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SUM RULES FOR NEUTRINO OSCILLATIONS

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A b s t r a c t

Some rules for neutrino oscillations are obtained. The effects due to neutrino masses are described without assuming that m_ν is a small parameter.

Introduction

The problem of neutrino oscillations has been under discussion for more than twenty years since the pioneering papers by B.Pontecorvo [1,2]. Recently the problem attracted much attention. (For the review of present status see [3-5]). The phenomenology of neutrino oscillations has been the subject of a large number of papers (see e.g. refs. [6-11] and references therein). In spite of this vast literature and in spite of the obvious simplicity of the subject it nevertheless seems inexhaustable.

In this paper we derive some new sum rules for neutrino oscillations (see eqs. (19) and (20)). These sum rules refer to the total probability of transition of a neutrino of given flavour into all possible flavors of neutrinos and antineutrino respectively. The most interesting consequences of the sum rules follow in the case of Majorana neutrino masses. Another minor point is that expressions we obtain do not assume that the neutrino mass m_ν is small compared to its energy E. (Usually in the literature only the first nonvanishing terms in the m_ν/E expansion are taken into account).

We consider a two stage process. At the first stage a charged lepton ℓ_i of a given flavor i ($\ell_i = e, \mu, \tau, \ldots$) collides with an infinitely heavy nucleus, which is situated at point $\vec{x}_1$. The neutrino ν_i which is produced at this point by charged current has the energy equal to the energy of the lepton ℓ_i. Due to the flavor-nondiagonal terms in the neutrino mass Lagrangian the state ν_i is a superposition of mass eigenstates φ_A with the same energy, but with different momenta and helicities. At point $\vec{X}_2$ these superposition collides with another heavy nucleus and a charged lepton ℓ^-_K or an antilepton ℓ^+_K are produced. This choice of quasiexperimental conditions allows to discuss the properties of oscillations in most transparent form. The adaptation of results to the conditions of real experiments is rather obvious. Below we derive amplitudes and effective cross-sections for the above two stage processes. By summing these cross-sections over flavors of final leptons and antileptons respectively we obtain the above mentioned sum rules.

In the next section the derivation of the general form of the S-matrix for two stage process $\ell^-_i \to \nu \to \ell^\pm_K$ is presented. In the subseq uent sections sum rules for some special cases of neutrino mass lagrangian are considered.

1. The amplitude of the process $\ell^-_i \to \nu \to \ell^\pm_K$ and neutrino oscillations

The scheme of the two stage process $\ell^-_i \to \nu \to \ell^\pm_K$ is shown on fig. 1, where $\vec{X}_1 = 0$ and $\vec{X}_2 = \vec{L}$. We denote by $\vec{p}_i, \varepsilon_i, \lambda_i$ $(\vec{p}_K, \varepsilon_K, \lambda_K)$ - the momentum, energy and helicity of the initial (final) lepton. Due to the large mass of the nucleus $\varepsilon_i = \varepsilon_K = \varepsilon$. We assume that nuclei are spinless. Then only the vector hadronic currents give contribution to our processes. The amplitudes depend on three angles

α, ϑ and φ, the connection of which with vectors $\vec{p}_i$, $\vec{p}_k$ and $\vec{L} = \vec{x}_2 - \vec{x}_1$ is shown on fig. 2.

Consider at first the process $\ell_i^- \to \nu \to \ell_k^-$ without violation of leptonic charge. S-matrix of this process is equal to

$$S = -\frac{1}{2!}\left(\frac{G}{\sqrt{2}}\right)^2 \cdot$$
$$\cdot \int d^4x\, d^4y \langle \ell_k^-; N_1', N_2' | T\{ j_\mu(x) J_\mu^+(x) j_\nu^+(y) J_\nu(y)\} | N_1, N_2; \ell_i^- \rangle, \qquad (1)$$

where N_1, N_2 and N_1', N_2' characterize the initial and final nuclei states, G - Fermi constant, $j_\mu(x)$ - charged leptonic current

$$j_\mu(x) = \sum_n \bar{\nu}_n(x)\gamma_\mu(1+\gamma_5)\ell_n(x) \qquad (2)$$

(the sum is taken over leptonic flavours $\ell_n = e, \mu, \tau, \ldots$), $J_\mu(x)$ - charged hadronic current. As is well known [1,2] oscillations take place under the following conditions: neutrino with the flavour is not an eigenstate of hamiltonian but is represented by the superposition of diagonal states

