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INITIAL-STATE INTERACTIONS, FACTORIZATION, AND THE DRELL-YAN PROCESS*

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ABSTRACT

We show that initial state interactions violate the factorization conjecture for the Drell-Yan process order by order in perturbation theory. We also discuss the effects of elastic and inelastic initial state interactions on the observed cross sections.

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I. Introduction

Factorization theorems play a central role in the analysis of many hadronic processes in that they allow one to write observable quantities as the convolution of a non-perturbative, process-independent piece with a perturbative, process-specific piece.[1] For the case of the Drell-Yan process,[2] proofs of the factorization conjecture remain incomplete. In this paper, we show that initial state interactions in the Drell-Yan process violate factorization order by order in perturbation theory, and we discuss the effects of such interactions on the observed cross sections.

Figure 1 shows the basic Drell-Yan process for anti-baryon-baryon collisions: a quark from one baryon annihilates with an anti-quark from the anti-baryon to produce a time-like virtual photon, which in turn produces a lepton pair of invariant mass-squared Q^2. QCD factorization is the statement that, at large Q^2, the cross section $d\sigma/dQ^2$ is given, up to terms of $O(1/Q^2)$, by the convolution of the absolute square of a hard process (simple Feynman diagram) with evolved (scale-breaking) structure functions. This statement is depicted in Fig. 2 for the basic process and some $O(\alpha_s)$ radiative corrections. The dashed vertical lines cut through the final state (with conversion of the massive photon into a pair understood); the inner Feynman diagram is the square of the hard process; and the hadronic wave function "blobs" squared make up the structure functions. For example, the basic process gives a contribution

$$\frac{d\sigma}{dQ^2} = \sum_q \frac{4\pi\alpha^2}{3} \frac{e_q^2}{n_c} \frac{1}{Q^4} \int dx_1 dx_2 \delta(x_1 x_2 - \frac{Q^2}{s}) [q(x_1,Q^2)\bar{q}(x_2,Q^2) + (1 \leftrightarrow 2)], \quad (1)$$

where $q(x_1,Q^2)$ and $\bar{q}(x_2,Q^2)$ are the quark and anti-quark structure functions respectively.

Factorization theorems, as they are usually stated, also relate various hadronic processes to one another (universality). In particular, they state that the structure functions $q(x,Q^2)$, $\bar{q}(x,Q^2)$, $g(x,Q^2)$ (gluon) that appear in the Drell-Yan formula are the <u>same</u> structure functions as those measured in deep-inelastic scattering. For deep-inelastic scattering on a nuclear target, the virtual photon interacts with a given charged constituent with a strength that is independent of the location of the constituent within the nucleus. Thus, we are led immediately to the statement of nucleon number additivity for structure functions:

$$\left(q(x,Q^2)\right)_A = A\left(q(x,Q^2)\right)_N, \text{ etc. (away from } x=1) \ . \qquad (2)$$

Here A is the nucleon number and the subscripts A and N denote the nucleus and nucleon, respectively.

Nucleon number additivity, in the context of factorization, has important consequences for the Drell-Yan process. It implies that quarks on the back face of a nuclear target are just as likely as quarks on the front face to annihilate with a projectile anti-quark. That is, factorization seems to imply that nuclear matter, at least for the Drell-Yan process, is infinitely penetrable.

On the other hand, we know that the projectile interacts strongly with the matter in the target. Total cross sections for hadrons on a nuclear target go like $A^{2/3}$, which indicates that the projectile does not penetrate much past the nuclear surface before an interaction occurs. In exclusive channels, multiple scattering (nuclear enhancement) appears to be important. Even in experiments that measure the Drell-Yan cross section, one must allow for the production of secondary hadrons and depletion of the beam as it passes through a macroscopic length of target. In short, a projectile's wave function must be profoundly disturbed as the projectile passes through the target.

II. Initial State Interactions

In order to resolve this apparent conflict between strong-interaction phenomenology and QCD factorization, we investigate the interaction between target and projectile constituents (initial state interactions) in QCD perturbation theory. At first sight, it might appear that perturbation theory is an inappropriate tool for the study of strong-interaction physics. However, it does give us a consistent field theoretic framework - incorporating principles like unitarity and causality - within which to check our ideas. As we shall see, the principles that emerge from our analysis are rather general and probably transcend the limits of perturbation theory. Furthermore, factorization, if it is correct, must hold in perturbation theory, so any exceptions we find perturbatively represent valid counter-examples to the "theorems."

Our analysis makes use of the light-cone (infinite-momentum frame) perturbation theory.[3] This is merely a convenience.

All of our results can, of course, be obtained by starting with the usual Feynman rules, picking an appropriate Lorentz frame, and carrying out the contour integration over one of the components (usually P_0 or P_-) of each loop momentum.

Figure 3 shows some examples of initial state interactions for the process of meson-baryon Drell-Yan production. Figures 3(a) and 3(b) show, respectively, examples of active-spectator elastic and bremsstrahlung initial state processes, and Fig. 3(c) shows an example of a spectator-spectator initial state process. The spectator-spectator interactions were considered previously by Cardy and Winbow and DeTar, Ellis, and Landshoff[4], and were shown to cancel, essentially because of unitarity. In this analysis we concentrate on the active-spectator interactions. For simplicity, we dicuss explicit calculations for the case of meson-meson collisions. The generalization to the Drell-Yan process for other types of hadronic collisions is straightforward.

