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ANAIS PROCEEDINGS

COBEM 81

VI CONGRESSO BRASILEIRO DE ENGENHARIA MECANICA

RIO DE JANEIRO, 15 – 18 de dezembro de 1981

TRABALHO PAPER N.º A-23 P.P. 233 - 244 PUC/RJ

THERMAL RESISTANCE OF A CONVECTIVELY COOLED PLATE WITH APPLIED HEAT FLUX AND VARIABLE INTERNAL HEAT GENERATION

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SUMÁRIO

Analisa-se aqui o problema de condução de calor em uma placa plana submetida a uma geração interna de calor não-uniforme, com uma de suas bordas resfriadas por convecção, enquanto parte da borda oposta a esta é submetida a um fluxo de calor. O restante desta, assim como as outras duas bordas, estão perfeitamente isoladas. O problema é resolvido pelo método de diferenças finitas e os resultados são analisados em função do módulo de Biot, parâmetros geométricos da placa, a relação entre geração interna e fluxo de calor e o perfil da distribuição da geração interna. (autor).

SUMMARY

The conductive heat transfer in a rectangular plate with nonuniform internal heat generation, with one end convectively cooled and a part of the opposite end subjected to external heat flux is considered. The remaining part of this end as well as the other two sides are thermally insulated. The governing differential equation is solved by a finite difference scheme. The variation of the thermal resistance with Biot modulus, the plate geometry, the internal heat generation parameter and the type of profile of internal heat generation is discussed. (author).

1. Introduction

During the thermal design and analysis of satellites, when the object is to keep the temperature of the various electronic components below the maximum operating temperature, a knowledge of the equivalent thermal resistance of the mounting plates of electronic banks is essential. Sometimes, the mounting plates may be convectively cooled by a fluid. Here we analyze the problem of a thin rectangular plate, with a portion of one side in contact with an external wall, receiving a specific flux, the opposite side being convectively cooled by a fluid at a constant temperature. The remaining sides are insulated. It is also assumed that heat dissipation of the electronic components can be represented as a nonuniform internal heat generation. The geometry is shown in figure 1.

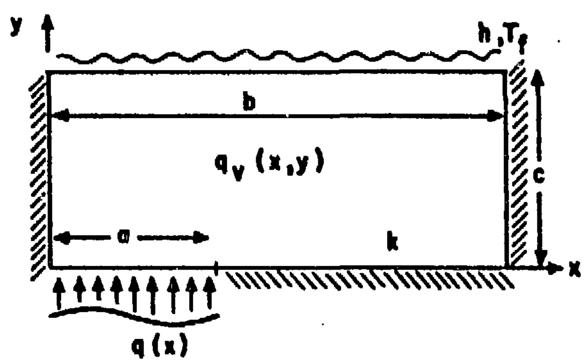

Fig. 1. The geometric configuration

Oliveira and Forslund [1], considered a problem of similar geometry with constant external heat flux and with no internal heat generation. Schneider, Yovanovich and Cane [2], extended the same problem to the case of nonuniform heat flux, but with no internal heat generation. They considered three different profiles for the external heat flux and concluded that the nature of this profile has no influence on the thermal resistance. Venkataraman, Oliveira Filho and Cardoso [3] extended this problem to include constant internal heat generation and obtained the solution analytically. In this case also, the thermal resistance was found to be insensitive to the nature of the external heat flux profile. The object of this work is to extend the work of reference [3] to include nonuniform internal heat generation. The governing differential equation is solved by a finite difference scheme. The accuracy check is made by comparing with the exact solution for the case of constant heat generation [3]. Following this, three different types of profiles are assumed for the internal heat generation and their effects on thermal resistance are studied. The last profile assumed, simulates closely the situation arising in a satellite. Finally, the results for thermal resistance are presented graphically as functions of Biot Modulus, plate geometry, relation of internal heat generation to external heat flux and the nature of the profile for the internal heat generation.