$$\nu_i = U_{iA}\varphi_A \quad ; \qquad (3)$$

the masses of diagonal states are different

$$m_A \neq m_B, \quad A \neq B. \qquad (4)$$

Substituting (3) into (2) we may express the product of leptonic currents in S-matrix as follows

$$\langle \ell_K^- | T\{j_\mu(x) j_\nu^+(y)\} | \ell_i^- \rangle =$$

$$= \bar{u}(p_K)\hat{O}_\mu \langle 0 | T(\varphi_A(x)\bar{\varphi}_B(y)) | 0 \rangle \hat{O}_\nu u(p_i) \cdot \tag{5}$$

$$\cdot U_{iA}^+ U_{BK} e^{ip_K y - ip_i x},$$

where

$$\hat{O}_\mu = \gamma_\mu (1 + \gamma_5)$$

and $u(p_i)$, $u(p_K)$ are spinors of the initial and final leptons. T-product of diagonal fields φ_A and φ_B is obviously proportional to δ_{AB}

$$G_{AB}(x-y) = \langle 0 | T(\varphi_A(x)\bar{\varphi}_B(y)) | 0 \rangle =$$

$$= \int \frac{e^{-iq(x-y)}}{\hat{q} - m_A} \frac{d^4 q}{(2\pi)^4} \delta_{AB}. \tag{6}$$

Hadronic currents for heavy spinless nuclei are represented by δ -functions

$$J_\mu(x) \rightarrow \delta_{0\mu}\delta(\vec{x} - \vec{x}_1),$$

$$J_\nu(y) \rightarrow \delta_{0\nu}\delta(\vec{y} - \vec{x}_2). \tag{7}$$

By using transformation described in appendix 1 we obtain for the S-matrix (1) the expression

$$S = 2\pi\delta(\varepsilon_i - \varepsilon_K)\frac{\left(\frac{G}{\sqrt{2}}\right)^2}{\sqrt{2\varepsilon_i V}\sqrt{2\varepsilon_K V}} M,$$

$$M = \sum_A \frac{e^{ip_A L}}{2\pi L} U_{iA}^+ U_{AK} \cdot \bar{u}(p_K)(\varepsilon\gamma_0 + \tag{8}$$

$$+ p_A \vec{n}_L \vec{\gamma})(1 + \gamma_5) u(p_i).$$

Here $p_A = \sqrt{\mathcal{E}^2 - m_A^2}$, $\mathcal{E}_i = \mathcal{E}_K = \mathcal{E}$, $\vec{n}_L = \vec{L}/L$, V - normalization volume. Matrix element M is a sum over diagonal neutrino states. Formula (8) is valid both for Dirac and Majorana neutrino φ_A. Note that if for some A $m_A > \mathcal{E}$ then $p_A = i|p_A|$ and the corresponding contribution to M is exponentially small: $e^{ip_AL} = e^{-|p_A|L}$. In formula (8) for S the δ-function corresponding to momentum conservation is absent because in the scattering on heavy centre only the energy of scattered particle is conserved. The cross-section is expressed through M as follows

$$\frac{d\sigma}{d\Omega_K} = \frac{G^4}{64\pi^2} |M|^2 \frac{v_K}{v_i}, \qquad (9)$$

where $d\Omega_K$ is the solid angle of momentum $\vec{p}_K$, v_i and v_K are the velocities of leptons ℓ_i and ℓ_K.

S-matrix for the process $\ell_i^- \to \nu \to \ell_K^+$ may be obtained analogously

$$S = -\frac{1}{2!}\left(\frac{G}{\sqrt{2}}\right)^2 \cdot \qquad (10)$$
$$\cdot \int d^4x\, d^4y \langle \ell_K^+; N_1', N_2' | T\{j_\mu(x) J_\mu^+(x) j_\nu(y) J_\nu^+(y)\} | N_1, N_2; \ell_i^- \rangle.$$