III. Elastic Interactions

Let us consider first the elastic initial state interactions, the simplest example of which is shown in Fig. 4. If this type of interaction is to give a factorization-violating (leading twist) contribution to the Drell-Yan cross section, it must not be suppressed by powers of Q^2 relative to the basic process. That is, at fixed x_q, $x_{\bar{q}}$ it must give an s-independent contribution relative to the basic process. The various factors in this Feynman amplitude, in addition to those contained in the basic process, are as follows (for small y):

Energy denominator $\approx yr_\perp^2 - 2r_\perp \cdot \ell_\perp + i\epsilon$

indicated by a solid vertical line in Fig. 4;

gluon propagator denominator = $\ell_\perp^2$;

gluon spin sum multiplying quark, anti-quark convection currents $= \dfrac{r_\perp \cdot \ell_\perp}{y}$.

We work in the light-cone gauge ($A^+ = 0$) throughout in order to eliminate large ($O(\sqrt{s})$) cancelling contributions. Now, if the amplitude is to give an s-independent contribution to the cross section we must have

$$\int dy \frac{d^2\ell_\perp}{\ell_\perp^2} \frac{r_\perp \cdot \ell_\perp}{y} \frac{1}{r_\perp^2 y - 2r_\perp \cdot \ell_\perp + i\epsilon} \psi(x_q - y, k_\perp - \ell_\perp) = O(1) \; . \qquad (2)$$

$\ell_\perp^2 = -t(1-y)$ is limited by the hadronic wave function ψ to be of the order of a (hadronic mass)^2. Then, one might expect that Eq. (2) would be impossible to satisfy since the energy denominator contains a term $yr_\perp^2$ with $r_\perp^2 = s$. In fact, we need only choose y to lie in a sufficiently small range:

$$y \sim \sqrt{-t/s} \; . \qquad (3)$$

Thus, we see that the leading-twist contribution comes from the region near the pole (Glauber singularity) in the energy denominator. The singularity corresponds to classical on-shell scattering, i.e., propagation over infinite distance. This is to be contrasted with the mass singularities discussed in the usual treatments of factorization, which arise from the "collinear" region of momentum space. Unlike the mass singularities, the initial state corrections cannot be eliminated through the use of Ward identities or a particular choice of gauge.

One might worry that the $\ell_\perp$-integration in Eq. (2) seems to contain a logarithmic divergence in the small-$\ell_\perp$ region. In general, such divergences are cut-off by terms of $O(k_\perp/Q)$ that we have neglected in the gluon propagator. For the particular example shown in Fig. 4, the infrared divergence is also regulated because of a cancellation that occurs when one sums over the interactions of the gluon with all constituents of the color-neutral (singlet) hadronic system, as in Fig. 5.[5] This is simply the statement that a gluon cannot couple to a color neutral system when its wavelength is long compared to the size of the system. Note that, were it not for the $\ell_\perp$-dependence of the hadronic wave functions, which is due to the finite transverse size of the hadronic color charge distributions, this particular initial state interaction would be completely cancelled.[6]

Based on our analysis of the momentum transferred by the virtual gluon, we expect that the elastic initial state interactions smear the transverse momentum ($Q_\perp$) distribution of the Drell-Yan pair, but leave the longitudinal momentum fraction (x) distributions unchanged. The magnitude of the smearing of the $Q_\perp^2$ distribution is

$$\langle \ell_\perp^2 \rangle_N \sim \text{hadronic mass}$$

for a nucleon target, and

$$\langle \ell_\perp^2 \rangle_A \approx A^{1/3} \langle \ell_\perp^2 \rangle_N \sim A^{1/3} \text{ (hadronic mass)}$$

for a nuclear target, since the distance the projectile quark travels through the nucleus is proportional to $A^{1/3}$. That is, the projectile quark undergoes a random walk in transverse momentum space, with each step of magnitude $\sim \langle \ell_\perp^2 \rangle_N^{1/2}$. Such initial state interactions give an A-dependent contribution to $\langle Q_\perp^2 \rangle$ that might be incorrectly attributed to the "primordial" k_T of the hadronic constituents.[7]

There is evidence for such transverse momentum smearing in the CIP data shown in Fig. 6.[8] In the mass region ($M \gtrsim 3$ GeV) for which we expect the Drell-Yan mechanism to be dominant, the data show a trend toward increasing $\langle Q_\perp^2 \rangle$ with increasing nuclear size. Fitting these data to Eq. (4), we obtain $\langle k_\perp^2 \rangle^{1/2} \sim 200$ MeV. In Fig. 7 we show the NA3 data for the ratio of the Drell-Yan cross section on H_2 to the Drell-Yan cross section on Pt.[9] Taking $\langle k_\perp^2 \rangle_N = 200$ MeV and using the CIP data for the $Q_\perp$ dependence of the cross section, we estimate that at large $Q_\perp$ the initial state interactions enhance the Pt cross section by a factor of about 1.7 relative to the H_2 cross section. This is just within the NA3 error bars.

