \hat{b} \frac{M}{b} g_{LM}^s(b) \right] e^{-iM\phi_b} \quad (19)$$

where

$$g_{LM}^c(b) = \frac{(-1)^M}{ik} B_{LM} \int_0^\infty dq [q f_c(q) F_L(q) J_M(qb)] \quad \begin{array}{l} L+M \text{ even} \\ = 0 \quad L+M \text{ odd} \end{array}$$

and a similar expression for g_{LM}^s with f_c replaced by $f_s(q)$. In Eq.(19) the dash on g_{LM}^s denotes differentiation with respect to b and ϕ_b is the azimuthal angle for the impact parameter vector \vec{b} .

Using Eq.(19) we may write Eq.(11) as

$$S_{f0}(\vec{b}) = S_{f0}^{(1)} + S_{f0}^{(2)} \quad (20a)$$

with

$$S_{f0}^{(1)} = - \sum_{j=1}^A (1 - \Gamma_0)^{A-1} (\epsilon_{LM}^c + i \vec{\sigma} \cdot \hat{n} \epsilon_{LM}^{is}) e^{-iM\phi_b} \quad (20b)$$

and

$$S_{f0}^{(2)} = \frac{M}{b} \sum_{j=1}^A (1 - \Gamma_0)^{j-1} \vec{\sigma} \cdot \hat{b} (1 - \Gamma_0)^{A-j} \epsilon_{LM}^s e^{-iM\phi_b} \quad (20c)$$

Next from Eq.(5a) it follows that for any integer m

$$(1 - \Gamma_0)^m = X_m + i \vec{\sigma} \cdot \hat{n} Y_m \quad (21)$$

where

$$X_m = \frac{1}{2} \left\{ (1 - \Gamma_{0c} - i \Gamma_{0s}')^m + (1 - \Gamma_{0c} + i \Gamma_{0s}')^m \right\} \quad (22a)$$

$$Y_m = \frac{1}{2i} \left\{ (1 - \Gamma_{0c} - i \Gamma_{0s}')^m - (1 - \Gamma_{0c} + i \Gamma_{0s}')^m \right\} \quad (22b)$$

As shown in the Appdx, simplifying the expressions for $S_{f0}^{(1)}$ and $S_{f0}^{(2)}$ as given by Eqs.(20b) and (20c), using Eq.(21) and substituting Eq.(20a) in Eq.(1a) the inelastic scattering amplitude may be written as

$$F_{f0}(\vec{q}) = \mathcal{F}_{LM} + \frac{1}{2} (\sigma_+ \mathcal{F}_{LM}^{(-)} + \sigma_- \mathcal{F}_{LM}^{(+)}) \quad (23)$$

where $\sigma_{\pm} = \sigma_x \pm i \sigma_y$ and

$$\mathcal{F}_{LM}(\vec{q}) = i^{M+1} k e^{-iM\phi_q} \int_0^{\infty} db \{ b J_M(qb) A T_{1c}(b) \} \quad (24a)$$

$$\mathcal{F}_{LM}^{(\pm)}(q) = \pm i^{M+1 \pm 1} k e^{-i(M \pm 1)\phi_q} \times \int_0^{\infty} db \left[b J_{M \pm 1}(qb) \{ A T_{1s} \mp \frac{M}{b} \epsilon_{LM}^s(b) T_b \} \right] \quad (24b)$$

In the above equations J_M is the cylindrical Bessel function of order M and

$$T_{1c} = X_{A-1} \epsilon_{LM}^c(b) - Y_{A-1} \epsilon_{LM}^{is} \quad (25a)$$

$$T_{1s} = X_{A-1} \epsilon_{LM}^{is} + Y_{A-1} \epsilon_{LM}^c \quad (25b)$$

$$T_b = \sum_{j=1}^A (X_{j-1} X_{A-j} + Y_{j-1} Y_{A-j}) \quad (25c)$$

with X 's and Y 's defined as in Eqs.(22).

For obtaining expressions for the inelastic differential cross-section and the corresponding polarization it is convenient to write Eqs.(24a) and (24b) as

$$\mathcal{F}_{LM}(q) = e^{-iM\phi_q} G_{LM} \quad (26a)$$

$$\mathcal{F}_{LM}^{\pm}(q) = e^{-i(M \pm 1)\phi_q} G_{LM}^{\pm} \quad (26b)$$

It is easy to see that the following relations hold:

$$G_{L-M} = (-1)^M G_{LM} \quad (27a)$$

$$G_{L-M}^{\pm} = (-1)^{M+1} G_{LM}^{\mp} \quad (27b)$$

The expressions for the differential cross-section and polarization may now be obtained applying

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_M \text{Tr}(F_{f0} F_{f0}^{\dagger}) \quad (28a)$$

and

$$P = \frac{\text{Tr}(F_{f0} F_{f0}^{\dagger} \vec{\sigma} \cdot \hat{n}_0)}{d\sigma/d\Omega} \quad (28b)$$

where

$$\hat{n}_0 = \frac{\vec{q} \times \vec{k}}{|\vec{q} \times \vec{k}|}$$

Substituting Eqs.(26) in Eqs.(28a) and (28b) and using relations given by Eqs.(27) one finally obtains

$$\frac{d\sigma}{d\Omega} = \sum_{M=-L}^L (|G_{LM}|^2 + |G_{LM}^+|^2) \quad (29)$$

and

$$P = -2 \text{Im} \left(\sum_{M=-L}^L G_{LM}^+ G_{LM}^* \right) / \frac{d\sigma}{d\Omega} \quad (30)$$

