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THERMODYNAMIC ANALYSIS OF ELASTIC-PLASTIC DEFORMATION

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RÉSUMÉ

L'ensemble complet d'équations constitutives qui décrit complétement le comportement des matériaux aux conditiones de la déformation élasto-plastique, est obtenu sur la base d'analyse thermodynamique du proceś de déformation. L'analyse est effectuée apres que la decomposition matriculelle du graduent de déformation est introdulte dans la structure de la thermodynamique avec les variables internes. Nous décomposons la fonction de l'énergie libre, nous déduisons les expressions pour la contriante, l'entropie et le flux thermal, et nous établissons l'équation évolutive. Finalement, nous établissons les restrictions thermodynamiques du procés de déformation.

ABSTRACT

The complete set of constitutive equations which fully describes the behaviour of material in elastic-plastic deformation is derived on the basis of thermodynamic analysis of the deformation process. The analysis is done after the matrix decomposition of the deformation gradient is introduced into the structure of thermodynamics with internal state variables. ~~We decompose~~ The free energy function, derive the expressions for the stress response, entropy and heat flux, and establish the evolution equation. Finally, we establish the thermodynamic restrictions of the deformation process. (autor)

1. Introduction:

In this paper we use the thermodinamics with internal state variables developed in [1] and Lee's decomposition [2] to analyse the phenomenon of elastic-plastic deformation and the structure of the laws which describe that phenomenon.

We recall from [1] that the constitutive equations for the material with internal state variables can be postulated as:

$$\Psi = \Psi(F, \theta, A) \tag{1.1}$$

$$T = T(F, \theta, A) \tag{1.2}$$

$$\eta = \eta(F, \theta, A) \tag{1.3}$$

$$\underset{\sim}{q} = \underset{\sim}{q}(F, \theta, \underset{\sim}{g}, A) \tag{1.4}$$

$$\dot{A} = \hat{A}(F, \theta, A) \tag{1.5}$$

where Ψ is the Helmholtz free energy function per unit mass, F is the deformation gradient, θ is the absolute temperature, T is the Cauchy stress tensor, η is the specific entropy per unit mass and $\underset{\sim}{q}$ is the heat flux vector. The presence of $\underset{\sim}{g} = \operatorname{grad}\theta$ in (1.4) arises because there is no heat flow if there is no temperature gradient. By A we denote the set of internal state variables which in this paper can consist of several scalars and several second order tensors, i.e. $A = \{a, b, c, \ldots; \alpha_{ij}, \beta_{ij}, \gamma_{ij}, \ldots\}$ which describe the micro-structure of the material at a current instant and which account for the disclocation mechanisms which cause the plastic flow. The superimposed dot in (1.5) is the appropriate time derivative [4].

The forms of the constitutive equations are further restricted by the entropy principle and the principle of material objectivity [1], such that, instead of (1.1)-(1.5), we have:

$$\Psi = \Psi(C, \theta, A) \tag{1.6}$$

$$T = 2\rho F \frac{\partial \Psi}{\partial C} F^T \tag{1.7}$$

$$\eta = -\partial\Psi/\partial\theta \tag{1.8}$$

$$\underset{\sim}{q} = F\underset{\sim}{q}(C, \theta, F^T\underset{\sim}{g}, A) \tag{1.9}$$

$$\dot{A} = \hat{A}(C, \theta, A) \tag{1.10}$$

under the condition that

$$-\frac{1}{\rho}\,tr\left[\left(\frac{\partial\Psi}{\partial A}\right)^{T}\dot{A}\right]-\frac{1}{\rho\theta^{2}}\,\underset{\sim}{q}\cdot\underset{\sim}{g}\geqslant 0 \qquad (1.11)$$

In (1.6)-(1.11), ρ is the mass density and $C=F^{T}F$ is the right Cauchy-Green deformation tensor.

We now follow Lee [2] and introduce the intermidiate configuration by destressing the whole body from the current configuration and by reducing the temperature to the inital value. The intermidiate configuration comprises the pure plastic deformation, for thermal expansion and elastic strain components are both zero. One can then establish [2] at each point of the deformed body the decomposition

$$F=F_{e}F_{p} \qquad (1.12)$$

of the total deformation gradient F into its pure (thermo) elastic part F_e and pure plastic part F_p.

In the next section we investigate what consequences the decomposition (1.12) introduces into the structure of the laws (1.6)-(1.10).