2. Formulation

As shown in figure 1, the rectangular plate of width b, height c and thermal conductivity k is subjected to external heat flux per unit time q(x) over the contact width a. The remaining portion of this side as well as the lateral sides x = 0 and x = b are insulated. The topside y = c is cooled convectively by a fluid at constant temperature T_f and with a constant film coefficient of heat transfer h. The nonuniform internal heat generation per unit volume, per unit time in denoted by $q_v(x, y)$. The steady state temperature T(x, y) inside the plate is governed by the two-dimensional Poisson equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{-q_v(x, y)}{k} \qquad (1)$$

The boundary conditions are

$$y = 0, \quad 0 \leq x \leq a, \quad \frac{\partial T}{\partial y} = \frac{-q(x)}{k} \qquad (2)$$

$$a \leq x \leq b, \quad \frac{\partial T}{\partial y} = 0 \qquad (3)$$

$$y = c, \quad 0 \leq x \leq b, \quad \frac{\partial T}{\partial y} = -\frac{h'}{k}\left[T(x, c) - T_f\right] \qquad (4)$$

$$x = 0, \quad 0 \le y \le c, \quad \frac{\partial T}{\partial x} = 0 \tag{5}$$

$$x = b, \quad 0 \le y \le c, \quad \frac{\partial T}{\partial x} = 0 \tag{6}$$

Following [2], we nondimensionalize equation (2) using the following nondimensional variables:

$$\beta = \frac{x}{b}, \quad \eta = \frac{y}{b}, \quad \theta = \frac{k \ell (T - T_f)}{Q + Q_v} \tag{7}$$

Where ℓ is the thickness of the plate normal to the cross--section, Q is the total heat flow rate and Q_v is the total internal heat generation rate. Thus

$$Q = \ell \int_0^a q(x)\, dx, \quad Q_v = \ell \int_0^c \int_0^b q_v(x, y)\, dx\, dy \tag{8}$$

Using these nondimensional variables equations (1) to (6) become

$$\frac{\partial^2 \theta}{\partial \beta^2} + \frac{\partial^2 \theta}{\partial \eta^2} = \frac{- q_v(\beta, \eta)\, b^2 \ell}{Q + Q_v} \tag{9}$$

with the boundary conditions:

$$\eta = 0, \quad 0 \le \beta \le \varepsilon, \quad \frac{\partial \theta}{\partial \eta} = \frac{- q(\beta)\, b\, \ell}{Q + Q_v} \tag{10}$$

$$\varepsilon \le \beta \le 1, \quad \frac{\partial \theta}{\partial \eta} = 0 \tag{11}$$

$$\eta = \alpha, \quad 0 \le \beta \le 1, \quad \frac{\partial \theta}{\partial \eta} = - Bi\, \theta(\beta, \alpha) \tag{12}$$

$$\beta = 0, \quad 0 \le \eta \le \alpha, \quad \frac{\partial \theta}{\partial \beta} = 0 \tag{13}$$

$$\beta = 1, \quad 0 \le \eta \le \alpha, \quad \frac{\partial \theta}{\partial \beta} = 0 \tag{14}$$

Where $\varepsilon = \frac{a}{b}$, $\alpha = \frac{c}{b}$ and $Bi = \frac{hb}{k}$ is the Biot modulus. When the internal generation $q_v(\beta, \eta)$ is not a constant, the solution of equation (9) has to be obtained numerically. At this stage it is necessary to specify $q(\beta)$ and $q_v(\beta, \eta)$. As mentioned before, the nature of the profile of $q(\beta)$ has no influence on the thermal resistance [2, 3] and hence we take it as equal to a constant q. For $q_v(\beta, \eta)$ we consider the following three profiles:

Profile A: $q_v(\beta, \eta) = q_{v_A} \beta\eta(1 - \beta)(\alpha - \eta)$ (15)

Profile B: $q_v(\beta, \eta) = q_{v_B}(1 - \beta) e^{-\eta}$ (16)

Profile C: $q_v(\beta, \eta) = q_{v_C}$ for the region $\beta_1 \leq \beta \leq \beta_2$ and $\eta_1 \leq \eta \leq \eta_2$,
$= 0$ outside this region (17)

The last profile here closely simulates the conditions arising in satellite thermal analysis, and the zone of heat generation is taken approximately near the center. We define an internal heat generation parameter G as

$$G = \frac{\bar{q}_v\, b}{q} \tag{18}$$

Where $q_v = \dfrac{Q_v}{b\, c\, \ell}$ is the average internal heat generation rate per unit volume.