T-product of leptonic currents contains now the matrix element $\langle 0 | T(\bar{\varphi}_A^T(x) \bar{\varphi}_B(y)) | 0 \rangle$ that is different from zero only for Majorana neutrinos. In this case we have $\varphi_A = C\bar{\varphi}_A^T$ where $C = i\gamma_2\gamma_0$ and

$$\langle 0 | T(\bar{\varphi}_A^T(x) \bar{\varphi}_B(y)) | 0 \rangle = -C G_{AB}(x-y), \qquad (11)$$

where $G_{AB}(x-y)$ is given by formula (6). The similar calculation gives the matrix element of $\ell_i^- \to \nu \to \ell_K^+$ process

$$M(\ell_i^- \to \ell_K^+) = \sum_A \frac{e^{ip_A L}}{2\pi L} U_{iA}^T U_{AK} \cdot m_A \bar{u}(p_K)(1+\gamma_5)u(p_i). \tag{12}$$

We have used above the well-known property of Dirac spinors

$$u^T(-p_K)C = \bar{u}(p_K).$$

For further discussion it is convenient to use the spinors of definite helicity and split representation for γ - matricies. For example we have

$$M(\ell^-_{i,-1/2} \to \ell^-_{K,-1/2}) = \sum_A \frac{e^{ip_A L}}{2\pi L} U_{iA}^{+} U_{AK}(E_i^+ E_K^+) \cdot \left\{ (\varepsilon + p_A \cos\alpha) e^{-i\varphi/2} \cos\vartheta/2 + p_A e^{i\varphi/2} \sin\vartheta/2 \sin\alpha \right\}, \tag{13}$$

$$M(\ell^-_{i,-1/2} \to \ell^+_{K,+1/2}) = \sum_A \frac{e^{ip_A L}}{2\pi L} U_{iA}^T U_{AK}(E_i^+ E_K^+) m_A e^{-i\varphi/2} \sin\vartheta/2. \tag{14}$$

Here $p_A = \sqrt{\varepsilon^2 - m_A^2}$, $E_i^{\pm} = \sqrt{\varepsilon + \mu_i} \pm \sqrt{\varepsilon - \mu_i}$; μ_i - mass of lepton ℓ_i . The expressions for other amplitudes are given in appendix 2. In the limit $\varepsilon \gg \mu_i$, when μ_i may be neglected, only amplitudes (13), (14) are not equal to zero. To simplify the geometry let us take α = 0, that is $\vec{p}_i$ to be parallel to $\vec{L}$. The cross-sections are then easily calculated :

$$\sigma(\ell_i^- \to \ell_K^-) \sim \frac{G^4 \varepsilon^2}{L^2} \sum_{A,B} e^{i(p_A - p_B)L} U_{iA}^+ U_{AK} U_{KB}^+ U_{Bi} \cdot \left\{ \varepsilon^2 + p_A p_B + \varepsilon (p_A + p_B) v_i \right\} \frac{v_K}{v_i} \tag{15}$$

and

$$\sigma(\ell_i^- \to \ell_K^+) \sim \frac{G^4 \varepsilon^2}{L^2} \sum_{A,B} e^{i(p_A - p_B)L} U_{iA}^T U_{AK} U_{KB}^+ U_{Bi}^* m_A m_B \frac{v_K}{v_i} . \tag{16}$$

Formula (15) is a general one, whereas (16) is valid only in the case when diagonal neutrinos are Majorana particles. We assume weak currents to be left-handed. In principle it is not difficult to take into account possible contribution of right-handed currents. In following sections we consider some particular cases of neutrino mass matrix.

2. Left-handed Majorana neutrino

Let the neutrino mass Lagrangian be of the form

$$-\mathcal{L}_M = m_{iK}^L \nu_{iL}^T C \nu_{KL} + H.C. \tag{17}$$

Lagrangian (17) contains only nonsterile components. Matrix m_{iK}^L is a complex, symmetric N x N matrix depending on N (N + 1) parameters. It is diagonalized by N x N unitary matrix U_{Ai} which contains $N(N-1)/2$ angles, $N(N-1)/2$ physical phases and N unphysical phases. Let us calculate the total neutrino flux at point $\vec{X}_2 = \vec{L}$. The quantities

$\frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^-)$ and $\frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^+)$ characterize evidently the flux of neutrino and antineutrino of flavour K at this point.

