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INS-Rep.-433
October 1981

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Abstract;

The antisymmetric spin-orbit interaction(ALS) proposed for sd-shell nuclei is investigated. It is shown that the centroid energy of the $d_{5/2}$-$d_{3/2}$ interactions plays a crucial role in reproducing the excited band spectra of A=18-24 nuclei. An empirical effective interaction without ALS component is proposed to reproduce the observed spectra of light sd-shell nuclei.

It is well known that the full $(sd)^n$ shell model calculations[1] using the Kuo interaction[2] succeed in reproducing the ground bands in light sd-shell nuclei such as ^{24}Mg. This interaction, however, predicts the excited bands systematically at lower excitation energies compared with the observed spectra. In order to remedy this defect, various empirical two-body interactions have been proposed[1,3]. A characteristic feature common to these interactions is a strong antisymmetric spin-orbit(ALS) component in the matrix elements. A property peculiar to the ALS interaction is as follows; it has non-vanishing matrix elements only between two-particle states of different orbital symmetries[4,6], and does not conserve total spin S of the two-particle, namely

$$< L\ S=0\ T\ J\ |V_{ALS}|\ L\ S=1\ T\ J > \quad \neq 0. \tag{1}$$

In particular Feldmeier et al.[4] demonstrated that the splitting between the K=0 and the K=2 bands in ^{24}Mg can be reproduced by increasing the strength of ALS part of the Kuo and Brown matrix elements[5] by a factor of about 5. Thus an ALS interaction plays a significant role in reproducing the excited bands. However it has not been clear how the ALS interaction affects the entire shell model calculation to give a good result for these rotational bands. Furthermore it is also not clear whether or not the ALS component is indispensable to reproduce the band splittings. In this letter, brief answer for these two questions will be reported by investigating the excited band spectra of ^{24}Mg as an example.

In order to clarify the questions, first we propose two possible operator-forms of the ALS interaction. One of the forms is

$$V^{(1)}_{ALS}\ (1,2) = (\hat{\ell}_1 - \hat{\ell}_2)\cdot(\hat{\sigma}_1 - \hat{\sigma}_2), \qquad (2)$$

and the other is

$$V^{(2)}_{ALS}\ (1,2) = (\hat{\ell}_1 \times \hat{\ell}_2)\cdot(\hat{\sigma}_1 \times \hat{\sigma}_2), \qquad (3)$$

where $\hat{\ell}$ and $\hat{\sigma}$ are the orbital angular momentum and spin operators respectively. It is found that the empirical matrix elements[3] can be reproduced by adding the following correction term ΔV to the Kuo interaction[1],

$$\Delta V = \alpha V^{(1)}_{ALS} + \beta V^{(2)}_{ALS} + B_{T=0} + B_{T=1}, \qquad (4)$$

where B_T is the isospin-dependent monopole interaction. For example, the Chung and Wildenthal interaction[3] which is one of the most feasible effective interaction to describe light sd-shell nuclei is approximately reproduced by setting

$$\alpha = -0.03, \quad \beta = -0.2, \quad B_{T=0} = 0.2, \quad B_{T=1} = 0.35 (\mathrm{MeV}).$$

Since the ALS interaction $V^{(1)}_{ALS}$ in (2) can be rewritten as,

$$\Sigma_{i<j}\ V^{(1)}_{ALS}\ (i,j) = 2n\ \Sigma_i \hat{\ell}_i\cdot\hat{s}_i - 2\ \hat{L}\cdot\hat{S}, \qquad (5)$$

where $\hat{L} = \Sigma^n_{i=1}\hat{\ell}_i$, $\hat{S} = \frac{1}{2}\Sigma^n_{i=1}\hat{\sigma}_i = \Sigma^n_{i=1}\hat{s}_i$,

we can relate the ALS interaction to a one-body spin-orbit force. A matrix element of $V^{(1)}_{ALS}$ between n-particle states becomes $2n\ \Sigma_i < LSTJ \mid \ell_i\cdot s_i \mid L'S'TJ >$, if any one of

L, S, L′ or S′ is zero. Therefore $V^{(1)}_{ALS}$ behaves like a one-body spin-orbit force with strength propotional to 2n. This fact is closely related to the observation of Cole et al.[7] and Amiot et al.[8], who have shown that an increased one-body spin-orbit force works similarly as an ALS interaction in reproducing the band splitting in ^{24}Mg. Since the low-lying states of ^{24}Mg are predominantly S=0, their results are easily understood from the above discussion. On the other hand the second term in (5) has appreciable effects on the low-lying states in odd-odd and odd-A nuclei.