In the case of an Abelian theory there is an important cancellation in $d\sigma/dQ^2$ (but not $d\sigma/dQ^2 dQ_\perp^2$) between the square of the lowest order elastic initial state amplitude (Fig. 8(a)) and the interference of a two-gluon elastic exchange with the basic process (Fig. 8(b)).[10] Technically, the cancellation occurs as follows: once one has symmetrized the energy denominators of Fig. 8(b) with respect to the integration variables (gluon momenta), the denominators are identical to those of Fig. 8(a), except for a minus sign from moving one denominator across the final state cut and a factor of 1/2 from carrying out the symmetrization. The factor of 1/2 just cancels the factor of 2 coming from the two equal contributions of Fig. 8(b). Physically, the cancellation occurs because the initial state interactions can change the transverse momentum of the incoming quarks, but not the total incident flux. As long as one does not observe the lepton pair transverse momentum, the changes in quark transverse momentum are of no consequence. This cancellation at large Q^2 persists to all orders. In general, in the Abelian case, the Drell-Yan elastic initial state interaction graphs factorize into the absolute square of an elastic amplitude times the basic Drell-Yan process (Fig. 9). Since the elastic amplitude is, for an Abelian theory, the exponential of an imaginary eikonal phase, its absolute square is unity.[11]

By examining the momentum dependence of the hadronic wave functions in the expressions for the elastic contributions, we arrive at a condition that must be fulfilled if the cancellation is to occur:

$$s \gg \langle k_\perp^2 \rangle_N \, (M_N L_N)^2 \quad , \tag{5a}$$

where M_N and L_N are the nucleon mass and length, respectively. Condition (5a) is actually the statement that the beam be coherent over the region in which the target quark is confined:

$$(\Delta P_z{}^{\bar{q}})_{Lab} \, L_N \ll 1 \ . \tag{5b}$$

This condition is easily satisfied in most experiments.

In the case of a non-Abelian theory, such as QCD, the cancellation of elastic initial state effects outlined above fails because of the color algebra. Consider, for example, the color factors of the graphs of Fig. 8(a) and Fig. 8(b), which are shown schematically in Figs. 10(a) and 10(b), respectively. They differ by terms involving the commutator of two λ-matrices. In fact, the ratio of the color factor of Fig. 8(a) to that of Fig. 8(b) is $-(n_c^2-1)$, so that these two graphs do not cancel, as in the Abelian case, but add. Since the cancellation fails because of terms involving the commutator of λ-matrices, one might guess that graphs involving the triple gluon vertex, as in Fig. 11, could play a role in restoring the cancellation. However, such a graph contains one less Glauber singularity than the ladder graphs of the same order in α_s (Fig. 8). Thus, it has the wrong phase (pure imaginary) to contribute to the Drell-Yan cross section.

In general, the elastic interactions fail to cancel in a non-Abelian theory because the color rotation associated with the elastic exchanges fails to commute with the color matrix of the basic Drell-Yan process (Fig. 12). Thus, elastic initial state interactions, no matter how soft, can dramatically alter the Drell-Yan cross section by allowing color to "leak" from the active quarks to the spectators. An analysis of general initial state color rotations shows that this results in an initial state enhancement factor I_{el},

$$1 \leq I_{el} \leq n_c^2 \quad . \tag{6}$$

In (6), one factor of n_c is present because color "leakage" removes the constraint that the annihilating quark and anti-quark have opposite colors. The second factor of n_c accounts for the number of possible spectator colors for a given active-quark color.

An example of elastic initial state interactions in a nuclear target is shown in Fig. 13. In this example, the spectator quark is a constituent of a (color singlet) nucleon that does not contain an active quark. As a consequence, the color factors of the two graphs shown are identical, and they cancel as in the Abelian case. Thus, the elastic exchanges modify the nucleon cross section in a non-Abelian theory, but result in nuclear cross section that is still proportional to A.

IV. Gluon Bremsstrahlung

As we have seen, elastic initial state interactions have only a minor effect on the x-distributions of the annihilating constituents. However, one might expect inelastic reactions to alter the x-distributions significantly. For example, s-independent initial state bremsstrahlung could, in principle, remove an arbitrarily large fraction of the momentum of the incident particles. We shall see, however, that such bremsstrahlung is suppressed at large Q^2.

Let us consider the initial state bremsstrahlung graphs shown in Fig. 14. In discussing the initial-state bremsstrahlung we find it convenient to isolate the part of the amplitude that is induced by the active-spectator interaction. To this end, we note that the amplitude of Fig. 14(a), in which the gluon is emitted before any initial state interactions, has a numerator coupling $\epsilon_\perp \cdot j_\perp$, whereas all other diagrams lead to numerators of the form $\epsilon_\perp \cdot (j_\perp + \ell_\perp)$. Thus, we define the j-part of each amplitude to be the piece obtained by keeping only the $\epsilon_\perp \cdot j_\perp$ term in the numerator. In addition, we drop all cross terms of the form $\ell_\perp \cdot j_\perp$ in the energy denominators for the j-part. It turns out that the j-part is the correct leading-twist approximation for the bremsstrahlung amplitude for $j_\perp \gtrsim \sqrt{-t}$. We define the ℓ-part to be the remainder of the leading twist contribution to the amplitude. The ℓ-part is proportional to the momentum transfer $\ell_\perp$, and so contains the part of the bremsstrahlung that is induced by the active-spectator exchanges. It contributes in leading twist only for $j_\perp \lesssim \sqrt{-t} \ll Q$.