Due to the unitarity of U matrix

$$\sum_K U_{AK} U^+_{KB} = \delta_{AB} \qquad (18)$$

and we see that the total neutrino and antineutrino fluxes separatly do not oscillate:

$$\sum_K \frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^-) \sim \mathcal{E}^4 \frac{G^4}{L^2} \sum_A U^+_{iA} U_{Ai} (1+v_A^2+2v_A v_i) \frac{1}{v_i}, \qquad (19)$$

$$\sum_K \frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^+) \sim \mathcal{E}^4 \frac{G^4}{L^2} \sum_A U^T_{iA} U^*_{Ai} (1-v_A^2) \frac{1}{v_i}, \qquad (20)$$

$$\frac{\sum_K \frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^+)}{\sum_K \frac{1}{v_K}\sigma(\ell_i^- \to \ell_K^-)} = \frac{\sum_A U^+_{iA} U_{Ai}(1-v_A^2)}{\sum_A U^+_{iA} U_{Ai}(1+v_A^2+2v_A v_i)} . \qquad (21)$$

The sum rules (19), (20) are valid for all energies $\mathcal{E} \geqslant \mu_K$. The fact that these sum rules are valid separatly for $\ell_i^- \to \ell_K^-$ and $\ell_i^- \to \ell_K^+$ transitions means that there are no oscillations between neutrinos and antineutrinos in Majorana case. Note that the presence of antineutrino in the neutrino beam does not yet mean the appearance of $\nu \leftrightarrow \tilde{\nu}$ oscillations. Consider for example N = 2 (ν_e and ν_μ). Let the initial lepton be electron: $\ell_i^- = e^-$. The production of μ^- means then that $\nu_e \to \nu_\mu$ oscillation takes place, the increase of ν_μ being due to the decrease of ν_e. The sum of probabilities of ν_e and ν_μ presence in the beam is constant, as follows from formula (19) (more precisely proportional to $1/L^2$). The total probability to find $\tilde{\nu}_e$ or $\tilde{\nu}_\mu$ in the beam measured by e^+ and μ^+ production is also constant, so the oscillations take place only between ν_e and ν_μ and independently between $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$. The picture looks so as if electron produces ν_e already with $\tilde{\nu}_e$, $\tilde{\nu}_\mu$ admixture and neutrino and antineutrino oscillate later separatly. The amplitudes of $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$ admixture are proportional at point $\vec{X}_1 = 0$ to the elements m_{ee} and $m_{e\mu}$ of mass matrix respectively.

3. General case of Dirac plus Majorana mass terms

In general case neutrino mass lagrangian contains both Dirac and Majorana mass terms

$$-\mathcal{L}_M = m^D_{iK}\bar{\nu}_{iL}\nu_{KR} + m^L_{iK}\nu^T_{iL}C\nu_{KL} + m^R_{iK}\nu^T_{iR}C\nu_{KR} + + H.C. \tag{22}$$

Three terms in (22) may be written [8] in the form of 2N x 2N mass matrix

$$\mathcal{M} = \begin{pmatrix} \| m^L_{iK} \|^{*} & \| \frac{m^D_{iK}}{2} \| \\ \| \frac{m^D_{iK}}{2} \|^T & \| m^R_{iK} \| \end{pmatrix}. \tag{23}$$

The number of diagonal neutrino states is equal to 2N. Matrix U_{Ai} with 2N x N matrix elements is in this case a part of unitary 2N x 2N matrix K that connects diagonal and nondiagonal states:

$$K = \begin{pmatrix} U_{11} \cdots U_{1N} & V_{11} \cdots V_{1N} \\ U_{2N,1} \cdots U_{2N,N} & V_{2N,1} \cdots V_{2N,N} \end{pmatrix}. \quad (24)$$

Left-handed neutrinos ν_{iL} and right-handed neutrinos ν_{iR} (sterile neutrinos)of sort i are expressed in terms of diagonal neutrinos φ_A in the following way

$$\nu_{iL} = K^{+}_{iA} \varphi_{AL} \equiv U^{+}_{iA} \varphi_{AL}, \quad (25)$$

$$\nu_{iR} = K^{T}_{i+N,A} \varphi_{AR} \equiv V^{T}_{iA} \varphi_{AR}. \quad (26)$$

Only N x 2N matrix U_{Ai} enters the description of the processes $\ell^-_i \to \nu \to \ell^{\pm}_K$. Formulae (15) and (16) for cross sections are valid in this case also but the unitarity condition now has the form

$$K K^{+} = U_{AK} U^{+}_{KB} + V_{AK} V^{+}_{KB} = \delta_{AB}. \quad (27)$$

The second term in (27) corresponds to the contribution of sterile states (26) and consequently $\sum_K U_{AK} U^{+}_{KB} \neq \delta_{AB}$. The cross sections of $\ell^-_i \to \ell^-_K$ and $\ell^-_i \to \ell^+_K$ summed over K can therefore oscillate as neutrino produced at point $\vec{X}_1 = 0$ can transform into sterile states. The amplitude of nonsterile component therefore decreases and the cross section decreases also.