In order to study whether or not the ALS component is indispensable in reproducing band splittings, we have attempted a least-square fit to 161 experimental energies in A=18-24 nuclei without any ALS component, and determined a set of two-body matrix elements (we refer them to the NOALS interaction). The least-square fitting procedure is carried out employing the full $(sd)^n$ model space and imposing the condition

$$< L\ S=0\ T\ J \mid V \mid L'\ S=1\ T\ J > \ = 0.$$

It is remarkable that the NOALS interaction can satisfactorily reproduce the band splittings of these nuclei equally as effective interactions which contain large ALS components. The results for ^{24}Mg is shown in Fig.1 together with the calculated results with the Kuo[1], the modified Kuo[1] and the Chung-Wildenthal[3] interactions. The modified Kuo interaction differs from the Kuo interaction only in ALS part; its strength in the modified Kuo interaction is 4.9 times larger than that of the Kuo interaction. Fig. 1 shows that all of the modified Kuo, the Chung-Wildenthal and the NOALS interactions successfully reproduce the splitting between the K=0 and the K=2 bands. Therefore we recognize that the inclusion of ALS component in the effective interactions is not essential in reproducing the band splittings.

The common feature of the NOALS and the Chung-Wildenthal interactions is found in the diagonal matrix elements as shown in Fig.2. The diagonal matrix elements of the $d_{5/2}$-$d_{3/2}$ interactions are quite repulsive in these two interactions compared with those of the Kuo interaction. This property is not found systematically for the other matrix elements. Therefore the centroid energy $C(d_{5/2}d_{3/2})$ of the $d_{5/2}$-$d_{3/2}$ interactions which is generally defined as,

$$C(j_1j_2)=\Sigma_{TJ}(2T+1)(2J+1)\langle j_1j_2|V|j_1j_2\rangle_{TJ}\,/\,\Sigma_{TJ}(2T+1)(2J+1), \quad (6)$$

seems to be essential to reproduce band splittings.

The reason why the ALS component in the modified Kuo and the Chung-Wildenthal interactions give relatively repulsive centroid energy $C(d_{5/2}d_{3/2})$ can be found in the fact that the signs of the matrix elements $\langle d_{5/2}d_{3/2}|V_{ALS}|d_{5/2}d_{3/2}\rangle_{TJ}$ and $\langle d^2_{5/2}|V_{ALS}|d^2_{5/2}\rangle_{TJ}$ are opposite in these interactions. Using the spin-tensor decomposition technique[6], we find relations among the non-vanishing T=1 ALS diagonal matrix elements of the two-particle states in d-shell[9],

$$\langle d_{5/2}d_{3/2}|V_{ALS}|d_{5/2}d_{3/2}\rangle_{J=2}$$
$$= -\langle d^2_{5/2}|V_{ALS}|d^2_{5/2}\rangle_{J=2} - \langle d^2_{3/2}|V_{ALS}|d^2_{3/2}\rangle_{J=2}, \quad (7)$$

and

$$< d_{5/2}d_{3/2}|V_{ALS}|\ d_{5/2}d_{3/2}>_{J=4} = - < d^2_{5/2}|V_{ALS}|\ d^2_{5/2}>_{J=4} . \qquad (8)$$

From the above expressions, the diagonal matrix element $< d^2_{5/2}|V_{ALS}|\ d^2_{5/2}>_{J=2}$ for T=1 has an opposite sign to that of $< d_{5/2}d_{3/2}|V_{ALS}|\ d_{5/2}d_{3/2}>_{J=2}$, if the relation

$$< d^2_{5/2}|V_{ALS}|\ d^2_{5/2}>_{J=2} \times < d^2_{3/2}|V_{ALS}|\ d^2_{3/2}>_{J=2} > 0$$

is satisfied such as in the case of the Kuo interaction, or if the relation

$$| < d^2_{5/2}|V_{ALS}|\ d^2_{5/2}>_{J=2}|>| < d^2_{3/2}|V_{ALS}|\ d^2_{3/2}>_{J=2}|$$

is satisfied such as in the case of the Chung-Wildenthal interaction. The sign of $< d_{5/2}|V_{ALS}|\ d_{5/2}>_{J=4}$ is always opposite to that of $< d_{5/2}d_{3/2}|V_{ALS}|\ d_{5/2}d_{3/2}>_{J=4}$. We can find similar phase-relations for T=0, J=1 and 3 diagonal matrix elements.