Let us temporarily set aside the ℓ-parts and consider first the effect of the j-parts. The j-parts of the various graphs combine to give a convenient factorized form. For example, for the j-parts of Figs. 14(a) and 14(b) the energy denominators combine as follows:

$$B^{-1}(A^{-1} + C^{-1}) = A^{-1}C^{-1} . \qquad (7)$$

Here A is the denominator associated with the emission of a gluon, and C is the denominator associated with the elastic active-spectator scattering. In the case of a non-Abelian theory, we must also take into account the triple-gluon coupling graph, Fig. 14(c). Aside from the color factor, its j-part is identical to that of Fig. 14(a). The color factor is such that, when added to the color factor of Fig. 14(a), it yields the color factor of Fig. 14(b). A more general example of this sort of combinatorics is shown in Fig. 15. The graphs on a given row have identical energy denominators. Their color factors combine to give the color factor of the last row. Then, the energy denominators associated with each row can be added to give a factored result. Thus, we see that, in both the Abelian and non-Abelian theories, the j-parts combine to give a factored result of the form of a Drell-Yan amplitude with gluon emission (including wave function evolution) times an elastic initial state scattering amplitude (see Fig. 16).

The factorized structure is such that, for a non-Abelian theory, the color factor is always computed with the elastic scattering outside of the (real or virtual) bremsstrahlung, as in Fig. 14(b). Thus, the color traces are different for the cases of real and virtual emission (Fig. 17). This leads us to expect that $d\sigma/dQ^2$ is modified by an x-dependent, factorization-violating factor:

$$1 \leq I(Q^2, x_q, x_{\bar{q}}) \leq n_c^2 \; . \tag{7}$$

In $O(\alpha_s)$ the virtual graphs contribute a Sudakov double log:

$$-(2)\,\frac{\alpha_s}{4\pi}\left(\ln^2 \frac{Q^2}{\lambda^2}\right)^2 \times \text{(elastic amplitude)} \; .$$

In an Abelian theory this double log would be cancelled by a similar contribution from the real emission graphs. However, as pointed out by Mueller[12], in $O(\alpha_s^3)$ the color factor associated with the virtual emission is C_F, whereas the color factor associated with real emission is $C_F - \frac{1}{2}C_A$. Thus, there is a residual double log contribution

$$-\frac{\alpha_s C_A}{4\pi} \ln^2 \frac{Q^2}{\lambda^2} \times \text{(elastic amplitude)} \; .$$

It can be seen (most easily in axial gauge) that these double logs exponentiate to all orders in α_s to give (ignoring the running of the coupling constant)

$$\exp\left[-\frac{\alpha_s C_A}{4\pi} \ln^2 \frac{Q^2}{\lambda^2}\right] \times \text{(elastic amplitude)} \; . \tag{8}$$

Assuming that this formal resummation of the perturbation theory is justified, one is lead to conclude that the initial state factor is of the form

$$I(Q^2, x_q, x_{\bar{q}}) = (I_{e\ell} - 1)\,|S(Q^2, x_q, x_{\bar{q}})|^2 + 1 \; , \tag{9}$$

where

$$|S|^2 \sim \exp\left[-\frac{\alpha_s C_A}{4\pi}\ln^2\frac{Q^2}{\lambda^2}\right] \text{ in the limit of large } Q^2 .$$

That is, the initial state enhancement falls faster than any power of Q^2 for Q^2 large, so that the initial state effects are not in conflict with the factorization conjecture for $d\sigma/dQ^2$. Note, however, that the initial state corrections may still be phenomenologically important. Taking the infrared cutoff λ to be given by the hadronic size, we find that the initial state enhancement factor is

$$I \lesssim (n_c^2-1)e^{-(2 \text{ or } 3)} + 1$$

at present values of Q^2.

Finally, let us return to the discussion of the ℓ-parts. By definition, the ℓ-part bremsstrahlung is always internal to at least one of the elastic exchanges. As a consequence, it tends to be suppressed because of cancelling contributions from the Glauber singularities on either side of the gluon emission vertex. For example, the energy denominators C and B in Fig. 14(b) are of the form

$$B \cong (y-y_B + i\epsilon)\, s$$
$$C \cong (y-y_C + i\epsilon)\, s \quad , \qquad (10)$$

with

$$y_B-y_C = \mathcal{M}^2/s , \qquad \mathcal{M}^2 \approx \frac{J_\perp^2}{x_{\bar q}(1-x_{\bar q})} \quad ,$$

where $\mathcal{M}$ is the invariant mass of the anti-quark-gluon system. If the hadronic wave function $\psi(x_q-y,k_\perp-\ell_\perp)$ is a slowly varying function of y, then the leading twist contributions from $y \sim y_B$ and $y \sim y_L$ cancel in the integral over y. The dependence of $\psi(x)$ on the longitudinal momentum fraction of the constituent is controlled by the longitudinal size of the target: $\psi = (xML)$, where L is the length of the target. For example, in a non-relativistic bound state $x = (m+k_3)/M$, where m is the constituent mass and M is the bound state mass, and $\psi(k_3) \sim e^{ik_3L}$ for constituents at fixed separation L. $\psi(x-y,k_\perp-\ell_\perp)$ "slowly varying" then means

$$\mathcal{M}^2/(x_{\bar{q}}s) \ll (M_N L)^{-1} . \tag{11}$$

Since, as we noted previously, the leading twist contribution to the lepton-pair cross section due to the ℓ-part amplitudes comes from the region $\mathcal{M}^2 \lesssim \ell_\perp^2 \ll Q^2$, (11) implies that

$$< \ell_\perp^2 >/(x_{\bar{q}}s) \ll (M_N L)^{-1} . \tag{12}$$

This is a new condition for the validity of the QCD prediction of the $x_{\bar{q}}$-dependence of the cross section $d\sigma/dQ^2$.