The differential equation (9) can be written in terms of the parameter G for each of the profiles A, B and C. Thus for profile A, equation (9) becomes

$$\frac{\partial^2 \theta}{\partial \beta^2} + \frac{\partial^2 \theta}{\partial \eta^2} = \frac{-\alpha G}{\varepsilon + \alpha G} \; \frac{\beta\eta(1 - \beta)(\alpha - \beta)}{g_A(\alpha)} \tag{19}$$

$$\text{where } g_A(\alpha) = \int_0^1 \int_0^\alpha \beta\eta(1 - \beta)(\alpha - \beta)\, d\beta\, d\eta \tag{20}$$

and equation (10) becomes

$$\eta = 0, \quad 0 \leq \beta \leq \varepsilon, \quad \frac{\partial \theta}{\partial \eta} = -\frac{1}{\varepsilon + \alpha G} \tag{21}$$

All other boundary conditions remain the same. Similar expressions can be written for the profiles B and C. The nondimensional thermal resistance R of the plate is defined as

$$R = \frac{k\, \ell(\bar{T}_c - T_f)}{Q + Q_v} \tag{22}$$

where $\bar{T}_c$ is the average contact temperature of the lower plate given by

$$\bar{T}_c = \frac{1}{\varepsilon} \int_0^\varepsilon T(\beta, 0)\, d\beta \tag{23}$$

From equations (7), (22) and (23), the nondimensional resistance turns out to be

$$R = \frac{1}{\varepsilon}\int_0^{\varepsilon} \theta(\beta,\ 0)\, d\beta \tag{24}$$

Thus the nondimensional thermal resistance is equal to the nondimensional average contact temperature.

3. Numerical solution

For the finite difference solution, by discretizing the plate as shown in figure 2, we obtain the following difference equations for profile A.

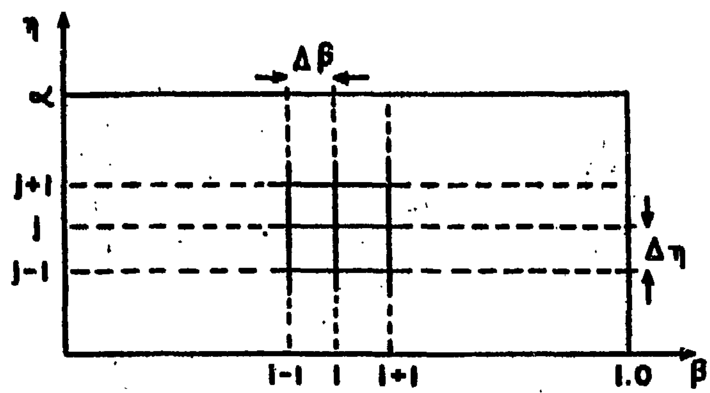

Fig. 2. Discretization of plate

$$\theta_{i,j} = \frac{\dfrac{\theta_{i+1,j}+\theta_{i-1,j}}{(\Delta\beta)^2} + \dfrac{\theta_{i,j+1}+\theta_{i,j-1}}{(\Delta\eta)^2} + \dfrac{\alpha G}{\varepsilon+\alpha G}\,\dfrac{[\beta\eta(1-\beta)(\alpha\ \beta)]_{i,j}}{g_A(\alpha)}}{\dfrac{2}{(\Delta\beta)^2}+\dfrac{2}{(\Delta\eta)^2}} \tag{25}$$

$$\eta = 0,\quad 0 \le \beta \le \varepsilon,\quad \theta_{i,0} = \frac{\Delta\eta}{\varepsilon+\alpha G} \tag{26}$$