III. CALCULATION AND DISCUSSION

The expressions derived in the previous section will now be used to study certain aspects of inelastic p-nucleus scattering at intermediate energies. In what follows we do not attempt to achieve a good fit to the existing data by varying the transition density or the NN amplitude. Rather our main aim is to make a comparison of the present approach with that proposed by Fäldt and Osland¹¹⁾ and to study the effect of considering the momentum transfer dependence of the NN amplitude neglected by these authors. For this we use the following parametrization for the NN amplitude obtained by Auger and Lombard³⁾:

$$f_c(q) = \frac{k\sigma}{4\pi} (1 + \alpha) e^{-\beta q^2/2} \quad (31)$$

$$f_s(q) = \frac{ik D_s (i + \alpha_s)}{8\pi M} e^{-\beta_s q^2/2} \quad (32)$$

with

$$\alpha = \alpha_0 + \alpha_1 q^2 \quad (33)$$

In the above expressions σ is the NN total cross-section, M the nucleon mass, β (β_s) the slope parameters, D_s the strength parameter for the spin dependent part of the amplitude and α (α_s) is the ratio of the real to the imaginary part of the amplitude. This parametrization of the amplitude differs from the generally used ones in that α in Eq.(31) is not constant rather it depends upon q through Eq.(33). The details regarding the above parametrization are given in Ref.3. The parameter values are (σ and α_0 have been averaged over their proton and neutron values):

$$\begin{aligned} \sigma &= 4.3 \text{ fm}^2, & \beta &= 0.25 \text{ fm}^{-2}, & \alpha_0 &= -0.24 \\ D_s &= 1.5, & \beta_s &= 0.4 \text{ fm}^{-2}, & \alpha_s &= 0.7 \end{aligned} \quad (34)$$

and

$$\alpha_1 = 0.1$$

Further, all calculations to be presented below have been made for p-⁴²Ca scattering at incident proton energy $E = 1.04$ GeV. For the target ground state density we have used the theoretical HFBCS density obtained by Beiner and Lombard¹³⁾. As regards the transition density we obtain it from the ground state density using the Tassie model¹⁴⁾

$$\rho_{tr}(r) = N_k r^{k-1} \frac{d\rho}{dr} \quad (35)$$

The normalization constant is fixed from $B(E2)$ and $B(M2)$ values.

In Fig.1 we show contributions of various magnetic substates to the $2^+(1.5 \text{ MeV})$ inelastic p-⁴²Ca scattering. Since in our approach only those M states which satisfy $L+M = \text{even}$ contribute to the scattering, the M values to be considered are: $M = \pm 2$ and 0 . The dashed curves are for the summed contributions for $M = \pm 2$ while the continuous ones include the contribution for $M = 0$ as well. It is seen that but for small momentum transfers the $M = \pm 2$ contribution dominates. In fact for polarization the $M = 0$ contribution is almost negligible for $q \gtrsim 0.5 \text{ fm}^{-1}$. Our results for the inelastic angular distribution are essentially the same as obtained by Fäldt and Osland¹¹⁾ (these authors do not study polarization). However, in principle, in their formulation even those states for which $L + M$ is odd make some finite contribution. But in practice such contributions are negligibly small being about 10^{-4} times the values shown¹¹⁾ in Fig.1.

A comparison of the calculated inelastic differential cross-sections given by the present approach and that of Fäldt and Osland¹¹⁾ is presented in Fig.2. To make the comparison meaningful we put $\beta = \beta_s = \alpha_1 = 0$ in our expressions as assumed by these authors. It is seen that the two calculations are quite close to each other. We have checked that the some difference as seen in Fig.2 is mainly because of the optical limit approximation used by Fäldt and Osland¹¹⁾ and not because of contributions from $L + M = \text{odd}$ states absent in our approach.

Next, in Figs.3a and 3b we present our study of the effect of neglecting the q -dependence in the NN amplitude on the inelastic scattering calculations. The dotted curves are obtained with the full q -dependence of the NN amplitude as shown in Eqs.(31) to (33). The continuous curves are the same as the dotted ones but with $\alpha_1 = 0$, i.e. neglecting the q -dependence in α . Similarly, the difference between the continuous curves and the dashed curves is that the latter are calculated assuming zero range NN force. In other words for the dashed curves all the three parameters β , β_s and α_1 are taken as zero and correspond to the study reported by Fäldt and Osland¹¹⁾. The following conclusions may be drawn from the calculations shown in Figs.3a and 3b. Assuming zero range for the NN force is a very poor approximation for inelastic scattering calculations. The situation is particularly worse for polarization where this approximation fails even qualitatively. It is interesting to note that the situation is somewhat different for polarization in elastic scattering calculation. In this case the zero range approximation gives relatively much better results (compare the continuous and the dashed curves in Fig.4). Next

comparing *) the dotted and the continuous curves in Figs. 3a and 3b we note that while neglecting q -dependence in α is a good approximation for the differential cross-section (except in narrow regions of minima) the same is not true for polarization which seems to depend quite sensitively on the q -dependence in α . (The situation is somewhat similar for elastic scattering polarization in this respect as shown in Fig. 4.) Thus our results show that a careful treatment of the q -dependence in the NN amplitude is necessary for a realistic analysis of inelastic scattering polarization data. In other words any attempt to extract information on nuclear transition densities from the medium energy proton inelastic scattering experiments must be preceded by an accurate determination of the NN amplitude over a reasonably large momentum transfer.

