BNL 31036
Conf-820142--2

March 4, 1982

BNL--31036

DE82 012023

PROTON DECAYS - 1982*

William J. Marciano

MASTER

*Invited talk presented at the 19th Orbis Scientiae Meeting, January 18-21, 1982.

The submitted manuscript has been authored under contract DE-AC02-76CH00016 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

MGU

PROTON DECAY - 1982*

William J. Marciano

Physics Department
Brookhaven National Laboratory
Upton, New York 11973

ABSTRACT

Employing the current world average $\Lambda_{\overline{MS}} = 0.160$ GeV as input, the minimal Georgi-Glashow SU(5) model predicts $\sin^2\hat{\theta}_W(m_W) = 0.214$, $m_b/m_\tau \simeq 2.8$ and $\tau_p \gtrsim (0.4 \sim 12) \times 10^{29}$ yr. The first two predictions are in excellent agreement with experiment; but the implied proton lifetime is already somewhat below the present experimental bound. In this status report, uncertainties in τ_p are described and effects of appendages to the SU(5) model (such as new fermion generations, scalars, supersymmetry etc.) are examined.

*Invited talk presented at the 19th Orbis Scientiae Meeting, January 18-21, 1982.

I. INTRODUCTION

I presented my first theoretical overview of proton decay at the 1980 Orbis Scientiae Meeting.[1] The main theme of my talk was that several detailed renormalization group analyses of the Georgi-Glashow[2] SU(5) model had been completed and we needed only to wait for experimental verification (or negation) of its predictions[3,4] regarding $\sin^2\hat{\theta}_W(m_W)$ and τ_p, the proton lifetime. Since then, the experimental value of $\sin^2\hat{\theta}_W(m_W)$ has been lowered and rendered rather precise, primarily by the inclusion of electroweak radiative corrections.[5] It is now in remarkably good agreement with the SU(5) prediction (see Sections II and III).

In the case of the proton lifetime, using $\Lambda_{\overline{MS}} = 0.5$ GeV as input (the prevailing 1980 value of the QCD mass scale suggested by scaling violations in deep-inelastic scattering experiments), the minimal SU(5) model predicted[1] $\tau_p \approx 10^{31} \sim 10^{32}$ yr. This lifetime was only about one or two orders of magnitude larger than the 1980 experimental bound[6] $\tau_p^{exp} > 10^{29} \sim 10^{30}$ yr. So, with several new dedicated proton decay experiments just beginning, we anxiously anticipated an exciting discovery.[7] Two of those experiments have now completed initial runs and published their findings. The Homestake gold mine collaboration has reported a bound[8]

$$\tau_p \gtrsim 3 \times 10^{31} \text{ yr} \quad (\mu^+ \text{ in final state}) \tag{1.1a}$$

for proton decays with a final state muon. In some models where proton decay is mediated by scalar mesons, one expects μ^+ rather than e^+ in the decay products and (1.1a) is then a stringent constraint. However, in the standard minimal SU(5) model roughly 10% of all proton decays lead to a μ^+. In such a theory (1.1a) becomes

$$\tau_p \gtrsim 3 \times 10^{30} \text{ yr} . \tag{1.1b}$$

I should remark that this collaboration did observe a few proton decay candidates; but since those events were consistent with anticipated backgrounds, they interpret their results in terms of a bound. A second new experiment which was quickly assembled and began running in a remarkably short time at the Kolar gold field has published 3 proton decay candidate events.[9] Excluding one because of its proximity to the apparatus edge, they find that their two remaining events correspond to $\tau_p \simeq 8 \times 10^{30}$ yr. Several subsequent months of running have recently yielded a fourth very clean new candidate.[10] It is, however, consistent with the anticipated background due to neutrinos. A very conservative interpretation of these findings is that they roughly correspond to the bound

$$\tau_p \gtrsim 6 \times 10^{30} \text{ yr} . \tag{1.2}$$

The fact that both experiments find candidate proton decay events at a rate consistent with the 1980 SU(5) prediction should be encouraging news for advocates of Grand Unified Theories (GUTS). However, an amusing development occurred during the last two years which has had a dramatic effect on the SU(5) prediction for τ_p. Most experiments now favor a lower value for $\Lambda_{\overline{MS}}$. Indeed, the current world average[11] is about 0.16 GeV. (see Section II). Since the proton lifetime scales approximately as $\Lambda^4_{\overline{MS}}$, this reduction in $\Lambda_{\overline{MS}}$ lowers the minimal SU(5) model's prediction to $\tau_p = (0.4 \sim 12) \times 10^{29}$ yr. Does this mean that the SU(5) model is now ruled out? No, because there are still other sources of uncertainty in the SU(5) prediction (see Section IV). However, it is clear that the confrontation between experiment and SU(5) theory has already reached an exciting stage. If the minimal SU(5) model is correct and $\Lambda_{\overline{MS}} \simeq 0.16$ GeV, then proton decay probably has been observed or will certainly be seen during the coming year when the Brookhaven-Irvine-Michigan experiment gets underway.[7] That experiment is capable of pushing the bound on τ_p up to $10^{32} \sim 10^{33}$ yr. and can readily detect a more rapid rate of decay. They should thereby determine the fate of SU(5).

Given the above motivation for closely scrutinizing the SU(5) model's predictions, I have organized my talk as follows: In Section II I present updated experimental values and uncertainties for α, $\sin^2\hat{\theta}_W(m_W)$ and $\Lambda_{\overline{MS}}$, the fundamental $SU(3)_C \times SU(2)_L \times U(1)$ coupling parameters. Then in Section III the SU(5) renormalization group analysis initiated by Georgi, Quinn and Weinberg[3] is reviewed. Using $\Lambda_{\overline{MS}}$ and α as input, predictions for $\sin^2\hat{\theta}_W(m_W)$ and τ_p (as well as other related parameters) are given. The minimal SU(5) model's prediction for m_b/m_τ is also illustrated. In Section IV uncertainties in τ_p are discussed and effects due to intermediate mass scalars are described. Supersymmetry implications for SU(5) predictions are outlined in Section V. Finally, in Section VI some conclusions are presented.