The suppression of radiation over a finite length can be understood in terms of the uncertainty principle. The induced bremsstrahlung changes the spectator laboratory momentum by an amount $\Delta p_z^{spec} = \mathcal{M}^2 M_N/(x_{\bar{q}}s)$. In order to detect the radiation specifically induced by the active-spectator interactions, one must have $\Delta p_z^{spec} L > 1$. This leads immediately to (12) as the condition for no induced radiation in the target.[13]

Note that for very long targets induced radiation does occur. Thus, we understand why depletion of the incident beam and the production of secondary hadrons occur in a macroscopic target. In the case of a nucleus, an estimate of the condition for no induced radiation is

$$Q^2 \geq x_q M_N L_A \langle \ell^2 \rangle_A = x_q M_N (1.2\ \text{fm}) A^{1/3} \langle \ell^2 \rangle_N A^{1/3} \approx 0.25\ \text{GeV}^2 A^{2/3}, \quad (13)$$

where we have used $\langle \ell^2 \rangle_N^{1/2} \sim 200$ MeV for the average momentum exchange in a quark-nucleon collision. Note that for a uranium target one requires $Q^2 \gtrsim 10\ \text{GeV}^2$ before radiation losses can be neglected.

V. Other Processes

Finally, we note that initial (and final) state interactions of the sort we have investigated in the context of the Drell-Yan process are expected to affect many other hadronic reactions. An example that is closely related to the Drell-Yan process is direct photon production at large p_T. As in the case of lepton pair production, we expect an enhancement in the cross section due to initial state interactions. At very large p_T the relative correction should be $\sim \langle \ell^2 \rangle_N A^{1/3}/p_T^2$. Jet and single particle inclusive reactions ($A+B \to C+X$) should exhibit similar momentum-smearing effects. For example, in the case of jet fragmentation processes in deep-inelastic scattering $\ell A \to \ell' HX$, the final state collisions modify the transverse momentum distributions of the produced hadrons. In addition, we expect the inelastic final state collisions of soft particles to increase hadron multiplicity. Generally, all large p_T inclusive hadronic processes should be affected by initial and final state processes. Exclusive processes are expected to be unaffected, since, for these, the hard process involves all the constituents - that is, there are no spectators.

VI. Conclusions

In summary, we predict two important effects arising from initial state interactions in the Drell-Yan process at large Q^2: (1) a new constribution to $d\sigma/dQ^2$ compared to standard factorization predictions, and (2) a smearing of the transverse momentum distribution $d\sigma/dQ^2dQ_{\perp}^2$. Although the leading twist color enhancement of $d\sigma/dQ^2$ is probably suppressed by a Sudakov form factor, it may be numerically important at present energies. In addition, we find a new condition (12) for the validity of factorization predictions for $d\sigma/dQ^2$. In spite of the initial state collisions, we expect $d\sigma/dQ^2$ on a nuclear target to be proportional to A. We note that these predictions do not depend critically upon the detailed nature of the color-changing active-spectator interaction,[14] and that they seem to be based on rather general concepts like conservation of flux (unitarity) and the uncertainty principle, which apply outside the domain of perturbation theory. Thus, we expect the effects of initial and final state interactions to occur quite generally in inclusive hadronic processes.

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5. This type of cancellation renders all graphs finite in the Abelian case. However, in a non-Abelian theory, the cancellation is, in general, spoiled because of the different color factors associated with the potentially cancelling graphs.

6. Indeed, for two-dimensional QCD, in which the hadrons have no transverse size, the initial state interactions do cancel, and one obtains the parton model result [J. Kripfganz and M. G. Schmidt, Nucl. Phys. B125, 323 (1977)]. For a further discussion of this point in the context of two-dimensional QCD see J. H. Weis, Acta Phys. Polon. B9, 1051 (1978).

7. See, for example, R. P. Feynman and R. D. Field, Phys. Rev. B15, 2590 (1977).

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10. The lowest order initial state amplitude is pure imaginary, so there is no $O(\alpha_s)$ contribution.

11. See, for example, Hung Cheng and Tai Tsun Wu, Phys. Rev. D1, 2775 (1970).

12. A. H. Mueller, Columbia University preprint CU-TP-213 (October 1981).

13. This argument is very similar to the discussion of a "formation zone" for radiation due to a classical current given by L. Landau and I. Pomeranchuk, Doklady Academii Nauk USSR 92, 535 (1953). We thank L. Stodolsky for bringing this work to our attention.

14. For example, any elastic or inelastic interaction with an amplitude $T \sim sf(t)$, where $tf(t) \to 0$ for t large, leads to our results.