4. Dirac neutrinos

In the case when only Dirac mass terms are different from zero the diagonal neutrino states are Dirac states. The analysis of this possibility is rather simple - the cross section of the process $\ell_i^- \to \ell_K^-$ is given by (15) and the process $\ell_i^- \to \ell_K^+$ is forbidden. As in the case of left-handed Majorana neutrinos the cross sections satisfy the sum rule (19).

In this paper we did not use the assumption of CP-conservation so our results are valid for general form of complex matrix U allowing for CP-nonconservation.

5. Concluding remarks

Let us formulate the main conclusions. The consideration of two step process $\ell_i^- \to \nu \to \ell_K^\pm$ gives the possibility to take into account neutrino masses and to obtain the expressions for the oscillating cross sections. In the case of Dirac and left-handed Majorana neutrino we obtain the sum rule (19) for the quantities $\frac{1}{\nu_K}\sigma(\ell_i^- \to \ell_K^\pm)$. In the left-handed Majorana neutrino case there is an additional antineutrino admixture leading to $\ell_i^- \to \ell_K^+$ process. Both components (neutrino and antineutrino) oscillate independently, the cross sections $\sigma(\ell_i^- \to \ell_K^+)$ satisfying the sum rule (20) analogous to (19). Note that $\nu_i \to \nu_K$ oscillations may be absent but $\ell_i^- \to \ell_K^+$ transitions take place. For example if neutrino mass lagrangian consists of just one term $m_{e\mu}\nu_e^T C \nu_\mu$ then there are no oscillations but $e^- \to \mu^+$ and $\mu^- \to e^+$ transitions are possible.

In the most general case when neutrino mass Lagrangian contains both Dirac and Mahorana mass terms the oscillations into sterile states are possible.

The sums $\sum_K \frac{1}{v_K} \sigma(\ell_i^- \to \ell_K^\pm)$ then oscillate due to the presence of left-handed antineutrinos and right-handed neutrinos which do not take part in weak interactions. If right--handed currents are added sum rules analogous to considered above may be obtained. All our conclusions are valid in the general case when CP is not conserved.

Appendix 1

The calculation of the matrix element of $\ell_i^- \to \nu \to \ell_K^\pm$ transition

Let us calculate S-matrix for the process of fig. 1

$$S = -\frac{1}{2!}\left(\frac{G}{\sqrt{2}}\right)^2 \cdot$$

$$\cdot \int d^4x\, d^4y \langle \ell_K^-; N_1', N_2' | T\{ j_\mu(x) J_\mu^+(x) j_\nu^+(y) J_\nu(y)\} | N_1, N_2; \ell_i^- \rangle . \quad (1.\ 1)$$

Substituting (6) and (7) into (1.1) and integrating over d^3x and d^3y we obtain

$$S = -\frac{(G/\sqrt{2})^2}{\sqrt{2\varepsilon_i V}\,\sqrt{2\varepsilon_K V}} \int \bar{u}(p_K)\hat{O}_0 G_{AB}(\vec{L}, x_0 - y_0)\hat{O}_0 u(p_i) \cdot$$
$$\cdot U_{iA}^+ U_{BK} e^{i\vec{p}_i \vec{x}_1 - i\vec{p}_K \vec{x}_2} e^{i\varepsilon_i y_0 - i\varepsilon_K x_0} dx_0\, dy_0 . \quad (1.2)$$

Charged leptons are described here by plane waves

$$\ell = \sum_{p,\lambda} \frac{1}{\sqrt{2\mathcal{E}V}} \left(a_{p,\lambda} u(p,\lambda) e^{-ipx} + b^{+}_{p,\lambda} u(-p,-\lambda) e^{ipx} \right), \quad (1.3)$$

where V is the volume of normalization, $\hat{O}_o = \gamma_o(1+\gamma_5)$.

The constant factor $exp(i\vec{p}_i\vec{X}_1 - i\vec{p}_K\vec{X}_2)$ is omitted.