II. $SU(3)_C \times SU(2)_L \times U(1)$ COUPLINGS

The standard model of strong and electroweak interactions is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)$ (i.e. QCD × the Weinberg-Salam model). Associated with this model are three independent fundamental gauge couplings g_3, g_2 and g_1. They are generally reparametrized and discussed in terms of $\Lambda_{\overline{MS}}$, $\alpha = e^2/4\pi$ and $\sin^2\theta_W = e^2/g_2^2$. It is interesting that the experimental values of all three couplings have changed since 1980, in part due to the calculation of higher order radiative corrections. Because of their importance in obtaining SU(5) predictions and testing that model, I present an updated phenomenological profile of these fundamental couplings.

i) The fine structure constant α

In 1980, the Josephson effect provided the best purely experimental measurement of α. Now the quantized Hall effect is beginning to become competitive. The average value of α obtained from those two types of experiments is[12]

$$\alpha = 1/137.035965(12) \quad \text{(Josephson \& Hall effects)} \ . \tag{2.1}$$

Recently, Kinoshita and Lindquist[13] presented a numerical calculation of the $O(\alpha^4)$ corrections to $(g_e - 2)/2$, the anomalous magnetic moment of the electron. Using their calculation and the very precise experimental value obtained by the University of Washington group[14], Kinoshita and Lindquist found[13]

$$\alpha = 1/137.035993(10) \quad ((g_e - 2)/2) \ . \tag{2.2}$$

The slight disagreement between (2.1) and (2.2) is probably a spurious effect that will eventually go away. Alternatively, it might be a faint signal of new weak interaction phenomena or perhaps a first indication of lepton structure, very speculative but exciting possibilities.

What is of interest to us (for use in the SU(5) model) is the short distance coupling $\hat{\alpha}(m_W)$, the QED running coupling defined by $\overline{\mathrm{MS}}$ (modified minimal subtraction) using dimensional regularization with the 't Hooft unit of mass $\mu = m_W$, the W boson mass.* A renormalization group analysis gives[4,15,16]

$$\hat{\alpha}^{-1}(m_W) = \alpha^{-1} - \frac{2}{3\pi}\sum_f Q_f^2 \ln(m_W/m_f) + \frac{1}{6\pi} + \cdots \tag{2.3}$$

where the sum is over all fermions with mass $m_f < m_W$ and electric charge Q_f. The ... are meant to indicate higher order terms which, although not illustrated, are numerically included. For three generations of fermions with $m_t = 20$ GeV and $m_W = 83$ GeV one finds from (2.1) or (2.2) and a dispersion relation analysis of $e^+e^- \to$ hadrons (to determine the "effective" light quark masses)[15,16]

$$\hat{\alpha}^{-1}(m_W) = 127.54 \pm 0.30 \ , \tag{2.4}$$

where the uncertainty comes primarily from the light quark sector.

---

* We choose to compare all couplings at $\mu = m_W$, which is a particularly convenient scale since in the standard theory it is the dividing point between $SU(3)_C \times SU(2)_L \times U(1)$ and the effective $SU(3)_C \times U(1)_{em}$ theory.

ii) $\sin^2\hat{\theta}_W(m_W)$

In the standard $SU(2)_L \times U(1)$ electroweak model $\sin^2\theta_W^0 \equiv e_0^2/g_{2_0}^2 = g_{1_0}^2/(\frac{5}{3}g_{1_0}^2 + g_{2_0}^2)$ is an infinite bare parameter. Defining the renormalized weak mixing angle $\sin^2\hat{\theta}_W(m_W)$ by $\overline{MS}$ with $\mu = m_W$, and computing the electroweak radiative corrections, one can extract $\sin^2\hat{\theta}_W(m_W)$ from experiment. At present the most precise determination of $\sin^2\hat{\theta}_W(m_W)$ comes from a measurement of $R_\nu \equiv \sigma(\nu_\mu + N \to \nu_\mu + X)/\sigma(\nu_\mu + N \to \mu^- + X)$. After including radiative corrections one finds (averaging existing data)[5,15,17,18]

$$\sin^2\hat{\theta}_W(m_W) = 0.215 \pm 0.014 + \frac{3\alpha}{16\pi\sin^2\hat{\theta}_W(m_W)}\frac{m_t^2}{m_W^2} \tag{2.5}$$

where m_t is the t quark's mass.

So, assuming that $m_t^2/m_W^2 << 1$, (2.5) becomes

$$\sin^2\hat{\theta}_W(m_W) = 0.215 \pm 0.014 \quad . \tag{2.6}$$

Note that if $m_t \gtrsim m_W$ (or if there is a large mass difference in some other as yet undiscovered weak isodoublet) $\sin^2\hat{\theta}_W(m_W)$ may be somewhat larger. From a combination of R_ν and $R_{\bar{\nu}}$ data leaving the ρ parameter free and thus allowing for such effects, one finds[18,19]

$$\rho = 1.010 \pm 0.020 \tag{2.7a}$$

$$\sin^2\hat{\theta}_W(m_W) = 0.236 \pm 0.030 \tag{2.7b}$$

which yields the (not very tight) constraint $m_t \lesssim 400$ GeV.