It may be added that although our conclusions are based on calculations using NN parameter values which are by no means uniquely determined still they are very likely to remain unaltered when a set of more accurately determined parameter values become available.

*) Before proceeding further a clarifying remark on our calculation with $\alpha_1 = 0$ is appropriate. The parametrization of the NN amplitude as given by Eqs. (31) and (33) has been found to be consistent with the available NN elastic scattering data as well as with the p - ^4He elastic scattering experiments ³⁾. It accounts for the different q -dependences for the real and imaginary parts of the spin independent amplitude. Naturally the value of the parameter β in the parametrization in which α is taken independent of q would be somewhat different from that used here (some other parameter values might also be slightly affected). Therefore, our calculation with $\alpha_1 = 0$ does not exactly correspond to that using q -independent α parametrized NN amplitude. However, the difference between the values of β in the two parametrizations is rather small being roughly 16% less for the case of q -independent α parametrization. Since small variations in the value of β are not critical to the calculation, therefore, for a qualitative study of this kind a comparison between the calculations with $\alpha_1 \neq 0$ and with $\alpha_1 = 0$ would largely show the effect of assuming the same q -dependence for the real and imaginary parts of the NN amplitude.

IV. CONCLUDING REMARKS

In this work we have extended our previously proposed effective profile function expansion approach for evaluating the Glauber amplitude to the case of proton-nucleus inelastic scattering with spin dependent interaction. An obvious advantage, in addition to some others as discussed in the text, of expanding the Glauber S-matrix in terms of the effective profile γ is that the z -ordering problem which complicates the computational work considerably is no longer present in evaluating the first two terms of the expansion which contribute dominantly to the elastic and the inelastic scatterings, respectively. Of course, the z -ordering problem is present in the evaluation of the terms which are of higher order in γ but such terms being dependent upon the two-body and the other higher correlations are expected (on the basis of spin independent calculation) to make only a small contribution to the total scattering provided that the momentum transfer covered is not very large. Even if it becomes necessary to consider the higher order terms in the expansion one does not expect, at the present, to go beyond the two-body correlation terms. If so one faces a z -ordering problem only in two co-ordinates rather than A co-ordinates as in the original problem. One further consideration: spin dependent part of the basic amplitude being relatively small, spin and i -spin structure of the two-body correlation function only crudely known, there seems to be little point in worrying about the z -ordering problem while evaluating the two-body correlation term (these remarks do not apply to very light nuclei). We, therefore, feel that the z -ordering problem may be ignored in evaluating this higher order term without introducing any significant error in the calculation.

Having obtained a suitable approach for evaluating the Glauber amplitude we have next studied the sensitivity of the inelastic scattering calculation to the momentum transfer dependence of the basic NN amplitude neglected in the work of Faldt and Osland ¹¹⁾. In the parametrization of the amplitude employed in this study the momentum transfer dependence enters in two ways. First, through the usual exponential factors governed by β and β_s and second, through the q -dependence in α . Our finding that the polarization in inelastic scattering calculation is particularly sensitive to both, demands on the one hand a careful treatment of the NN amplitude in such calculations and on the other it also emphasizes the need for a careful determination of the NN amplitude upto fairly large momentum transfers in this energy region.

In this appendix we give in some detail derivation of Eq.(23) from Eq.(1a) using Eqs.(20a) and (21).

Since $\vec{\sigma} \cdot \hat{n}$ commutes with itself, it follows from Eqs.(20b) and (21) that

$$S_{r0}^{(1)} = -A (T_{1c} + i \vec{\sigma} \cdot \hat{n} T_{1s}) e^{-iM_0 b} \quad , \quad (A.1)$$

where T_{1c} and T_{1s} are the same as defined in Eqs.(25a) and (25b).