$$\varepsilon \le \beta \le 1,\quad \theta_{i,0} = \theta_{i,1} \tag{27}$$

$$\eta = \alpha,\quad 0 \le \beta \le 1,\quad \theta_{i,n_\eta+1} = \theta_{i,n_\eta}(1 - Bi\,\Delta\eta) \tag{28}$$

$$\beta = 0,\quad 0 \le \eta \le \alpha,\quad \theta_{0,j} = \theta_{1,j} \tag{29}$$

$$\beta = 1,\quad 0 \le \eta \le \alpha,\quad \theta_{n_\beta+1,j} = \theta_{n_\beta,j} \tag{30}$$

Where n_β and n_η are the number of divisions corresponding to i and j respectively. This system of linear algebraic equations are solved by the method of residues by starting with the initial guess of

$$\theta_{i,j} = - \frac{G}{\varepsilon + \alpha G} \frac{\eta^2_{i,j}}{2} - \frac{\eta_{i,j}}{1 + \alpha G} + \frac{\alpha G}{\varepsilon + \alpha G} \left(\frac{1}{Bi} + \frac{\alpha}{2}\right) + \frac{1}{1 + \alpha G} \left(\alpha + \frac{1}{Bi}\right) \qquad (31)$$

which represents the temperature distribution for the one dimensional case in the η direction, such that

$$\lim_{\substack{a \to 0 \\ q \to \infty}} \frac{q\, a\, \ell}{Q} = 1 \quad \text{and} \quad \lim_{\substack{b \to 0 \\ q_v \to \infty}} \frac{q_v\, b\, c\, \ell}{Q} = 1$$

4. Results and discussions

For the calculation of the thermal resistance and the temperature, a maximum residual of 10^{-3} was used, after testing that with this residue, the maximum error turns out to be 5% when compared with the analytical solution of [3], for the case of constant heat generation. The maximum error occurs for $\alpha = 0.2$ and $G = 1.0$, (figure 3), for most of the other cases the error is around 0.1%

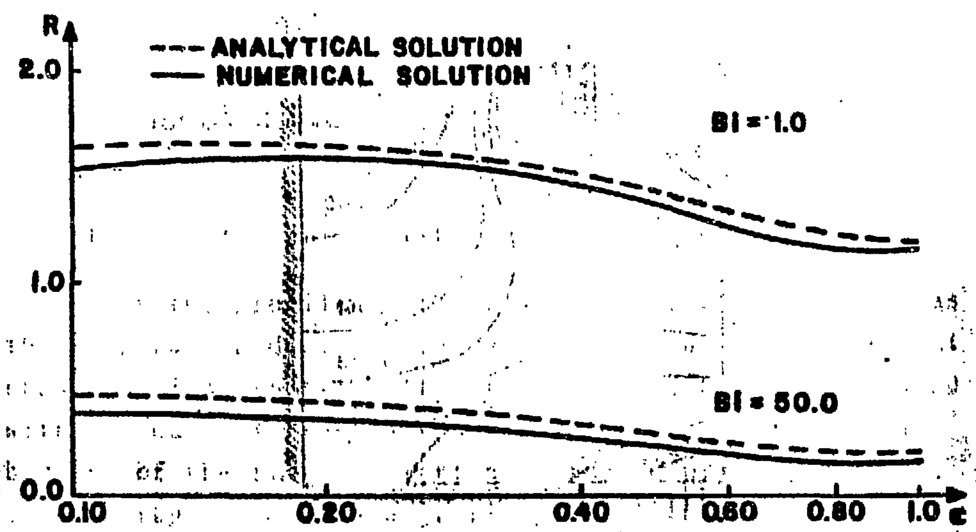

Fig. 3. Comparison between numerical and analytical solution for $\alpha = 0.2$, $G = 1.0$.

The isotherms are plotted for the case of the profile C in figures 4 and 5, where the are of the heat generation and its location is shown to scale. The thermal resistance for the profiles A, B and C are plotted against ε for G = 0.01, 1.0 and 50.0 for various α and Bi in figures 6, 7 and 8.