Employing the result in (2.6), one obtains the rather precise mass predictions for the $W^\pm$ and Z^0 bosons[4,5,15,16,20]

$$m_W = \frac{38.5 \text{ GeV}}{\sin\hat{\theta}_W(m_W)} = 83.0 \pm 2.4 \text{ GeV} \tag{2.8a}$$

$$m_Z = \frac{77.1 \text{ GeV}}{\sin 2\hat{\theta}_W(m_W)} = 93.8 \pm 2.0 \text{ GeV} \tag{2.8b}$$

When m_W or m_Z is precisely measured, the formulas in (2.8) (which include radiative corrections) will provide the best available determination of $\sin^2\hat{\theta}_W(m_W)$. Indeed, it should then be known to within 1% accuracy (as compared with the present-day 6% uncertainty).

iii) $\Lambda_{\overline{MS}}$ - The QCD Mass Scale

The QCD coupling $\hat{\alpha}_3(\mu) = \hat{g}_3^2(\mu)/4\pi$ is also defined by $\overline{MS}$. It is traditionally parametrized in terms of the mass scale $\Lambda_{\overline{MS}}$, such that for $m_c < \mu < m_b$ (i.e., an effective 4 flavor regime)[21]

$$\hat{\alpha}_3(\mu) = \frac{12\pi}{25\ \ln(\mu^2/\Lambda^2_{\overline{MS}})}\left[1 - \frac{462}{625}\ \frac{\ln\ln(\mu^2/\Lambda^2_{\overline{MS}})}{\ln(\mu^2/\Lambda^2_{\overline{MS}})}\right]. \tag{2.9}$$

Using this formula, scaling violations in deep-inelastic scattering now tend to give $\Lambda_{\overline{MS}} \simeq 0.16$ GeV.[11] In addition, Lepage and Mackenzie[22] have computed the QCD corrections to Upsilon decay. Their analysis when compared with experiment indicates $\Lambda_{\overline{MS}} \simeq 0.100$ GeV. (I personally believe that the Upsilon decay analysis presently represents the best perturbative determination of $\Lambda_{\overline{MS}}$.) At the level of non-perturbative strong coupling analysis, lattice calculations[23] of the hadronic spectrum also indicate $\Lambda_{\overline{MS}} \simeq 0.1$ GeV. Clearly, $\Lambda_{\overline{MS}}$ has been lowered considerably since 1980 when 0.5 GeV was the accepted value. In this talk I will employ the world average quoted by A. Buras[11]

$$\Lambda_{\overline{MS}} = 0.160\ ^{+0.100}_{-0.080}\ \text{GeV} \tag{2.10}$$

which for $m_t = 20$ GeV and $m_W = 83$ GeV translates into

$$\hat{\alpha}_3(m_W) = 0.1088\ ^{+0.0087}_{-0.0110}\ . \tag{2.11}$$

However, I emphasize that the smaller values ($\Lambda_{\overline{MS}} \simeq 0.100$ GeV $\rightarrow \hat{\alpha}_3(m_W) \simeq 0.1015$) are beginning to be favored.

To summarize the results of this section, one finds in the standard model with $m_t = 20$ GeV and $m_W = 83$ GeV

$$\hat{\alpha}^{-1}(m_W) = 127.54 \pm 0.30$$

$$\sin^2\hat{\theta}_W(m_W) = 0.215 \pm 0.014 \qquad (2.12)$$

$$\hat{\alpha}_3(m_W) = 0.1088 \begin{array}{l} + 0.0087 \\ - 0.0110 \end{array}$$

or using $\hat{\alpha}_2(m_W) = \hat{\alpha}(m_W)/\sin^2\hat{\theta}_W(m_W)$ and $\hat{\alpha}_1(m_W) = 3\hat{\alpha}(m_W)/5\cos^2\hat{\theta}_W(m_W)$, the central values in Eq. (2.12) correspond to

$$\hat{\alpha}_2(m_W) = 0.0365$$

$$\hat{\alpha}_1(m_W) = 0.0166 \qquad (2.13)$$

III. SU(5) PREDICTIONS

In Grand Unified Theories (GUTS) such as the Georgi-Glashow SU(5) model,[2] the three bare couplings are equal $g_{3_0} = g_{2_0} = g_{1_0}$ (which implies $\sin^2\theta_W^0 = 3/8$). However, the effective low energy couplings $\hat{\alpha}_i(m_W) = \hat{g}_i^2(m_W)/4\pi$, i = 1,2,3 differ by large finite radiative corrections. Such effects can be readily computed using renormalization group techniques. This approach was initiated in the pioneering work of Georgi, Quinn, and Weinberg[3] and subsequently refined and extended by others.[1,4,15,16,24] Assuming no new masses between m_W and m_S, the super-heavy unification mass scale (i.e., all particles added to the standard model in order to complete the SU(5) multiplets are taken to have mass m_S), one finds for n_g fermion generations and N_H light Higgs isodoublets (with masses $\lesssim m_W$) the following relationships[15,26]

$$\frac{\hat{\alpha}(m_W)}{\hat{\alpha}_3(m_W)} = \frac{3}{8}\left[1 - \frac{66 + N_H}{6}\ \frac{\hat{\alpha}(m_W)}{\pi}\ln\frac{m_S}{m_W} + \frac{\hat{\alpha}(m_W)}{2\pi} + \frac{\hat{\alpha}(m_W)}{4\pi}\left\{\frac{272 - \frac{176}{3}n_g}{11 - \frac{4}{3}n_g}\ln\frac{\hat{\alpha}_3(m_S)}{\hat{\alpha}_3(m_W)}\right.\right.$$

$$\left.\left. + \frac{-\frac{136}{3} + \frac{40}{3}n_g + \frac{11}{3}N_H}{\frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6}N_H}\ln\frac{\hat{\alpha}_2(m_S)}{\hat{\alpha}_2(m_W)} + \frac{\frac{4}{3}n_g + \frac{3}{5}N_H}{\frac{4}{3}n_g - \frac{1}{10}N_H}\ln\frac{\hat{\alpha}_1(m_S)}{\hat{\alpha}_1(m_W)}\right\}\right] \tag{3.1}$$

$$\sin^2\hat{\theta}_W(m_W) = \frac{3}{8}\left[1 - \frac{110 - N_H}{18}\ \frac{\hat{\alpha}(m_W)}{\pi}\ln\frac{m_S}{m_W} + \frac{5\hat{\alpha}(m_W)}{18\pi} + \frac{\hat{\alpha}(m_W)}{4\pi}\left\{\frac{-\frac{16}{9}n_g}{11 - \frac{4}{3}n_g}\ln\frac{\hat{\alpha}_3(m_S)}{\hat{\alpha}_3(m_W)}\right.\right.$$

$$\left.\left. + \frac{\frac{680}{9} - \frac{236}{9}n_g - \frac{19N_H}{9}}{\frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6}N_H}\ln\frac{\hat{\alpha}_2(m_S)}{\hat{\alpha}_2(m_W)} + \frac{\frac{16}{9}n_g - \frac{1}{5}N_H}{-\frac{4}{3}n_g - \frac{1}{10}N_H}\ln\frac{\hat{\alpha}_1(m_S)}{\hat{\alpha}_1(m_W)}\right\}\right] \tag{3.2}$$