The integrations over X_o and y_o are standard

$$\frac{X_o+y_o}{2} = T \quad , \quad x_o - y_o = t$$

$$\int e^{iT(\mathcal{E}_K-\mathcal{E}_i)} dT = 2\pi\, \delta(\mathcal{E}_K - \mathcal{E}_i),$$

$$\int e^{it(\mathcal{E}-q_o)} dt = 2\pi\, \delta(\mathcal{E}-q_o).$$

After integration over q_o S-matrix becomes

$$S = - \frac{(G/\sqrt{2})^2\, 2\pi\delta(\mathcal{E}_i - \mathcal{E}_K)}{\sqrt{2\mathcal{E}_i V}\ \sqrt{2\mathcal{E}_K V}} \cdot$$

$$\cdot\, \bar{u}(p_K)\, \hat{O}_o \int \frac{e^{i\vec{q}\vec{L}}}{\hat{q}-m_A} \frac{d^3q}{(2\pi)^3}\, \hat{O}_o\, u(p_i)\, U^{+}_{iA} U_{AK}; \quad (1.4)$$

$$\hat{q} = q_\mu \gamma_\mu\,, \quad q_o = \mathcal{E}.$$

Integrating over $|\vec{q}|$ we must take into account the poles of (1.4). On fig. 3 we show the contour of integration obtained using the Feynman rules. Standard integration gives

$$\int e^{i\vec{q}\vec{L}} \frac{\hat{q}+m_A}{\varepsilon^2-\vec{q}^{\,2}-m_A^2} \frac{d^3q}{(2\pi)^3} = \tag{1.5}$$

$$= \frac{1}{4\pi L}\left[\varepsilon\gamma_0 + m_A - (p_A + \frac{i}{L})\vec{n}_L\vec{\gamma}\right]e^{ip_AL}.$$

The term i/L is almost always much less than p_A and may be omitted. It may be essential for microscopic lengths, for example, in 2β -decay. Now (8) is trivially obtained. The cross section is expressed through the square of S-matrix

$$d\sigma = |S|^2 \frac{V d^3p_K}{(2\pi)^3}\left(\frac{V}{v_i}\right)\frac{1}{T}.$$

Finally we have

$$\frac{d\sigma}{d\Omega_K} = \frac{G^4}{64\pi^2}|M|^2\frac{v_K}{v_i}.$$

The calculation of $\ell_i^- \to \nu \to \ell_K^+$ S-matrix element is analogous to calculation considered above.

Appendix 2

Helicity amplitudes of $\ell_i^- \to \nu \to \ell_K^\pm$ transitions

The calculation of helicity amplitudes is convenient to perform in split representation for γ -matrix

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \ \gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}.$$

The particles of definite helicity $\lambda = \pm 1/2$ and momentum

p are described in this representation by the following spinors

$$\mathcal{U}(p,\lambda=-1/2)=\frac{1}{\sqrt{2}}\begin{pmatrix}E_-\chi_-\\E_+\chi_-\end{pmatrix},$$

$$\mathcal{U}(p,\lambda=+1/2)=\frac{1}{\sqrt{2}}\begin{pmatrix}E_+\chi_+\\E_-\chi_+\end{pmatrix},$$

where $E^i_\pm=\sqrt{\varepsilon+m_i}\pm\sqrt{\varepsilon-m_i}$. ε - energy.

$$\chi_-=\begin{pmatrix}-e^{-i\varphi/2}\sin\vartheta/2\\e^{i\varphi/2}\cos\vartheta/2\end{pmatrix},\quad \chi_+=\begin{pmatrix}e^{-i\varphi/2}\cos\vartheta/2\\e^{i\varphi/2}\sin\vartheta/2\end{pmatrix}.$$

Besides (13) and (14) there are following helicity amplitudes

$$M(\ell^-_{i,-1/2}\to\ell^-_{K,+1/2})=\sum_A \exp(ip_AL)/2\pi L\ \ U^+_{iA}U_{AK}\cdot$$
$$\cdot E^K_-E^i_+\{-p_Ae^{i\varphi/2}\cos\vartheta/2\sin\alpha+(\varepsilon+p_A\cos\alpha)e^{-i\varphi/2}\sin\vartheta/2\},$$