FIGURE CAPTIONS

1. The basic Drell-Yan amplitude for baryon-antibaryon collisions.
2. Contributions of the basic Drell-Yan process and some $O(\alpha_s)$ radiative corrections to the lepton-pair cross section. The dashed vertical line indicates the final state. Conversion of the virtual photon (saw-toothed line) to a lepton pair is understood.
3. Some examples of initial state interactions in the Drell-Yan process for meson-baryon collisions.
4. An active quark-spectator quark initial state interaction in the Drell-Yan process for meson-meson scattering.
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7. NA-3 data for the ratio of the Drell-Yan cross sections for pions on H_2 and Pt as a function of lepton pair transverse momentum.
8. Leading twist active-spectator elastic interactions in $O(\alpha_s^2)$. The contributions cancel in an Abelian theory.
9. Factorization of elastic active-spectator interactions in an Abelian theory.
10. Color factors for (a) Fig. 8(a) and (b) Fig. 8(b).
11. An example of an elastic initial state interaction in $O(\alpha_s^2)$ involving the triple gluon vertex.

12. Non-cancellation of the elastic initial state factors in a non-Abelian theory because of the color constraint due to the Drell-Yan basic process. The color indices are denoted by a, b, c, d, with summation over repeated indices understood.
13. Examples of elastic active-spectator interactions in pion nucleus scattering. In these examples the spectator quark is a constituent of a nucleon that does not contain an active quark (spectator nucleon).
14. Examples of initial state bremsstrahlung amplitudes.
15. An example of the factorization of the j-parts of initial state bremsstrahlung amplitudes. Diagrams on a given row have the same energy denominators. Color factors on a row combine to give the color factor of the last row. Energy denominators combine to give a factored result.
16. General factorization of the j-parts of the Drell-Yan bremsstrahlung amplitudes into an elastic initial state factor times "ordinary" QCD radiative corrections. It is understood that the initial state color matrices appear to the left of all other color matrices.
17. Examples of real and virtual bremsstrahlung with initial state interactions in $O(\alpha_s^3)$.

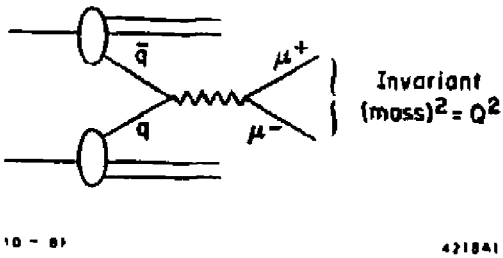


Fig. 1

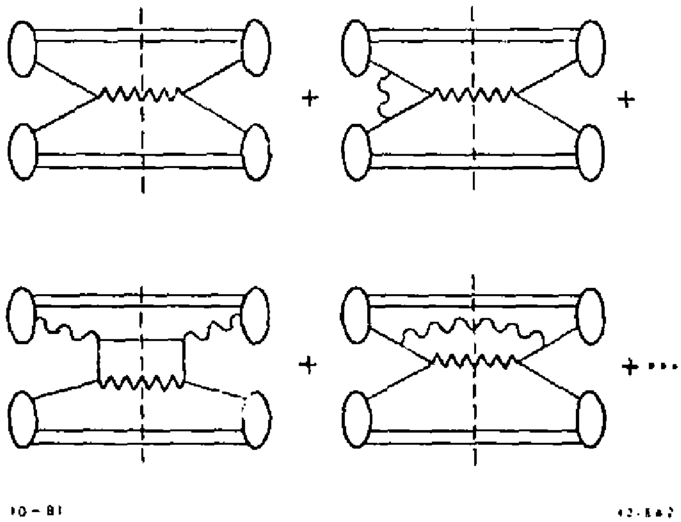


Fig. 2

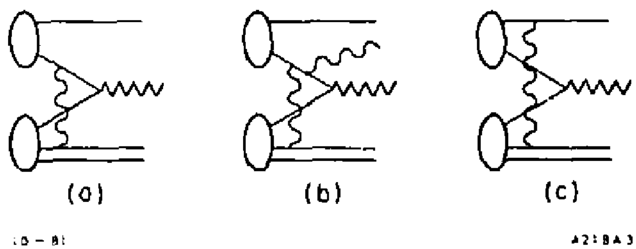
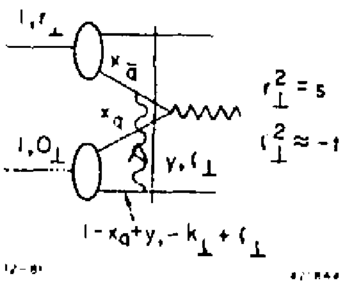


Fig. 3



1, r⊥
1, 0⊥
$x_{\bar{a}}$
x_a
$r_\perp^2 = s$
$\ell_\perp^2 \approx -t$
y, ℓ⊥
1 − x_a+y, − $k_\perp$ + $\ell_\perp$