Next the crucial factor in Eq.(20c) which needs careful evaluation is the sum

$$G = \sum_{j=1}^A (1 - \Gamma_0)^{j-1} \vec{\sigma} \cdot \hat{b} (1 - \Gamma_0)^{A-j} \quad (A.2)$$

Applying Eq.(21) we can write Eq.(A.2) as

$$\begin{aligned} G &= \sum_{j=1}^A X_{j-1} X_{A-j} \vec{\sigma} \cdot \hat{b} + i \sum_j X_{j-1} Y_{A-j} (\vec{\sigma} \cdot \hat{b}) (\vec{\sigma} \cdot \hat{n}) \\ &\quad + i \sum_j Y_{j-1} X_{A-j} (\vec{\sigma} \cdot \hat{n}) (\vec{\sigma} \cdot \hat{b}) \\ &\quad - \sum_j Y_{j-1} Y_{A-j} (\vec{\sigma} \cdot \hat{n}) (\vec{\sigma} \cdot \hat{b}) (\vec{\sigma} \cdot \hat{n}) \quad . \quad (A.3) \end{aligned}$$

Now using

$$\begin{aligned} (\vec{\sigma} \cdot \hat{b}) (\vec{\sigma} \cdot \hat{n}) &= i \vec{\sigma} \cdot \hat{k} \\ &= -(\vec{\sigma} \cdot \hat{n}) (\vec{\sigma} \cdot \hat{b}) \quad . \quad (A.4) \end{aligned}$$

and

$$\begin{aligned} (\vec{\sigma} \cdot \hat{n}) (\vec{\sigma} \cdot \hat{b}) (\vec{\sigma} \cdot \hat{n}) &= i (\vec{\sigma} \cdot \hat{n}) (\vec{\sigma} \cdot \hat{k}) \\ &= -\vec{\sigma} \cdot \hat{b} \quad . \quad (A.5) \end{aligned}$$

we have from Eq(A.3)

$$\begin{aligned} G &= \sum_{j=1}^A (X_{j-1} X_{A-j} + Y_{j-1} Y_{A-j}) \vec{\sigma} \cdot \hat{b} \\ &\quad + i \sum_{j=1}^A (Y_{j-1} X_{A-j} - X_{j-1} Y_{A-j}) \vec{\sigma} \cdot \hat{k} \quad . \quad (A.6) \end{aligned}$$

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It is easy to see that the second term in Eq.(A.6) involving $\vec{g} \cdot \hat{k}$ is zero. So that we can write Eq.(20c) as

$$S_{f0}^{(2)} = \frac{M}{b} \epsilon_{LM}^s(b) e^{-iM\phi_b} T_b \vec{\sigma} \cdot \hat{b} \quad (A.7)$$

where T_b is the same as defined by Eq.(25c).

Further adding Eqs.(A.1) and (A.2) we can write Eq.(20a) of the text as

$$\begin{aligned} \langle \Psi_f | S | \Psi_0 \rangle = & -A T_{1c} e^{-iM\phi_b} + \sigma_+ \left(-\frac{A}{2} T_{1s} + \frac{M}{2b} T_b \epsilon_{LM}^s \right) e^{-i(M+1)\phi_b} \\ & + \sigma_- \left(\frac{1}{2} A T_{1s} + \frac{M}{2b} T_b \epsilon_{LM}^s \right) e^{-i(M-1)\phi_b}, \quad (A.8) \end{aligned}$$

where

$$\sigma_{\pm} = \sigma_x \pm i\sigma_y.$$

Now substituting Eq.(A.8) in Eq.(1a) and using the relation

$$\int_0^{2\pi} e^{iqb \cos(\phi_a - \phi_b) - iM\phi_b} d\phi_b = 2\pi i^M J_M(qb) e^{-iM\phi_a}$$

Eq.(23) of the text can easily be obtained.

REFERENCES

- 1) J. Saudinos and C. Wilkin, Ann. Rev. Nucl. Sci. 24, 341 (1974).
- 2) I. Ahmad, Nucl. Phys. A247, 418 (1975).
- 3) J.P. Auger and R.J. Lombard, Ann. Phys. 115, 442 (1978).
- 4) G.D. Alkhazov, S.L. Belostotsky and A.A. Vorobyov, Phys. Rep. C42, 89 (1978).
- 5) G. Igo, Rev. Mod. Phys. 50, 523 (1978).
- 6) Y. Abgrall *et al.*, Nucl. Phys. A316, 389 (1979).
- 7) R.J. Glauber in Lectures in Theoretical Physics, Eds. W.E. Brittin and L.G. Dunham (Interscience, N.Y. 1959), Vol. 1, p.315.
- 8) P. Osland and R.J. Glauber, Nucl. Phys. A326, 255 (1979).
- 9) R.D. Viollier, Ann. Phys. 93, 335 (1975).
- 10) G. Alberi, M. Gmitro and L. Hambro, Nuovo Cimento 38A, 239 (1977).
- 11) G. Faldt and P. Osland, Nucl. Phys. A305, 509 (1978).
- 12) I. Ahmad and J.P. Auger, Nucl. Phys. A352, 425 (1981).
- 13) M. Beiner and R.J. Lombard, Ann. Phys. 86, 262 (1974).
- 14) L.J. Tassie, Austral. J. Phys. 9, 407 (1959).
- 15) G.D. Alkhazov *et al.*, Nucl. Phys. A274, 443 (1976).