2. Constitutive laws

First, we observe that the richt Cauchy-Green deformation tensors corresponding to the total and elastic deformation are connected through the relation

$$C=F_{p}^{T}C_{e}F_{p} \qquad (2.1)$$

This, one can show, leads to the indentity

$$F_{e}\frac{\partial\Psi}{\partial C_{e}}F_{e}^{T}=F\frac{\partial\Psi}{\partial C}F^{T} \qquad (2.2)$$

from which we see that the left hand side has exactly the form corresponding to pure elastic deformation F_e applied on the intermidiate configuration, providing that Ψ presents the Helmholtz function corresponding to that elastic defomation, [5]. Assuming that the previous plastic deformation doesn't significantly influence the elastic characteristics of the material [6] and in view of (2.2), we suggest, in place of (1.6), the following representation for the Helmholtz function for the elastic-plastic deformation

$$\Psi=\Psi_{e}(C_{e},\theta)+\Psi_{p}(\theta,F_{p},A) \qquad (2.3)$$

where Ψ_e coresponds to termo-elastic deformation and Ψ_p to pure plastic

deformation. The constitutive laws (1.7) and (1.8) consequently become:

$$T = 2\rho F_e \frac{\partial \Psi_e}{\partial C_e} F_e^T \qquad (2.4)$$

$$\eta = -\frac{\partial \Psi_e}{\partial \theta} - \frac{\partial \Psi_p}{\partial \theta} \qquad (2.5)$$

In this analysis we shall restrict ourselves to the initial isotropy of the material and the preservation of that isotropy in the course of deformation. The function Ψ_e is then an isotropic function of C_e and rinid body rotation of the intermidiate confiquration doesn't influence the stress response (2.4). With reqarde to the relation (2.5), the term $-\partial\Psi_e/\partial\theta$ expresses the elastic entropy carresponding to deformation F_e. Since part of that deformation is qenerated from the heat source produced by the dissipation of plastic work into the heat, the correspondinq entropy increase is included in $-\partial\Psi_p/\partial\theta$. The contribution to the total entropy associated with the dislocation structure and the plastic flow is repre-sented in (2.5) by the term $-\partial\Psi_p/\partial\theta$, [9]. In order that this is not influenced by the rigid body rotation of intermidiate confiquration, we must replace F_p by C_p in (2.3), i.e.

$$\Psi = \Psi_e(C_e, \theta) + \Psi_p(\theta, C_p, A) \qquad (2.6)$$

Let's now substitute the decomposition (1.12) into the constitutive equation (1.9). Since we reducethe temperature in the intermidiate confi-guration to the initial value, the heat flux $\underset{\sim}{q}$ is dependent only on elastic part F_e of the total deformation qradient F. We shall further assume that the thermal characteristics and the thermal response are not influe-nced by previous plastic flow. Assuminq also that the material is therm-ally isotropic as it was mechanically, and that it preserves its Istropy, we can replace the constitutive equation (1.9) with

$$\underset{\sim}{q} = F_e \underset{\sim}{\hat{q}}(C_e, \theta, F_e^T g) \qquad (2.7)$$

where $\underset{\sim}{\hat{q}}$ is an isotropic function of its arguments, i.e.

$$\underset{\sim}{\hat{q}}(QC_eQ^T, \theta, QF_e^T g) = Q\,\underset{\sim}{\hat{q}}(C_e, \theta, F_e^T g)\,. \qquad (2.8)$$

It is clear that (2.7) satisfies the principle of material objectivity. It also doens't depend on rotation of intermidiate (reference) confiqu-ration.

Finally, we consider the evolution equation (1.10). Since the cha-

nge of the internal structure of the material is induced by the micro-proceses which cause plastic deformation [7,8], there is no change in the value of A unless there is additional plastic deformation. Hence, we can postulate, in place of (1.10), the evolution eguation in the form

$$\dot{A} = \hat{A}\,(D_P,\ \theta,\ C_P,\ A) \tag{2.10}$$

where $D_p = \mathrm{sym}\,(\dot{F}_p F_p^{-1})$ is the plastic streching tensor [3]. In particular the function $\hat{A}$ in (2.10) has to be such that whenever D_p vanishes, $\dot{A}$ vanishes as well. Restricting further ourselves to the case of time-independent plasticity (i.e. no viscous effects), the law (2.10) has to be homogenous of degree one in $\dot{A}$ and D_p, which suggests for it the following expression

$$\dot{A} = \mathcal{A}\,(\theta, C_P, A) : D_P \tag{2.11}$$

or, more specifically (since $A = \{a, b, c, \ldots; \alpha_{ij}, \beta_{ij}, \gamma_{ij}, \ldots\}$), for the scalars, such as a,

$$\dot{a} = \mathcal{A}_{ij}\,(\theta, C_P, A)\, D^P_{ij} \tag{2.12}$$

and for the tensors, such as α_{ij},

$$\overset{\nabla}{\alpha}_{ij} = \mathcal{A}_{ijmn}\,(\theta, C_P, A)\, D^P_{mn} \tag{2.13}$$

where $(\overset{\nabla}{\ })$ is the appropriate corrotational derivative, [4].