Using the $\hat{\alpha}_i(m_W)$ in Eqs. (2.12) and (2.13) as input, one can self-consistently solve the renormalization group Eqs. for $\hat{\alpha}_i(m_S)$ (which give $\hat{\alpha}_i(m_S) = 0.0242$ for $n_g = 3$) in conjunction with either (3.1) or (3.2) and thereby determine m_S/m_W. Presently, one first employs (3.1) to find m_S/m_W and then "predicts" $\sin^2\hat{\theta}_W(m_W)$ from (3.2). However, when $\sin^2\hat{\theta}_W(m_W)$ is eventually measured to within 1%, we will reverse this procedure and predict $\Lambda_{\overline{MS}}$.

After m_S is determined, the SU(5) prediction for the proton lifetime, τ_p, can be estimated, since m_S is the physical mass of the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge bosons which mediate proton decay. Including radiative enhancements,[27] one finds

$$\tau_p \simeq 2C \times 10^{-29}\ (m_S/\text{GeV})^4\ \text{yr} \tag{3.3}$$

where the constant C depends on $\Lambda_{\overline{MS}}$, n_g and most importantly the particular proton decay matrix element calculation one chooses to believe. C ranges from a low of 1 found by Berezinsky, Ioffe and Kogan[28] to a high of 30 obtained independently by Donoghue[29] and Golowich[30] (for $\Lambda_{\overline{MS}} = 0.16$ GeV and $n_g = 3$). These extremes illustrate the present degree of uncertainty in the calculation of τ_p for a given m_S. (See Section IV for further discussion.)

From the formulas in Eqs. (2.8), (3.1), (3.2) and (3.3) (with the range of C mentioned above), one finds for $n_g = 3$ and $N_H = 1$ the minimal SU(5) model's predictions illustrated in Table I. (By minimal I mean only Higgs' 5 and 24 plets are included.)[15]

$\Lambda_{\overline{MS}}$ (GeV)	m_S (GeV)	$\sin^2\hat{\theta}_W(m_W)$	m_W (GeV)	m_Z (GeV)	τ_p (yr)
0.10	1.3×10^{14}	0.2164	82.8	93.6	$(1 \sim 30) \times 10^{28}$
0.16	2.1×10^{14}	0.2136	83.3	94.1	$(4 \sim 120) \times 10^{28}$
0.20	2.7×10^{14}	0.2124	83.5	94.3	$(1 \sim 30) \times 10^{29}$
0.40	5.5×10^{14}	0.2084	84.3	94.9	$(2 \sim 60) \times 10^{30}$
0.50	6.9×10^{14}	0.2070	84.6	95.1	$(5 \sim 150) \times 10^{30}$

Table I: Minimal SU(5) predictions for a given $\Lambda_{\overline{MS}}$

The agreement between the SU(5) prediction for $\sin^2\hat{\theta}_W(m_W)$ in Table I (when $\Lambda_{\overline{MS}} = 0.16$ GeV) and the experimental value $\sin^2\hat{\theta}_W(m_W)^{exp} = 0.215 \pm 0.014$ (see Eq. (2.6)) is extremely impressive. However, the corresponding value of τ_p is below the experimental bound in Eq. (1.2). How serious is this discrepancy? That question will be addressed in Section IV.

I conclude this section by updating another of the SU(5) model's successes, its prediction for m_b/m_τ. As pointed out by Georgi and Glashow,[2] if only a 24 and 5 of Higgs are used to break the SU(5) gauge symmetry (the so-called minimal model), then the quark-lepton bare mass relations $m_d^0 = m_e^0$, $m_s^0 = m_\mu^0$, $m_b^0 = m_\tau^0$ naturally follow. These lowest order relationships are strongly renormalized by higher order self-energy corrections; a feature first studied by Chanowitz, Ellis and Gaillard[31] and further analyzed by Buras, Ellis, Gaillard and Nanopoulos.[27] Their renormalization group analysis yields[27]

$$m_b/m_\tau \simeq 2.8 \tag{3.4}$$

for the minimal SU(5) model with 3 generations of fermions and $\Lambda_{\overline{MS}} = 0.16$ GeV. This is to be compared with the experimental value

$$m_b/m_\tau \simeq 2.6 \sim 2.9 \quad (\text{Exp}) \ . \tag{3.5}$$

The agreement is very good ($\Lambda_{\overline{MS}} = 0.1$ GeV $\rightarrow m_b/m_\tau \simeq 2.6$). Appending additional generations tends to increase this SU(5) prediction. For a given $\Lambda_{\overline{MS}}$, each new generation of fermions (with masses $\simeq m_W$) increases the m_b/m_τ prediction by about 7%. (However, $n_g = 4$ and $\Lambda_{\overline{MS}} = 0.1$ GeV implies $m_b/m_\tau \simeq 2.8$ just as in (3.4); so one can't strongly argue that 3 generations is favored by such an analysis.) Unfortunately, the minimal SU(5) model also predicts $m_s/m_d \simeq m_\mu/m_e = 207$ whereas current algebra estimates seem to indicate $m_s/m_d \simeq 22$. The solution to this dilemma may require the introduction of additional Higgs multiplets[2] such as a $\underline{45}$. The effect of additional Higgs scalars on SU(5) predictions will be further examined in Section IV.