Fig. 4

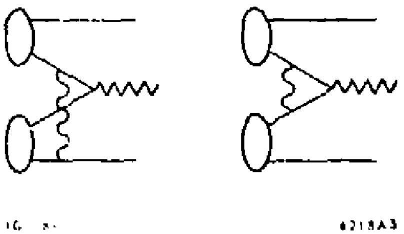


Fig. 5

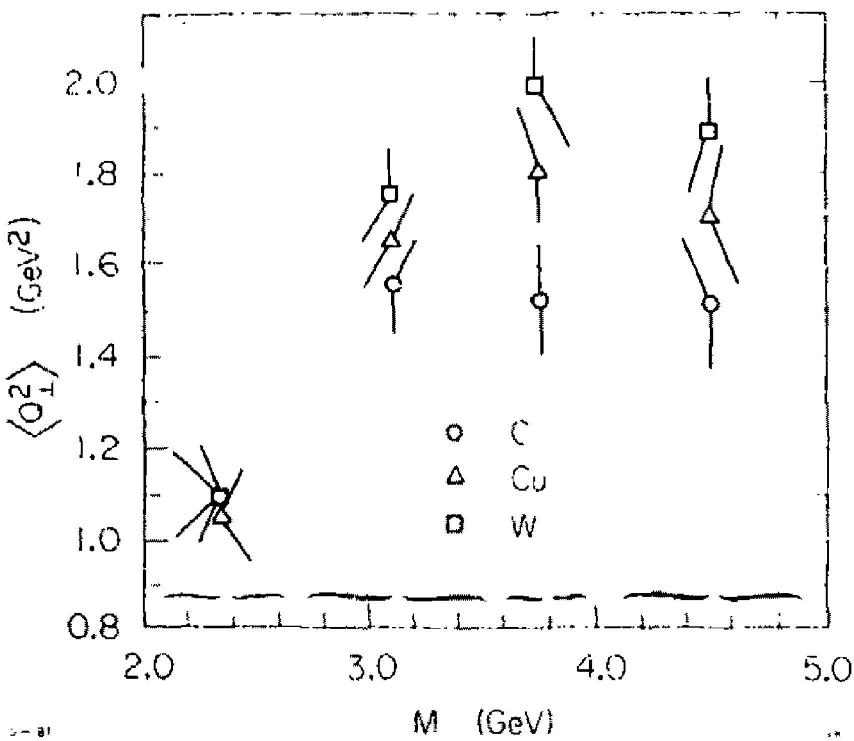


Fig. 6

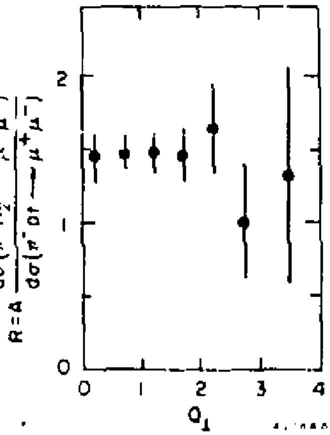

$R = A \frac{d\sigma(\pi^- H_2 \to \mu^+\mu^-)}{d\sigma(\pi^- Pt \to \mu^+\mu^-)}$
2
1
0
0
1
2
3
4
$Q_\perp$

Fig. 7

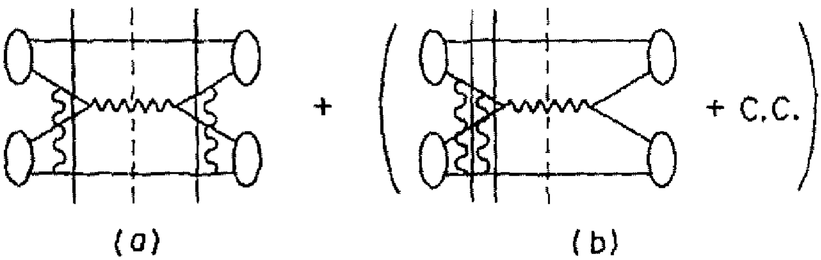
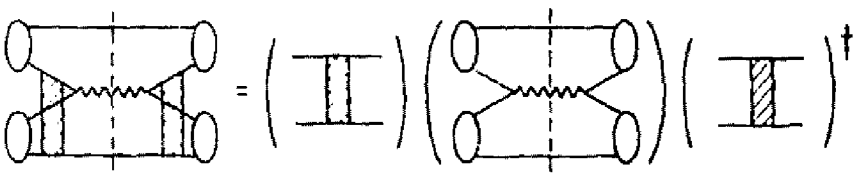


Fig. 8



4218A8

Fig. 9

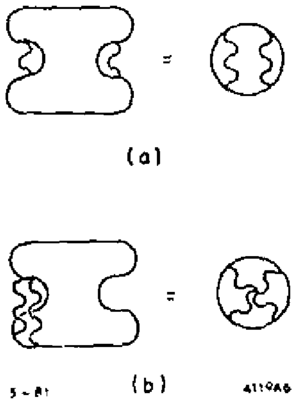


Fig. 10

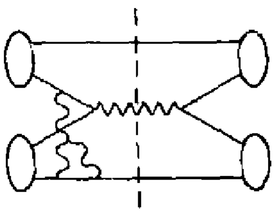
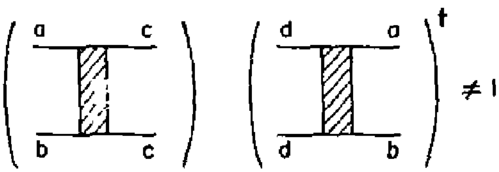
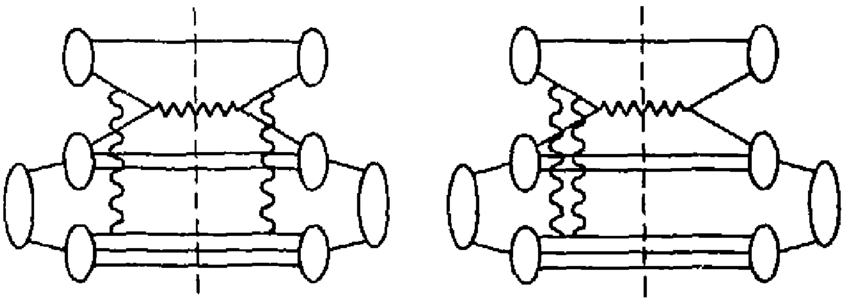