FIGURE CAPTIONS

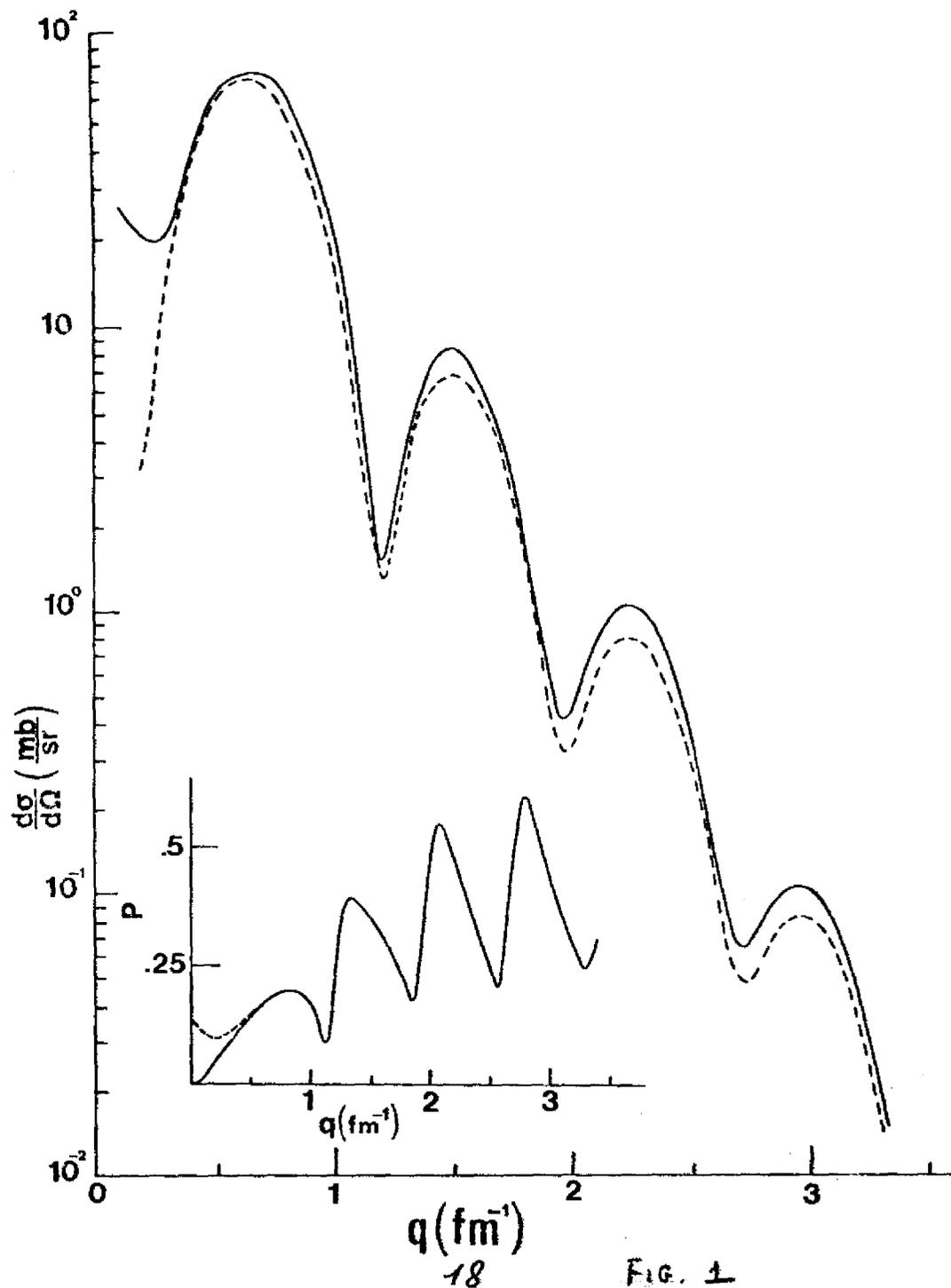
Fig.1 Contributions of various magnetic substates to the 2^+ (1.52 MeV) inelastic $p-^{42}\text{Ca}$ scattering at 1.044 GeV.

Fig.2 Comparison of the calculated differential cross-sections for the excitation of ^{42}Ca (1.52 MeV; 2^+). The dashed curve is calculated following the approach of Faldt and Osland ¹¹⁾ while the continuous one corresponds to the present approach taking zero range for the NN interaction.

Fig.3a Differential cross-sections for the excitation of ^{42}Ca (1.52 MeV; 2^+) at 1.044 GeV. Dashed curve is obtained using the NN parameters as given in the text but with $\beta = \beta_s = 0$ and $\alpha_1 = 0$. The continuous curve is the same as the dashed curve but with $\beta = 0.25 \text{ fm}^{-2}$, $\beta_s = 0.4 \text{ fm}^{-2}$ and $\alpha_1 = 0$. The dotted curve is the same as the continuous one but with $\alpha_1 = 0.1$. The larger dots show experimental data points of Ref.15.

Fig.3b Same as in Fig.3a but for polarization.

Fig.4 Calculated polarization for elastic $p-^{42}\text{Ca}$ scattering at 1.044 GeV. the description of the curves is the same as in Fig.3a.