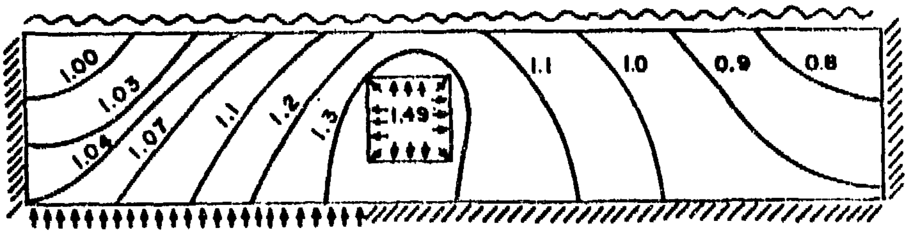

Fig. 4. Isotherms for profile C for α = 0.2, Bi = 1.0, G = 50.0

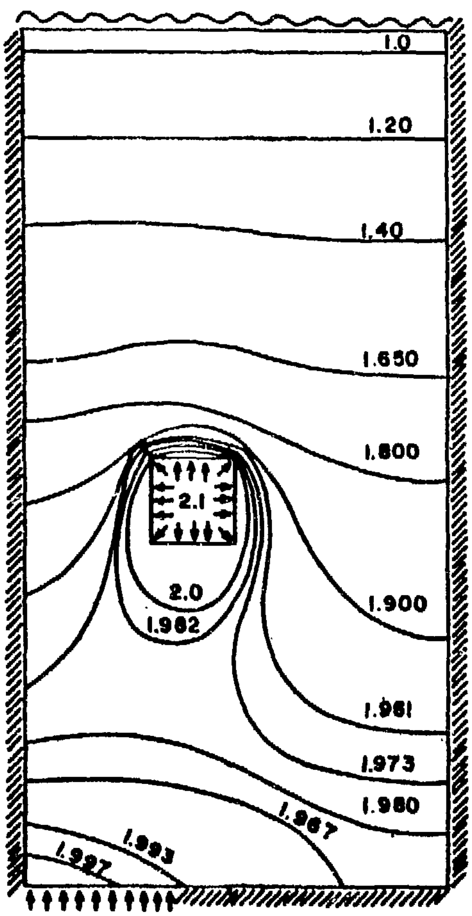

Fig. 5. Isotherms for profile C for α = 0.2, Bi = 1.0, G = 50.0

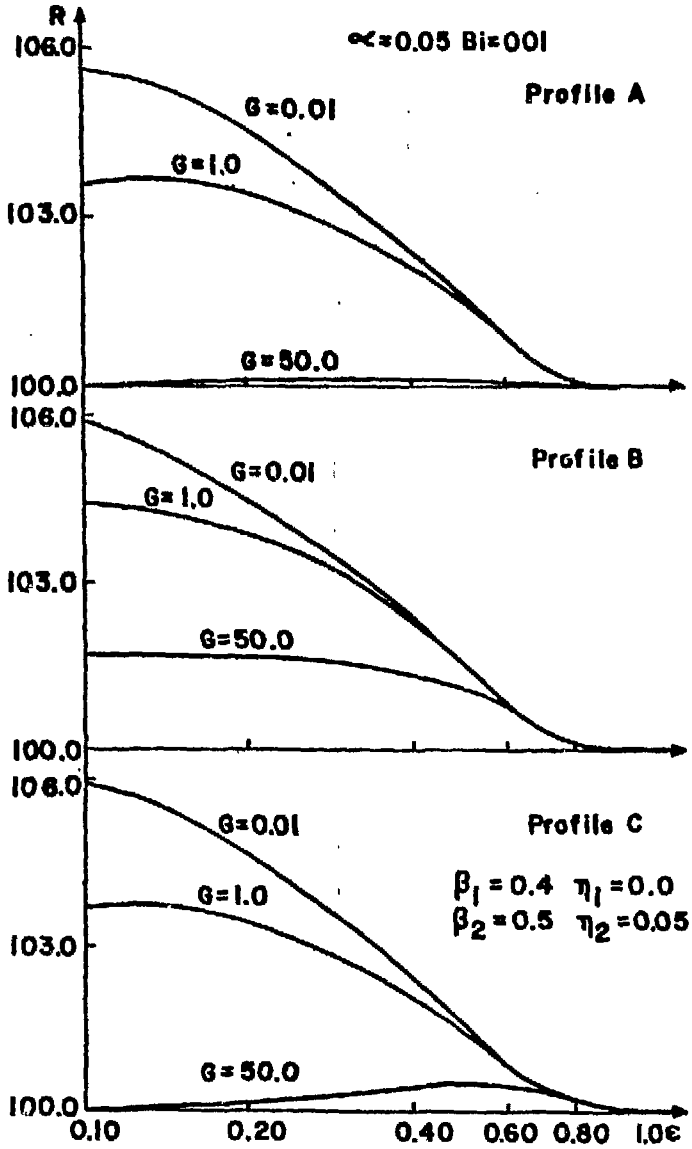

Fig. 6. Thermal resistance for α = 0.05, Bi = 0.01

For the profile A, the thermal resistance as well as the temperature distribution does not show any significant change in its behaviour (figures 6, 7 and 8) when compared with the case of constant heat generation [3]. This is because of the small variation of the q_v profile at the contact region. For the profile B, the thermal resistance values are higher than those for constant generation [3]. This is because of the concentration of the internal heat generation near the contact edge, thereby causing a higher temperature and hence the increased resistance. For the profile C, the thermal resistance increase becomes significant for large values G, and values of ε and α for which the zone of heat generation is near the contact surface. This can be visualized by the isotherms for α = 0.2 and 2.0 (figures 4 and 5). For α = 2.0, the region of heat generation is relatively far from the contact region and