$$M(\ell^-_{i,+1/2}\to\ell^-_{K,-1/2})=\sum_A \exp(ip_AL)/2\pi L\ \ U^+_{iA}U_{AK}\cdot$$
$$\cdot E^K_+E^i_-\{p_Ae^{-i\varphi/2}\cos\vartheta/2\sin\alpha+(\varepsilon-p_A\cos\alpha)e^{i\varphi/2}\sin\vartheta/2\},$$

$$M(\ell^-_{i,+1/2}\to\ell^-_{K,+1/2})=\sum_A \exp(ip_AL)/2\pi L\ \ U^+_{iA}U_{AK}\cdot$$
$$\cdot E^i_-E^K_-\{(\varepsilon-p_A\cos\alpha)e^{i\varphi/2}\cos\vartheta/2-p_Ae^{-i\varphi/2}\sin\alpha\sin\vartheta/2\},$$

$$M(\ell^-_{i,-1/2}\to\ell^+_{K,-1/2})=\sum_A \exp(ip_AL)/2\pi L\ U^T_{iA}U_{AK}m_AE^i_+E^K_-\cdot$$
$$e^{-i\varphi/2}\cos\vartheta/2,$$

$$M(\ell^-_{i,+1/2}\to\ell^+_{K,+1/2})=\sum_A \exp(ip_AL)/2\pi L\ U^T_{iA}U_{AK}m_AE^i_-E^K_+\cdot$$
$$e^{i\varphi/2}\cos\vartheta/2,$$

$$M(\ell^-_{i,+1/2}\to\ell^+_{K,-1/2})=\sum_A \exp(ip_AL)/2\pi L\ U^T_{iA}U_{AK}m_AE^i_-E^K_-\cdot$$
$$e^{i\varphi/2}\sin\vartheta/2.$$

It is easy to check our calculations assuming all masses equal to zero and α = 0. Then the cross section of $\ell^-_i\to\nu_i\to\ell^-_i$ process satisfy the equation

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_1(\vartheta_1 = 0)}{d\Omega_1} \cdot \frac{1}{L^2}\, \frac{d\sigma_1}{d\Omega} ;$$

where σ_1 is the cross section of $\ell_i^- \to \nu_i$ transition.

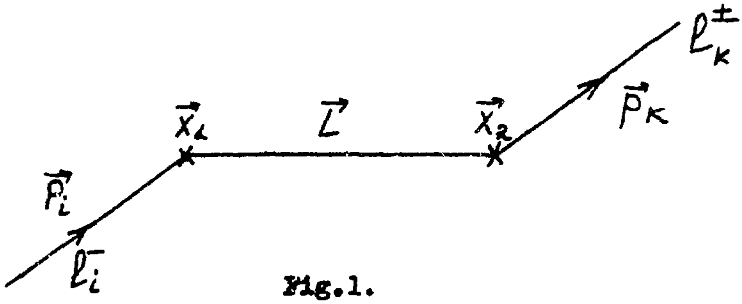

Fig.1.

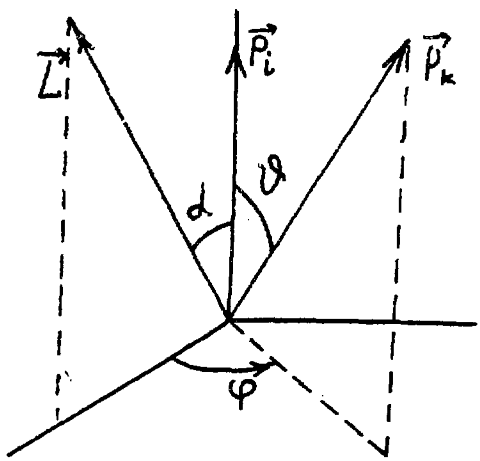

Fig.2.

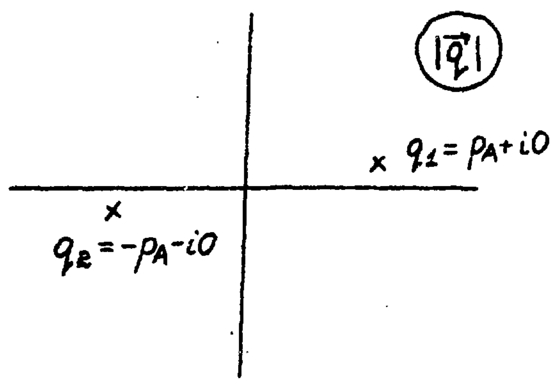

Fig.3.

R E F E R E N C E S

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