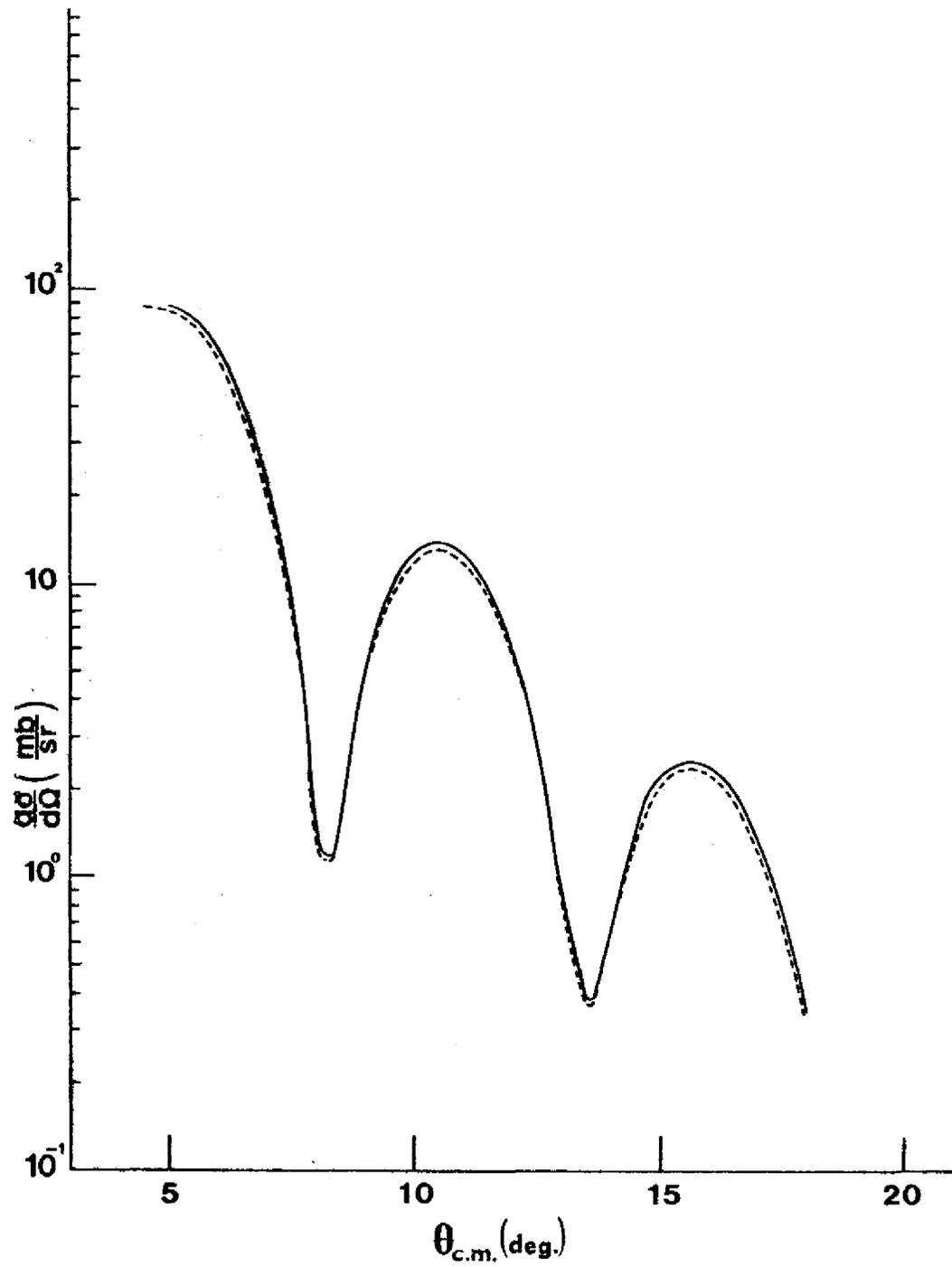


FIG. 2 19

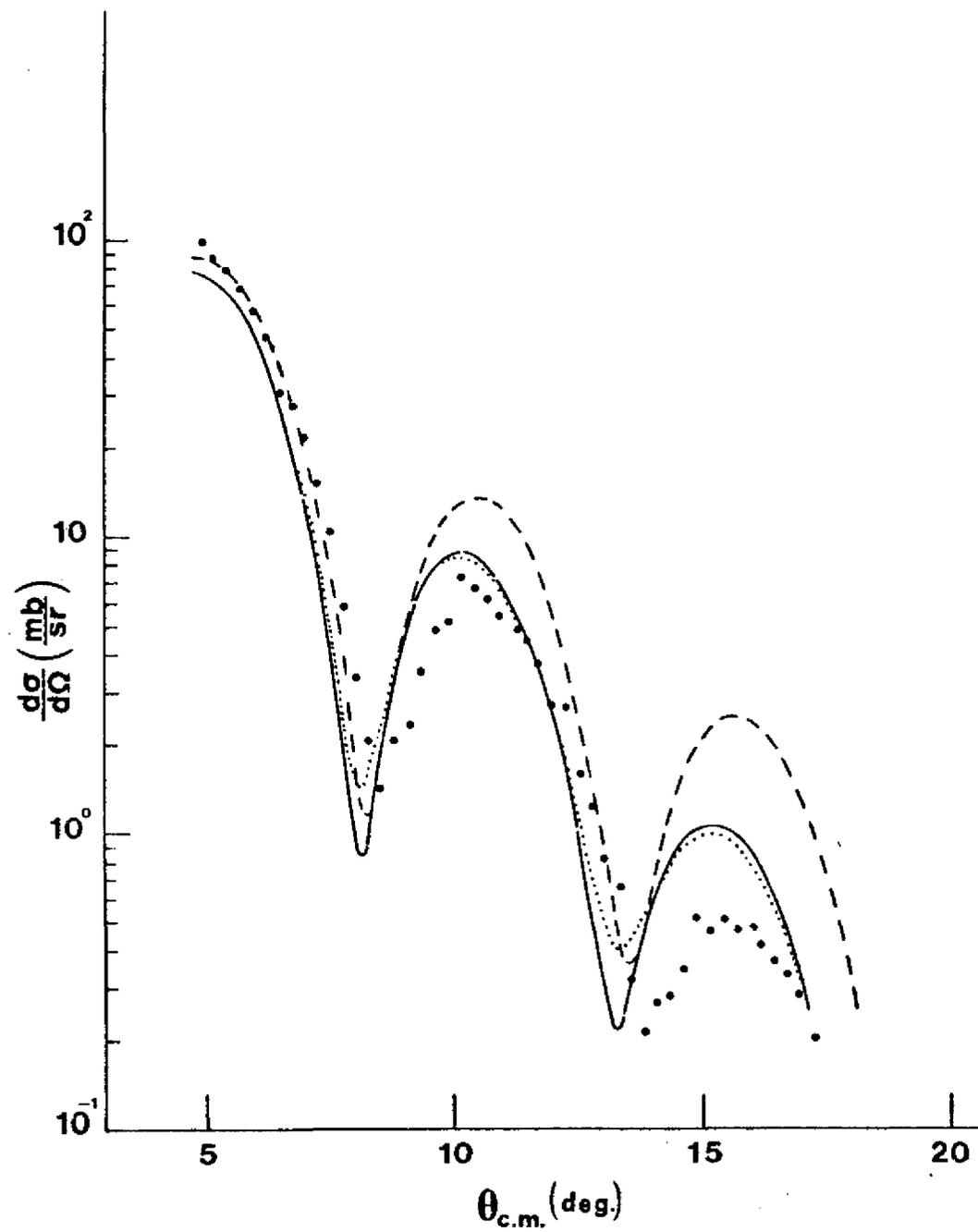


FIG. 3a 20