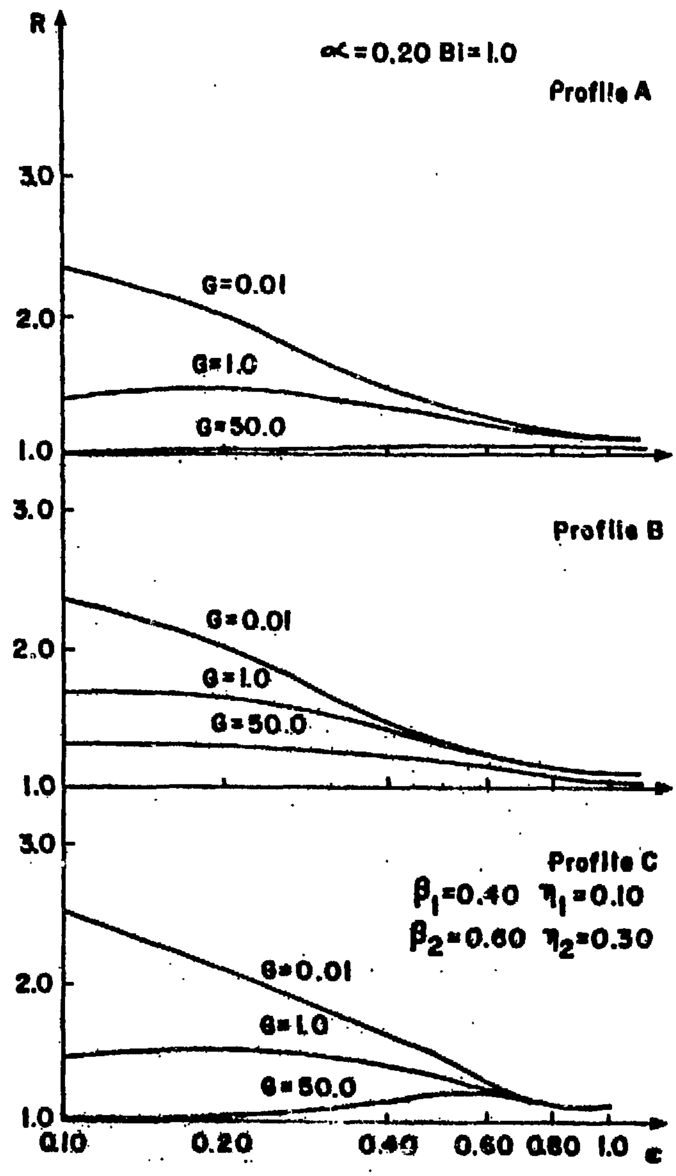

Fig. 7. Thermal resistance for α = 0.2, Bi = 1.0

hence for moderate values of G, the contact surface temperature remains approximately constant. Thus the thermal resistance remains constant with ε. For α = 0.2, the region of heat generation (0.4 ≤ $\dot{\beta}$ ≤ 0.5) is close to the contact surface. Thus for G = 50 there is a considerable increase in resistance for ε in the range of 0.4 to 0.6.

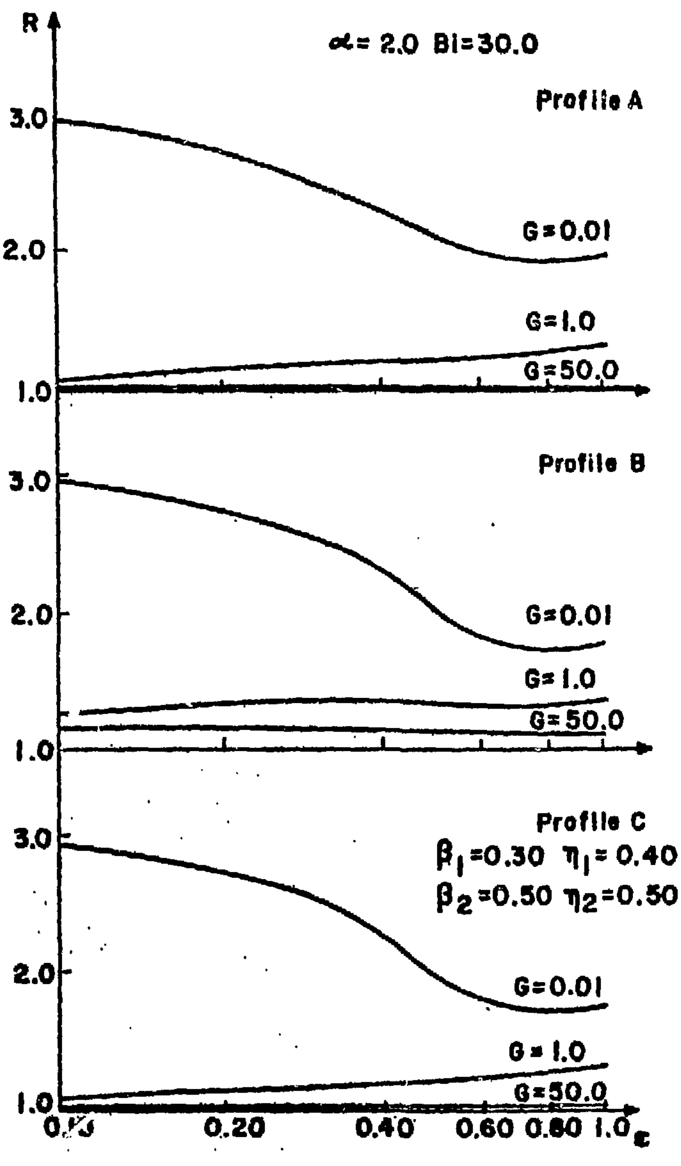

Fig. 8. Thermal resistance for α = 2.0, Bi = 30.0

Thus the behaviour of the thermal resistance is directly dependant on the type of profile used, thus necessitating specific analysis for each case. However for the three cases analyzed here, the influence of some of the parameters is similar to the case of constant heat generation:

- The resistance decreases with increase of G.
- The maximum value of resistance occurs for minimum value of α, ε, G and Bi.
- For small values of α (of the order of 10^{-2}) and whatever value of G, the thermal resistance tends to the value $\alpha + \frac{1}{Bi}$ when ε tends to 1.

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