IV. UNCERTAINTIES IN τ_p

The current world average $\Lambda_{\overline{MS}} = 0.160$ GeV implies a proton lifetime of about $\tau_p \gtrsim (0.4 \sim 12) \times 10^{29}$ yr. in the minimal SU(5) model (i.e., 3 generations of fermions and $\underline{5}$ and $\underline{24}$ Higgs multiplets). This prediction is already somewhat below the present experimental bound $\tau_p^{exp} \gtrsim 6 \times 10^{30}$ yr; so it is worth reexamining the uncertainties in τ_p. That will be the topic of this section.

i) Proton Decay Matrix Elements: To determine τ_p in the SU(5) model requires several ingredients. First the effective baryon number violating Hamiltonian is calculated. It is proportional to $\hat{\alpha}_i(m_S)/m_S^2$ and includes enhancement factors due to radiative corrections.*[27] Then the proton decay matrix elements of the effective Hamiltonian must be evaluated and transition rates calculated. Unfortunately, there is some uncertainty regarding the wavefunction overlap of quarks inside the nucleus and opposite views regarding the importance of the three quark fusion mechanism. To illustrate the differences that exist, I have given in Table II the minimal SU(5) model predictions obtained by several distinct groups.

Group	τ_p (yr)
Berezinsky-Ioffe-Kogan[28]	4×10^{28}
Goldman-Ross[4]	6×10^{28}
Jarlskog-Yndurain[32]	9×10^{28}
Ellis-Gaillard-Nanopoulos-Rudaz[33]	14×10^{28}
Donoghue[29]	120×10^{28}
Golowich[30]	123×10^{28}

Table II. Comparison of different groups' predictions for τ_p using $\Lambda_{\overline{MS}} = 0.16$ GeV $\rightarrow m_S = 2.1 \times 10^{14}$ GeV as input.

The bag model calculations by Donoghue and Golowich give the longest lifetimes. Their estimates are actually upper bounds since they only include two particle final states and neglect 3 quark fusion contributions. In contrast, Berezinsky et al., claim that the three quark fusion mechanism dominates and obtain from

*The enhancement factors increase the proton decay rate by a factor of 12 for $\Lambda_{\overline{MS}} = 0.4$ GeV; however, they monotonically decrease to about 7.5 for $\Lambda_{\overline{MS}} = 0.1$ GeV. Such changes have been incorporated in the τ_p predictions of Table I.

it the shortest lifetime prediction illustrated in our table. The truth probably lies somewhere between these extremes. The calculations of τ_p illustrated in Table II vary by about a factor of 30. Hopefully, further theoretical analysis can reduce this uncertainty in the future.

ii) $\underline{\Lambda_{\overline{MS}} \text{ and } \hat{\alpha}(m_W)}$: Although $\Lambda_{\overline{MS}} = 0.16$ GeV is favored, there is still some degree of uncertainty in the parameter. Using $\Lambda_{\overline{MS}} = 0.16 {}^{+0.10}_{-0.08}$ GeV $\rightarrow$ $\hat{\alpha}_3(m_W) = 0.1088 {}^{+0.0087}_{-0.0110}$ and $\hat{\alpha}^{-1}(m_W) = 127.54 \pm 0.30$, one finds

$$m_S = (2.1 {}^{+1.7}_{-1.2}) \times 10^{14} \text{ GeV} . \tag{4.1}$$

This range in the allowed values of m_S becomes magnified in τ_p, since it is proportional to m_S^4. So the uncertainty in $\Lambda_{\overline{MS}}$ and $\hat{\alpha}(m_W)$ implies about a factor of 10 uncertainty in τ_p. Taking the geometric mean of the predictions in Table II and accepting the range for m_S in (4.1), one finds

$$\tau_p = 2 \times 10^{29\pm2} \text{ yr} \tag{4.2}$$

which can be consistent with experiment. However, the + 2 uncertainty in (4.2) is rather generous; so I feel that a bound of $\tau_p \gtrsim 10^{32}$ yr would rule out the minimal SU(5) model.

iii) $\underline{m_t}$: In my analysis, I took $m_t = 20$ GeV, what if the actual value is much larger? It turns out that in a leading log approximation the SU(5) predictions are independent of m_t.[15] The next to leading log dependence is very weak. Hence, this uncertainty is insignificant. I must, however, re-emphasize that a very large m_t changes the phenomenological determination of $\sin^2\hat{\theta}_W(m_W)$. (See Eq. (2.5).) The requirement that the SU(5) prediction for $\sin^2\hat{\theta}_W(m_W)$ be compatible with experiment gives the constraint $m_t \lesssim 240$ GeV.

iv) <u>N_H Higgs Doublets:</u> If there are $N_H > 1$ light Higgs isodoublets such that the physical scalars coming from these multiplets have mass $\simeq m_W$ (all other scalars still have mass m_S), then the proton lifetime decreases by a factor of[15]

$$10^{-0.76(N_H-1)} \quad . \tag{4.3}$$

Hence, one cannot merely add new light Higgs doublets (say as parts of <u>5</u>-plets) to the minimal SU(5) model, without worsening the conflict with τ_p^{exp}.

v) <u>Generation Mixing:</u> By introducing <u>45</u> plets (in addition to the <u>5</u>), one relaxes constraints on the fermion mixings. A very contrived choice of mixing angles could then increase the proton lifetime τ_p.[25] (i.e., the proton might prefer to decay into a τ^+ rather than an e^+; but is kinematically unable to do so.) This scenario is too unnatural and contrived to be considered a viable possibility; so I will not consider it further.

vi) <u>A 4th Generation:</u> Is there a fourth generation of fermions? It has been suggested that to explain the observed baryon asymmetry of the universe within the framework of SU(5) may require an additional generation of heavy fermions.[34] What would their effect be on the low energy predictions of SU(5)? We already mentioned in Section III, that a fourth generation of fermions (with charged fermion masses $\gtrsim m_W$) increases m_b/m_τ by $\simeq$ 7% or less. Not a very significant effect. In the case of $\sin^2\hat{\theta}_W(m_W)$ and τ_p the modifications are even smaller.[15,35] A fourth generation reduces $\sin^2\hat{\theta}_W(m_W)$ by 0.0004 or less. It increases τ_p by 30% or less. Similar changes occur when a fifth generation is added (eg. For $n_g = 5$, the predictions for τ_p in Table I increase by a factor $\leqslant 2$.) Obviously, the SU(5) predictions are fairly insensitive to the addition of one or two new generations of fermions.