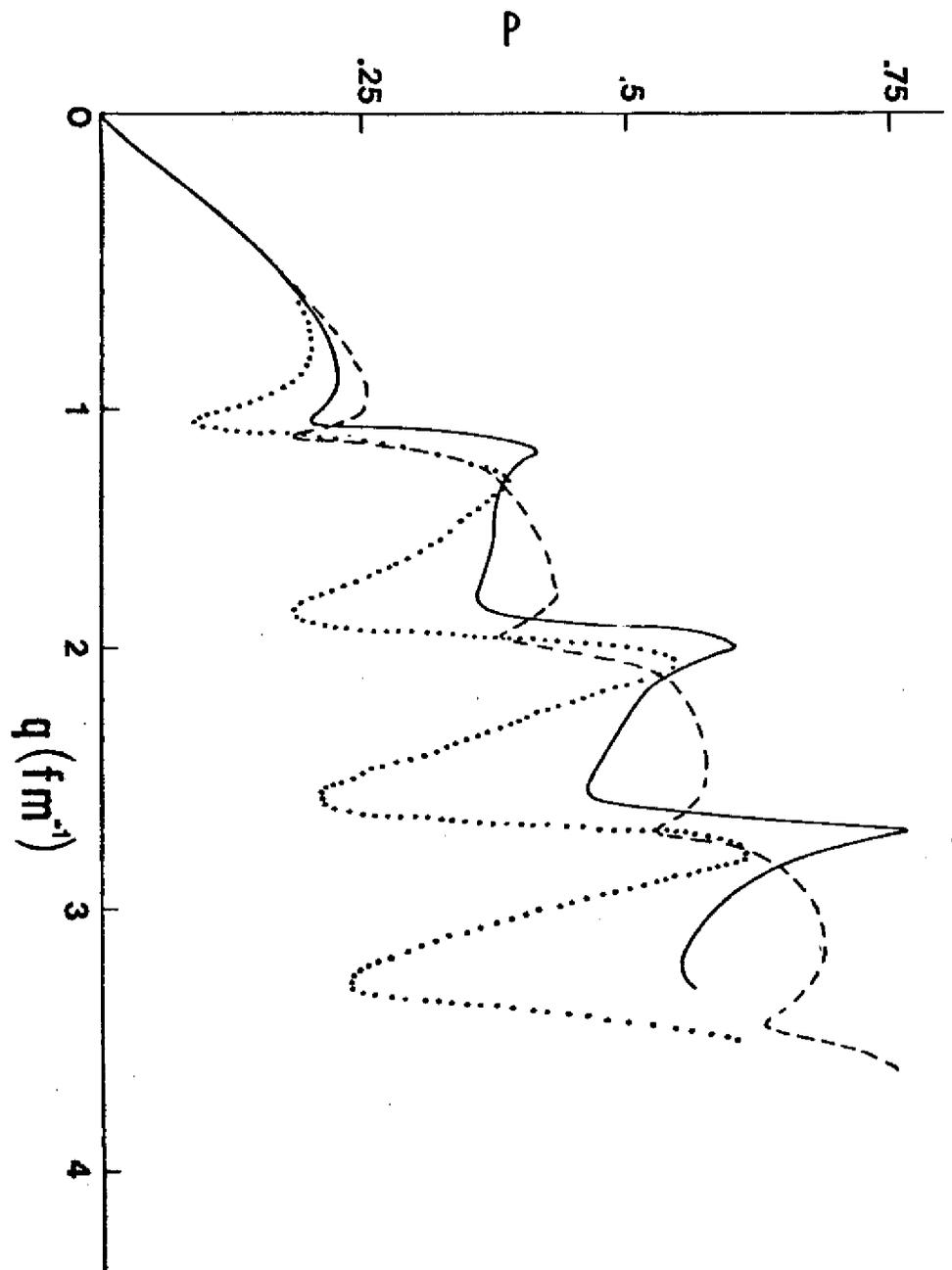


Fig. 36

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