Suppression of Magnetic Islands by RF-Driven Currents

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Abstract

The quasilinear theory for the saturation of nonlinear tearing modes is modified to include rf driven currents. It is shown that the presence of lower hybrid driven currents can strongly suppress the growth of magnetic islands.

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It is now generally believed that tokamak disruptions are caused by the nonlinear growth of magnetic islands. In this paper we show that the growth of such islands can be suppressed by the presence of rf driven currents. If some portion of the equilibrium current in a tokamak is driven by rf waves, the current deposition profile due to these waves may be strongly affected by the presence of a magnetic island. The theory for the nonlinear regime of the tearing mode must be altered to take into account the resulting perturbation of the electric field. In the linear regime, currents due to the mode are much more sharply peaked near the singular layer, and the effect considered in this paper may be neglected.

We work in a cylinder, with the magnetic field given in term of a helical flux function ψ,

$$\underset{\sim}{B} = \underset{\sim}{\nabla}\psi \times \hat{z} + B_z\left(\hat{z} - \hat{\theta}\,\frac{r}{q(r_s)R}\right) \quad , \tag{1}$$

where $q = q(r_s)$ at the rational surface and R is the major radius. The rf current drive comes in through Ohm's law because of the fact that the rf current has no associated electric field. We have

$$\underset{\sim}{J} = \left(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}\right)/\eta + \underset{\sim}{J}_{rf} \quad , \tag{2}$$

where $\underset{\sim}{J}$ is the total current and $\underset{\sim}{J}_{rf}$ is the current supported by rf waves.

Our treatment of the nonlinear tearing mode follows that of White et al., with the ordering $B_z \gg B_\theta$ assumed.[1] It follows from Faraday's law that

$$\left\langle\frac{\partial\psi}{\partial t}\right\rangle = \langle\eta(j - j_{rf})\rangle + E_w \quad , \tag{3}$$

where $\langle\ \rangle$ denotes a flux surface average, j is the z component of the current, and E_w is the imposed electric field at the plasma edge. (We have imposed the boundary condition $\psi = 0$ at the plasma edge.) Inertia may be neglected in the nonlinear regime, giving

$$\nabla^2\psi = -j(\psi) - \frac{2B_{zo}}{Rq(r_s)} \quad . \tag{4}$$

Expand $\psi = \psi_o(r) + \varepsilon\ \psi_1(r)\cos(kz + m\theta)$. The perturbation gives an island of width

$$w = 4\{\psi_1(r_s)\ \varepsilon/\psi_o''(r_s)\}^{1/2} \quad , \tag{5}$$

where r_s is the location of the rational surface. The growth of the island may be determined by evaluating Eq. (3) at the x-point and o-point and subtracting,

$$\frac{dw}{dt} = -\frac{4\eta(r_s)}{w\psi_o''(r_s)}\left[j(r_x) - j(r_o) + j_{rf}(r_o) - j_{rf}(r_x)\right] \quad , \tag{6}$$

where r_o and r_x are the locations of the o-point and x-point respectively. To solve for the current density, Taylor expand $j(\psi)$ in the interior of the island to first order in ψ, substitute into Eq. (4), and match to the exterior solution. The result may be expressed as

$$j(r_x) - j(r_o) = .41\ \psi_o''\ \Delta'\ w(1 - w/w_m) \quad , \tag{7}$$

where Δ', as usual, denotes the jump in the logarithmic derivative of the exterior solution for ψ at the singular surface, and where w_m, the saturation width of the island in the absence of rf currents, is a complicated expression which depends on $\psi_o(r)$ and the equilibrium current density.[1] We may take the equilibrium contribution to $j_{rf}(r_o) - j_{rf}(r_x)$ to be zero because the island is rotating rapidly, on the time scale of its growth, relative to the fixed external waveguide or antenna. It is, however, necessary to self-consistently include the perturbation in j_{rf} due to the growth of the magnetic island.

We see from Eq. (6) that the growth of the magnetic island may be either enhanced or suppressed, depending on the direction of the perturbed rf current. The perturbation of j_{rf} will depend on what kind of wave is used to drive the current. The attractiveness of a given scheme for maintaining a steady-state reactor may be strongly affected by whether magnetic island growth is enhanced or suppressed.

Equation (6) suggests that we consider the use of rf currents to suppress the growth of magnetic islands, independently of the question whether a reactor can be economically maintained in steady state. We are primarily interested in suppressing the growth of the $n = 1$, $m = 2$ island. Since the $q = 2$ surface lies in the outer region of the tokamak plasma, we need only drive rf currents in this region, where the equilibrium current density is relatively low. This makes the efficiency of the rf current drive less of an issue. Also, we do not require the current drive to work at all in the higher density inner region of the plasma.

We calculate $j_{rf}(r_o) - j_{rf}(r_x)$ for an example of present experimental interest, that of currents driven by lower hybrid waves. The current deposition is affected both directly by the magnetic field perturbations (the group velocity trajectories being closely aligned with the magnetic field), and also by the accompanying temperature perturbation. Because the magnitude of the driven current is very sensitive to the local electron temperature, the effect of the temperature perturbation dominates.