vii) Nuclear Physics Effects: All proton decay experiments involve nuclei rather than free nucleons. What are the nuclear physics effects on τ_p? This is a rather complicated subject. Sparrow[36] has estimated an effective suppression of about 50% in the rate for $p^+ \rightarrow e^+\pi^0$ due to real π^0 absorption. On the other hand, Dover, Goldhaber, Trueman and Chau[37] find some decay rate enhancements due to virtual π^0 absorption. One expects nuclear effects to introduce about a factor of 2 uncertainty[38] in τ_p, although some controversy regarding this matter still exists. A detailed study of nuclear physics effects will become a high priority once proton decay is truly established.

viii) Higgs Scalars: The minimal SU(5) model contains a real $\underline{24}$ and a complex $\underline{5}$ of Higgs scalars. To simplify the renormalization group analysis and get definite predictions we generally assume that all residual physical scalars have mass m_S except for the usual neutral member of the isodoublet component of the $\underline{5}$ which has mass $\sim m_W$. What if we relax this constraint? Or what happens if new scalar multiplets are added? I will discuss a few possibilities.

The simplest modification of the minimal SU(5) analysis is to allow the color triplet charge $\pm$ 1/3 components of the $\underline{5}$ to have arbitrary mass $m_H \neq m_S$. Since these scalars can mediate proton decay, they can not be too light. The bound in Eq. (1.2) implies (roughly) $m_H \gtrsim 10^{10}$ GeV. Furthermore, the Coleman-Weinberg[39] mass generation mechanism suggests $m_H/m_S \gtrsim 0.1$; otherwise fine-tuning of parameters is required. For m_H arbitrary, one finds[40] that τ_p is increased by about a factor of $(m_S/m_H)^{4/67}$. So, $m_S/m_H = 10$ leads to a 15% increase in τ_p. The extreme possibility $m_S/m_H \simeq 10^4$ implies a 70% increase in τ_p. The corresponding change in the weak mixing angle prediction

$$\Delta\sin^2\hat{\theta}_W(m_W) = \frac{11\hat{\alpha}(m_W)}{201\ \pi} \ln(m_H/m_S) \tag{4.4}$$

is not very significant.

Another possibility is to add a complex 45 of Higgs scalars to the minimal model. Indeed, I already mentioned that such a 45 may be required to have the low mass fermion spectrum come out right. The 45 decomposes under $SU(3)_C \times SU(2)_L \times U(1)$ as follows:

$$\underline{45} = (1,2,-1) + (3,1,2/3) + (3,3,2/3) + (\bar{3},1,-8/3) + (\bar{3},2,7/3) + (\bar{6},1,2/3) + (8,2,-1) \qquad (4.5)$$

where the last number denotes the U(1) hypercharge with $Y = 2Q - 2T_3$. In principle, each of the 7 components in (4.5) could have different independent masses. Labeling their masses by m_i, $i = 1,2..7$ (corresponding to the order in (4.5)), I find that the 45 modifies the prediction for τ_p by a factor of (approximately)

$$\left(\frac{m_1\, m_3^3\, m_4^4\, m_5^7}{m_2\, m_6^6\, m_7^8}\right)^{4/67} \qquad (4.6)$$

which $\sin^2\hat{\theta}_W(m_W)$ changes by

$$\Delta\sin^2\hat{\theta}_W(m_W) \simeq -\frac{\hat{\alpha}(m_W)}{804\,\pi} \ln\left(\frac{m_1^{44}\, m_3^{534}}{m_2^{44}\, m_4^{159}\, m_5^{94}\, m_6^{197}\, m_7^{84}}\right) \qquad (4.7)$$

For all m_i degenerate, there is no change in the predictions for τ_p and $\sin^2\hat{\theta}_W(m_W)$ (in leading log approx.). However, for arbitrary m_i, the changes may be significant. What guides do we have regarding the values of m_i? One expects m_2, m_3, m_4, $m_5 \gtrsim 10^{10}$ GeV since they can mediate proton decay. In addition, the Coleman-Weinberg[39] effect suggests that no two masses should differ by more than a factor of $\simeq 10$. Using this constraint and maximizing (4.6), one finds that τ_p can increase by a factor of 8 while $\Delta\sin^2\hat{\theta}_W(m_W) = -0.002$. These are not terribly large changes. However, without the Coleman-Weinberg constraint, the modifications can be much larger. For example, if

$m_6 \underset{\sim}{\sim} m_W$ while all other $m_i = m_S$ (a case considered by Ibanez[41]), then τ_p increases by $\underset{\sim}{\sim} 3 \times 10^4$ while $\Delta\sin^2\hat{\theta}_W(m_W) \underset{\sim}{\sim} - 0.017$. The increase in τ_p is significant; but so is the reduction in $\sin^2\hat{\theta}_W(m_W)$. So the weak mixing angle can provide a very useful constraint on mass hierarchies in the Higgs sector.

It has been suggested[42] that there may also be <u>10</u>, <u>15</u>, <u>50</u>, <u>75</u> etc multiplets of Higgs scalars. If all multiplet members are approximately degenerate, the SU(5) predictions are only slightly modified. However, for totally arbitrary masses, the predictability is lost. As a working constraint, I will accept the Coleman-Weinberg induced mass effect to imply that scalar mass ratios should be $\lesssim 10$. Then one does not expect much flexibility in $\sin^2\hat{\theta}_W(m_W)$ and anticipates (at most) a factor of 10 uncertainty in τ_p.

V. SUPERSYMMETRY

A possible way of increasing the proton lifetime is to impose a supersymmetry constraint on the theory.[43] One assumes that every boson (fermion) of the standard model has a supersymmetric fermion (boson) partner which has not yet been observed. In such supersymmetric extensions of grand unified theories, coupling constant renormalizations change; hence the unification mass m_S extracted using low energy experimental data as input may be significantly altered. Since τ_p is proportional to m_S^4, it is very sensitive to such changes. Indeed, a rough estimate by Dimopoulos, Raby and Wilczek[44] found (neglecting Higgs multiplets) that $\tau_p \underset{\sim}{\sim} 10^{45}$ yrs while $\sin^2\hat{\theta}_W(m_W)$ was essentially unchanged in supersymmetric extensions of SU(5). However, it was later noted that m_S and hence τ_p exhibits a very strong dependence on N_H the number of relatively light Higgs isodoublets in the model.[45] Since in realistic supersymmetric theories (if such things exist) $N_H = 2,4...$ an even number due to the anomaly cancellation requirement for their fermionic partners, τ_p generally turns out to be much smaller than the 10^{45} yr estimate and $\sin^2\hat{\theta}_W(m_W)$ is somehat larger than the ordinary SU(5) prediction.