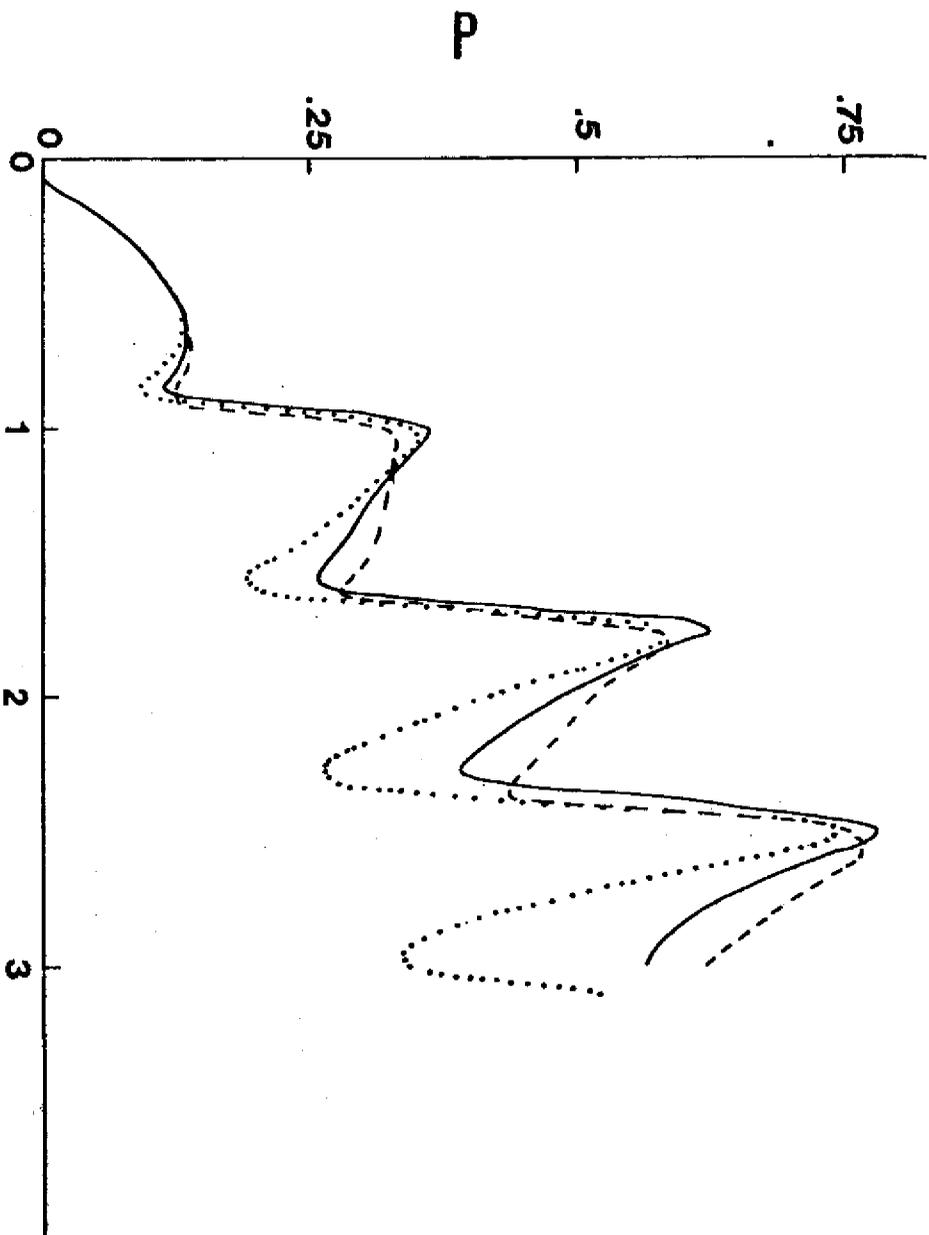


Fig. 4

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