The current driven by unidirectional lower hybrid waves has been calculated from a one-dimensional averaged quasi-linear theory,[2] and the result verified on a two-dimensional Fokker-Planck code.[3] The strongest dependence on temperature comes in through an $\exp(-v_{p1}^2/v_{te}^2)$ factor, where the quasilinear diffusion coefficient is taken to be large for phase velocities between v_{p1} and v_{p2}, and zero for phase velocities less than v_{p1} or greater than v_{p2}. For small temperature perturbations this gives

$$\delta j/j = (v_{p1}^2/v_{te}^2)\ \delta T_e/T_e \quad . \tag{8}$$

The exponential factor in the expression for j decreases rapidly for $v_{p1} \gg v_{te}$, and we cannot expect to drive substantial currents for v_{p1}/v_{te} much greater than four.

The island growth described by Eqs. (6) and (7) occurs on a flux diffusion time scale. Because the ohmic heating time is much smaller than the flux diffusion time (by a factor of about 8) we can regard the temperature as being in local equilibrium as the island grows. The heating will in fact occur at the much faster rate due to dissipation of rf waves if j_{rf} is comparable to the equilibrium current density. Inside the island, the temperature is constant on each flux surface because of the large parallel thermal conductivity. In the absence of strong impurity radiation, there is no energy sink in the island, so heat must flow outward, with the temperature higher at the o-point than on the separatrix. The magnitude of this temperature difference is inversely proportional to the local cross-field diffusion coefficient, which is strongly anomalous. We estimate $\chi_\perp \approx \eta_{eff}\, j_o^2\, a^2/T_{eo}$, where "a" is the minor radius of the plasma, where the subscript "o" refers to quantities evaluated at the magnetic axis, and where η_{eff} is an effective resistivity which includes the effect of rf wave dissipation. Further estimating the temperature difference between o-point and separatrix to be $\delta T \approx \eta_{eff}\, j^2 w^2/\chi_\perp$, we get an expression for δT_e in terms of the equilibrium quantities and η_{eff}. The effect is most prominent when the rf driven current is small enough that its contribution to the overall energy balance of the tokamak is negligible, but is comparable to the equilibrium current near the singular layer. We then find

$$\delta T_e/T_e \approx (\eta_{rf}/\eta)(j/j_o)^{4/3}(w^2/a^2) \quad , \tag{9}$$

where η_{rf} is the effective resistivity of the rf current. This temperature fluctuation could be further enhanced by a local thermal instability, which we will not consider in this paper. Note also that the effect would be enhanced by the use of a less efficient rf driver, so long as the efficiency is not so poor as to affect the overall economics. In a tokamak where some form of supplementary heating is used for the bulk of the plasma (including, possibly, heating due to some current drive mechanism other than that used near the singular surface), or where the plasma is heated due to fusion reactions, Eq. (9) would have an additional factor less than one to take this into account.

The effective rf resistivity is larger than the parallel ohmic resistivity by a factor of roughly v_{te}/v_d, where $v_d = j/ne$. For an ohmic discharge in PLT with $n \approx 10^{14}$ cm^{-3}, $I \approx 500$ kA, and $T_e \approx 1$ kev, v_{te}/v_d is of order 10^2. This ratio scales in general like $(\beta_p\, n)^{1/2}$ a.

The direction of the perturbed lower hybrid current is such that magnetic island growth is suppressed when the equilibrium rf current is in the same direction as the equilibrium ohmic current, and is enhanced when these currents are opposed. In the former case we can combine Eqs. (6) - (9) to find a saturated island width of

$$w = w_m/(1 + \alpha) \quad , \tag{10a}$$

where

$$\alpha = \frac{40}{\Delta' a} \frac{w_m}{a} \frac{\eta_{rf}}{\eta} \frac{j_s}{\psi_o''} \left(\frac{j_s}{j_o}\right)^{4/3} \quad , \tag{10b}$$

and where j_s is the equilibrium current at the singular layer. We evaluate α for a particular equilibrium whose nonlinear evolution was extensively studied by Waddell et al.,[4]

$$q = q_o[1 + (r/r_o)^8]^{1/4} \quad , \tag{11}$$

with $q_o = 1.37$ and $r_o = .6a$. The equilibrium was chosen to give a good description of the PLT temperature profile prior to a major disruption. Substituting into Eq. (10b), we find $\alpha \approx 5$ for the $m = 2$, $n = 1$ island and $\alpha \approx 2 \times 10^2$ for the $m = 3$, $n = 2$ island. The saturated island widths in the presence of the rf are roughly .06a and 4×10^{-4} a respectively. These are well below the widths at which destabilization due to nonlinear coupling is observed in Ref. 4. The stabilization could be further enhanced by raising the rf driven current density at the $q = 2$ rational surface above the ohmic current density given by Eq. (11).

We conclude that lower hybrid wave driven currents could be effectively used to suppress the growth of magnetic islands and prevent disruptions. This would entail the production of waves with relatively low phase velocities, which are electron Landau damped in the outer region of the plasma. The analysis suggests that rf current drive could also have important effects on sawtooth oscillations and on nonlinear coupling of tearing modes. These effects are outside the scope of this paper. An understanding of the MHD effects of rf current drive can potentially expand our ability to control the evolution of tokamak plasmas. It will also be important in interpreting the results of current drive experiments.

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