Assuming supersymmetry breaking occurs at a mass scale of about m_W, then a detailed renormalization group analysis yields the predictions in Table III[19,45]

	$\Lambda_{\overline{MS}}$ (GeV)	$\sin^2\hat{\theta}_W(m_W)$	m_S (GeV)	τ_p (yr)
$N_H = 2$	0.1	0.239	4.8×10^{15}	$(4 \sim 120) \times 10^{33}$
	0.2	0.235	1.1×10^{16}	$(9 \sim 270) \times 10^{34}$
	0.4	0.232	2.4×10^{16}	$(1.6 \sim 48) \times 10^{36}$
$N_H = 4$	0.1	0.260	2.6×10^{14}	$(4 \sim 120) \times 10^{28}$
	0.2	0.258	5.5×10^{14}	$(6 \sim 180) \times 10^{29}$
	0.4	0.255	1.2×10^{15}	$(1 \sim 30) \times 10^{31}$

Table III. Predictions of supersymmetric grand unified theories. N_H is the number of light Higgs isodoublets.

Of course, in specific supersymmetric models, τ_p may be substantially smaller than the estimates in Table III if super-heavy Higgs induced proton decay amplitudes are significant. In any case, some values of τ_p in Table III are within experimental reach (at least for $N_H = 4$).*

The clear distinction between the SU(5) model and its supersymmetric extensions lies in their predictions for $\sin^2\hat{\theta}_W(m_W)$. The values of $\sin^2\hat{\theta}_W(m_W)$ in Table III are significantly larger than their ordinary SU(5) counterparts. Are such supersymmetric models therefore already ruled out by the experimental constraint $\sin^2\hat{\theta}_W(m_W) = 0.215 \pm 0.014$ given in Eq. (2.6). The answer (as pointed out by G. Senjanović and myself[19]) is yes; unless $\rho > 1$. A 1% increase in ρ manifests itself as a 4% increase in the value of $\sin^2\hat{\theta}_W(m_W)$ extracted from R_ν data.[5,46] So, the supersymmetric predictions for $\sin^2\hat{\theta}_W(m_W)$

*For $N_H \geqslant 6$, the prediction for τ_p is well below the experimental bound and hence not illustrated.

can be reconciled with experiment if one introduces effects that cause deviations $\delta\rho \simeq 2 \sim 3\%$. One possibility is to have $m_t \gtrsim 240$ GeV (see Eq. (2.5)); there are, of course, many others. A more precise determination of ρ^{exp} and new independent measurements of $\sin^2\hat{\theta}_W(m_W)$ are clearly called for if one is to test the idea of supersymmetric grand unification.*

VI. CONCLUSIONS

The minimal SU(5) model with 3 generations of fermions, one light Higgs doublet and only two boson mass scales m_W and m_S predicts $\sin^2\hat{\theta}_W(m_W) = 0.214$ and $\tau_p \simeq (0.4 \sim 12) \times 10^{29}$ yr for $\Lambda_{\overline{MS}} = 0.16$ GeV. The weak mixing angle prediction is in excellent agreement with the experimental average $\sin^2\hat{\theta}_W(m_W)^{exp} = 0.215 \pm 0.014$; however, the implied proton lifetime is somewhat below the present bound $\tau_p^{exp} \gtrsim 6 \times 10^{30}$ yr. Taking a conservative view of the uncertainties involved in obtaining τ_p and allowing for the range of input values $\Lambda_{\overline{MS}} = 0.16\,^{+0.10}_{-0.08}$ GeV and $\hat{\alpha}^{-1}(m_W) = 127.54 \pm 0.30$, I found (using a geometric mean formula)

$$\tau_p \simeq 2 \times 10^{29\pm2}\ \text{yr}\ . \tag{6.1}$$

Additional uncertainties due to generation mixing, the value of m_t, new fermion generations, nuclear physics effects etc. appear to be small. Therefore, it seems that the minimal SU(5) model will be well tested by the coming proton decay experiments which will thoroughly explore the $\tau_p \simeq 10^{30} \sim 10^{32}$ regime.

Enlarging the Higgs sector of the SU(5) model with $\underline{45}$-plets or other more exotic representations diminishes the models predictability. For unconstrained physical scalar masses, the values of τ_p and $\sin^2\hat{\theta}_W(m_W)$ can become essentially arbitrary. However, using a hierarchy condition (i.e., the Coleman-

*Of course, if supersymmetry breaking occurs at a mass scale of order m_S none of the ordinary SU(5) predictions are changed. i.e., My discussion only applies to theories with supersymmetry breakdown at relatively low energies.

Weinberg effect) to restrict all new physical scalar mass ratios to be $\lesssim 10$, leads to about another factor of 10 uncertainty in τ_p and a rather negligible shift in $\sin^2\hat{\theta}_W(m_W)$. This additional order of magnitude uncertainty when combined with Eq. (6.1) demonstrates the importance of pushing experiments into the more difficult region $\tau_p \simeq 10^{32} \sim 10^{33}$ yr.

Supersymmetry extensions of grand unified theories are not yet on a firm theoretical footing. In any case, rather general renormalization group analysis indicate that their predictions for τ_p tend to exhibit a very sensitive dependence on the light Higgs scalar content. So, there is no definite prediction regarding τ_p. The best way of testing these models is to carry out new precise measurements of $\sin^2\hat{\theta}_W(m_W)$ which is predicted to be larger than in ordinary SU(5). In addition, one expects that such models should have $\rho > 1$ if they are to be reconciled with deep-inelastic neutrino scattering data. A very precise determination of ρ^{exp} (say to within 1%) should be a high priority.