Fig. 11



10 – 81 4218A10

Fig. 12



10-85 4218A11

Fig. 13

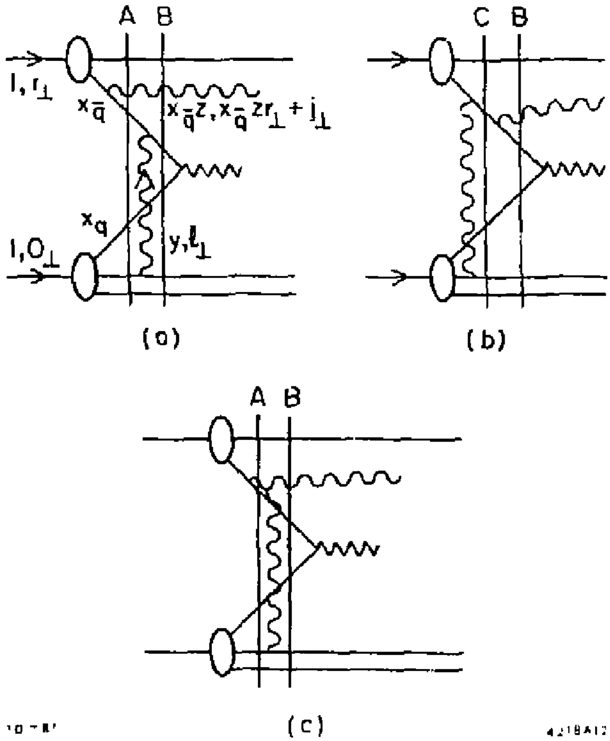
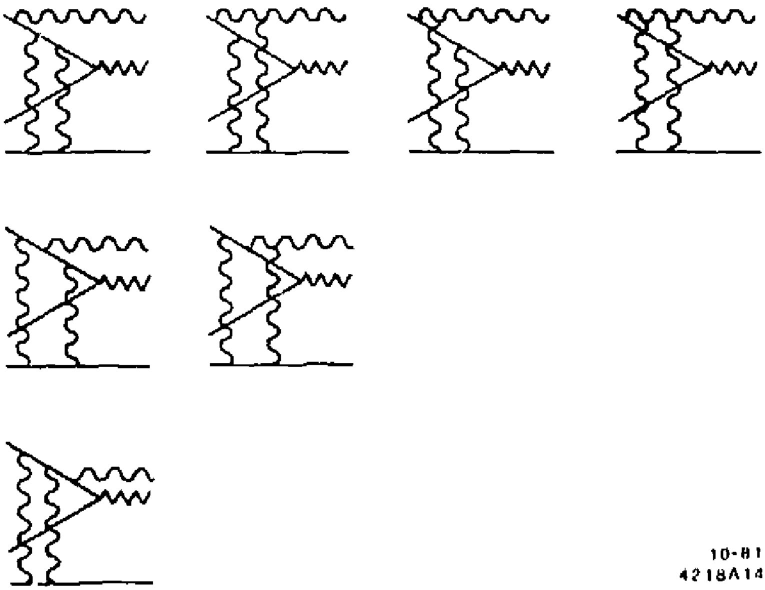

A B
$1, r_\perp$
$x_{\bar{q}}$
$x_{\bar{q}}z, x_{\bar{q}}zr_\perp + j_\perp$
x_q
$1, 0_\perp$
$y, \ell_\perp$
(a)
C B
(b)
A B
(c)
4218A12

Fig. 14



10-81
4218A14

Fig 15

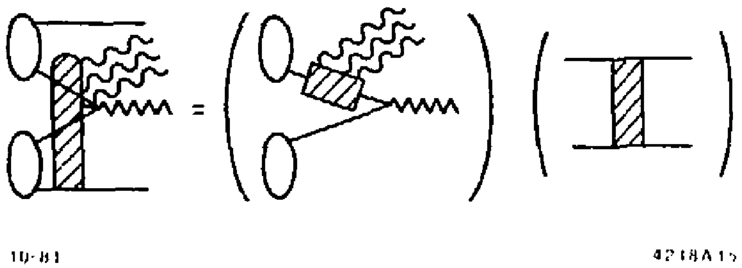


Fig. 16

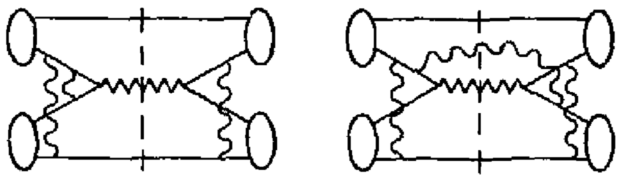


Fig. 17