3. Thermodynamic restrictions

Consider the energy equation, [10]

$$\dot{u} = \dot{w} - \tfrac{1}{\rho}\nabla \underset{\sim}{q} + r \tag{3.1}$$

where u is the specific internal energy per unit mass, r is the heat supply per unit mass and unit time, $-\frac{1}{\rho}\nabla \underset{\sim}{q}$ is the heat flux inflow per unit mass, and $\dot{w}$ is the stress power per unit mass

$$\dot{w} = \tfrac{1}{\rho}\,\mathrm{tr}\,(TD)\ , \tag{3.2}$$

D being the stretching tensor. In [2] it is shown that $\dot{w}$ can be splitted into the elastic an plastic part, such that:

$$\dot{w} = \dot{w}_e + \dot{w}_p \tag{3.3}$$

$$\dot{w}_e = \tfrac{1}{\rho}\,\mathrm{tr}\,[T(\dot{F}_e F_e^{-1})] \tag{3.4}$$

$$\dot{w}_p = \tfrac{1}{\rho}\,\mathrm{tr}\,(T D_p) \tag{3.5}$$

Therefore, (3.1) can be rewritten as

$$\dot{u} = \dot{w}_e + \dot{w}_p - \frac{1}{\rho}\nabla \underset{\sim}{q} + r \tag{3.6}$$

According to experiments [11], 90% of plastic stress power is dissipated into the heat. The rest is stored in the developing dislocation system. We may then consider that the energy dissipated during the plastic flow appears as a heat source in the elastic part of the deformation, and accordingly we separate (3.6) into two equations:

$$\dot{u}_e = \dot{w}_e + \gamma w_p - \frac{1}{\rho}\nabla \underset{\sim}{q} + r \tag{3.7}$$

$$\dot{u}_p = (1-\gamma)\dot{w}_p \tag{3.8}$$

where $\gamma = 0.9$. (In fact, γ is increasing from 0.9. to 1.0 with advancing of plastic flow).

Consider now the Clausius - Duhem inequality [10]

$$\dot{\eta} \geq -\frac{1}{\rho}\, div \frac{\underset{\sim}{q}}{\theta} + \frac{r}{\theta} \tag{3.9}$$

We recall that this inequality was already explored in arriving at the laws (1.6) - (1.10), under the condition (1.11). Nevertheless, we can begin once again from its initial from (3.9), by rewriting it as

$$\dot{\eta} \geq \frac{r+\gamma\dot{w}_p}{\theta} - \frac{1}{\rho}\, div \frac{\underset{\sim}{q}}{\theta} - \frac{\gamma\dot{w}_p}{\theta} \tag{3.10}$$

This we suggest to be splitted into two relations:

$$\dot{\eta}_e = \frac{r+\gamma\dot{w}_p}{\theta} - \frac{1}{\rho}\, div \frac{\underset{\sim}{q}}{\theta} \tag{3.11}$$

$$\dot{\eta}_p \geq -\frac{\gamma\dot{w}_p}{\theta} \tag{3.12}$$

Now, since $\psi_e = u_e - \theta\eta_e$, by simple operations, we obtain

$$\frac{\partial \psi_e}{\partial C_e} : \dot{C}_e = \dot{w}_e - \frac{1}{\rho}\nabla \underset{\sim}{q} + \frac{1}{\rho}\theta\, div \frac{\underset{\sim}{q}}{\theta} \tag{3.13}$$

and since first two terms in (3.13) cancel each other, this reduces to

$$\underset{\sim}{q} \cdot \underset{\sim}{g} = 0 \tag{3.14}$$

which is the classical condition for reversible processes, [5].

Further, since $\psi_p = u_p - \theta\eta_p$, we obtain by similar procedure as above,

$$\frac{1}{\rho}\, tr\,(T D_p) \geq \frac{\partial \psi_p}{\partial A} : \dot{A} \tag{3.15}$$

We also recall that from (1.11) we now have

$$\frac{\partial \Psi}{\partial A} : \dot{A} \leq 0 \qquad (3.16)$$

which corresponds to the internal dissipation inequality.

The relations (3.14),(3.15) and (3.16) are the thermodynamic restrictions imposed on the elastic-plastic deformation process.

4. Conclusions

The derived set of constitutive equations presents a basis for analysis of the behaviour of a material in elastic-plastic deformation. In particular, we see that the elastic part of deformation is governed by the classical finite-elasticity law, whereas the plastic part of deformation is of the rate (incremental) nature. Indeed, in the final formulation of the theory, one has to construct the law for the plastic streching and then to appropriately combine it with elastic law. This has been done in [3,4].

We note that in this analysis of time-independet material, the only restiriction imposed in the development of the theory is the elastic isotropy of material and preservation of that isotropy during the course of plastic deformation.

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