Finally, there are many alternative theories (such as SO(10)) which are bigger than the SU(5) model and can thus easily accommodate several intermediate mass scales between m_W and m_S. Such theories generally contain many free parameters and are, therefore, incapable of making precise predictions. They will, of course, become much more attractive if the coming generation of experiments turn out to be inconsistent with ordinary SU(5).

ACKNOWLEDGEMENT

The SU(5) calculations described in this talk were obtained in collaboration with A. Sirlin while the supersymmetry analysis was carried out with G. Senjanović. I also wish to thank C. Dover, M. Goldhaber and P. Langacker for informative discussions and comments on the subject of proton decay.

This research was supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

REFERENCES

1. W. Marciano, in "Orbis Scientiae, Recent Developments in High Energy Physics", Jan. 1980 edited by A. Perlmutter and L. Scott (Plenum, New York) p. 121.
2. H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).
3. H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
4. W. Marciano, Phys. Rev. D20, 274 (1979); T. Goldman and D. Ross, Phys. Lett. 84B, 208 (1979); Nucl. Phys. B171, 273 (1980).
5. A. Sirlin and W. Marciano, Nucl. Phys. B189, 442 (1981); C. Llewellyn Smith and J. Wheater, Phys. Lett. 105B, 486 (1981).
6. J. Learned, F. Reines and A. Soni, Phys. Rev. Lett. 43, 907 (1979).
7. For a review of ongoing and proposed proton decay experiments see: "Proceedings of the Second Workshop on Grand Unification" April 1981 edited by J. Leveille, L. Sulak and D. Unger (Birkhäuser, Boston).
8. M.L. Cherry et al., Phys. Rev. Lett. 23, 1507 (1981).
9. M. Krishnaswamy et al., Phys. Lett. 106B, 339 (1981).
10. Private communication from P. Langacker.
11. A. Buras, Proceedings of the 1981 Lepton Photon Symposium, Aug. 1981, FNAL preprint 81169-THY.
12. D.C. Tsui et al., Phys. Rev. Lett. 48, 3 (1982).
13. T. Kinoshita and W. Lindquist, Phys. Rev. Lett. 47, 1573 (1981).
14. R.S. Van Dyck, Jr., P.B. Schwinberg and H. Dehmelt, Bull. Am. Phys. Soc. 24, 758 (1979).
15. A detailed description of the SU(5) calculations is given in: W. Marciano and A. Sirlin, "Proceedings of the Second Workshop on Grand Unification", see Ref. 7, p. 151.

16. W. Marciano and A. Sirlin, Phys. Rev. Lett. 46, 163 (1981).

17. W. Marciano and A. Sirlin, Phys. Rev. D22, 2695 (1980).

18. J. Kim, P. Langacker, M. Levine and H. Williams, Rev. Mod. Phys. 53, 211 (1980); I. Liede and M. Roos, Nucl. Phys. B167, 397 (1980).

19. W. Marciano and G. Senjanović, "Predictions of Supersymmetric Grand Unified Theories" BNL preprint 30398, 1981.

20. A. Sirlin, Phys. Rev. D22, 971 (1980).

21. Cf. A. Buras, Rev. Mod. Phys. 52, 199 (1980).

22. P. Mackenzie and G.P. Lepage, Phys. Rev. Lett. 47, 1244 (1981).

23. H. Hamber and G. Parisi, Phys. Rev. Lett. 47, 1792 (1981).

24. S. Weinberg, Phys. Lett. 91B, 51 (1980); P. Binetruy and T. Schucker, Nucl. Phys. B178, 293, 307 (1981); L. Hall, Nucl. Phys. B178, 75 (1981); C. Llewellyn Smith, G. Ross and J. Wheater, Nucl. Phys. B177, 263 (1981); I. Antoniadis, C. Bouchiat and J. Iliopoulos, Phys. Lett. 97B, 367 (1980). Other references can be found in Ref. 25.

25. P. Langacker, Phys. Rep. 72, 185 (1981).

26. The results in Eqs. (3.1) and (3.2) have been updated to include the two loop contribution of Higgs scalars that was calculated by D.R.T. Jones, Phys. Rev. D25, 581 (1982).

27. A. Buras, J. Ellis, M.K. Gaillard and D. Nanopoulos, Nucl. Phys. B135, 66 (1978).

28. V. Berezinsky, I. Ioffe and Ya. Kogan, Phys. Lett. 105B, 33 (1981).

29. J. Donoghue, Phys. Lett. 72B, 53 (1977).

30. E. Golowich, Phys. Rev. D22, 1148 (1980).

31. M. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B128, 506 (1977).

32. C. Jarlskog and F. Yndurain, Nucl. Phys. B149, 29 (1979).

33. J. Ellis, M.K. Gaillard, D. Nanopoulos and S. Rudaz, Nucl. Phys. B176, 61 (1980).

34. G. Segre and M. Turner, Phys. Lett. 99B, 399 (1981).

35. M. Fischler and C. Hill, Nucl. Phys. B193, 53 (1981).

36. D. Sparrow, Phys. Rev. Lett. 44, 625 (1980).

37. C. Dover, M. Goldhaber, T.L. Trueman and L.-L. Chau, Phys. Rev. D24, 2886 (1981).

38. C. Dover, private communication.

39. S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

40. G. Cook, K. Mahanthappa and M. Sher, Phys. Lett. 91B, 369 (1981).

41. L. Ibánez, Nucl. Phys. B181, 105 (1981).

42. See Ref. 25 for a review.

43. For a review of supersymmetry see P. Fayet and S. Ferrara, Phys. Rep. 32C, 249 (1977).

44. S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24, 1681 (1981).

45. S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981); L. Ibánez and G. Ross, Phys. Lett. 105B, 439 (1981); M. Einhorn and D.R.T. Jones, Univ. of Mich. preprint UMHE 81-55, (1981).

46. W. Marciano, in Particles and Fields - 1979, edited by B. Margolis and D.G. Stairs, AIP Conference Proceedings No. 59 (American Institute of Physics, N.Y., 1980) p. 373.