Therefore if most of the matrix elements $< d^2_{5/2}|V_{ALS}|\ d^2_{5/2}>_{TJ}$ are negative, actually in the case of both the Kuo and the Chung-Wildenthal interactions, the ALS interactions induce a large difference between the centroid $C(d^2_{5/2})$ and $C(d_{5/2}d_{3/2})$. It can also be shown that the ALS interaction $V^{(1)}_{ALS}$ and $V^{(2)}_{ALS}$ in (2) and (3) can contribute to $C(d^2_{5/2})$ attractively and $C(d_{5/2}d_{3/2})$ repulsively.

On the other hand, in the NOALS interaction, the other parts of the effective interaction, such as tensor component, reproduces repulsive centroid energy $C(d_{5/2}d_{3/2})$. Details of discussions will be appeared in a separate paper.

In summary, a strongly repulsive centroid energy $C(\ d_{5/2}d_{3/2}\ )$ seems to be important in reproducing the rotational band splittings in light sd-shell nuclei. Although an enhancement of ALS component in the effective interactions can give such a repulsive centroid energy $C(\ d_{5/2}d_{3/2}\ )$, so to reproduce the observed band spectra, we have shown that it is also possible to obtain a repulsive centroid energy $C(\ d_{5/2}d_{3/2}\ )$ with only ordinary central, tensor and spin-orbit forces.

The author is grateful to K.-I. Kubo for his encouragement during the cource of this work. He would like to thank K. Ogawa, M.Nomura, A. Ikeda and K. Taruishi for their stimulating discussions and continuous interest in this work. He would express also his gratitude to M. Muraoka and T. Sebe for their hospitality. He is indebted to the financial support of Soryushi Syogakukai. The numerical calculations were carried out with FACOM M-180 II AD at I.N.S., University of Tokyo.

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Figure Captions

Fig. 1 Comparison of experimental and the full $(sd)^8$ shell-model calculated spectra of ^{24}Mg. Only the K=0 and the K=2 bands are presented. The single particle energies used with the four effective interactions were taken from the observed spectrum of ^{17}O, namely $\varepsilon(d_{5/2})=-4.15$ MeV, $\varepsilon(s_{1/2})=-3.28$MeV, $\varepsilon(d_{3/2})=0.93$MeV.

(a) the Kuo interaction; the arrows indicate the shiftings of the K=2 rotational band spectra calculated with the Kuo interaction,

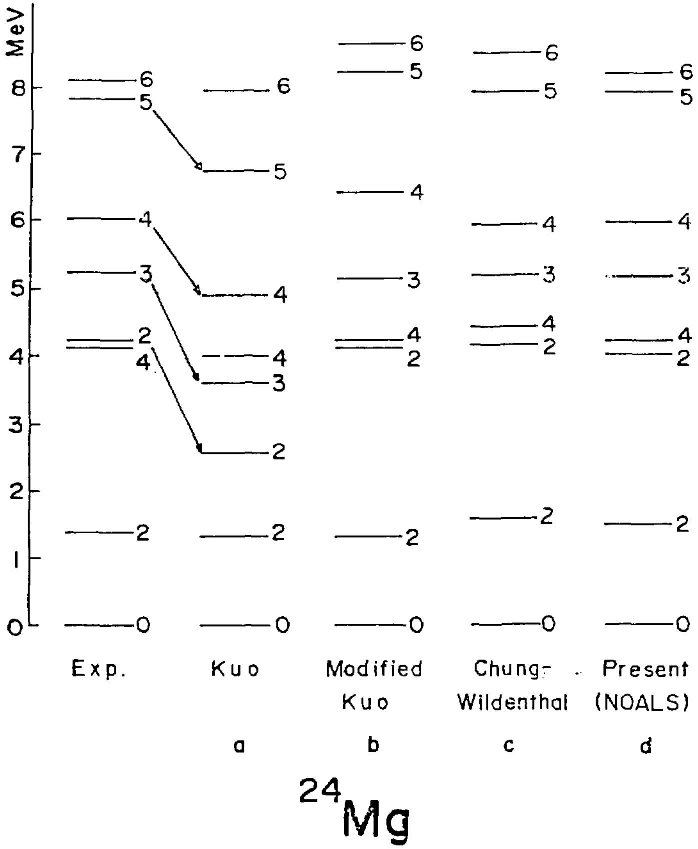
(b) the modified Kuo interaction,

(c) the Chung-Wildenthal interaction,

(d) present result(the NOALS interaction).

Fig.2 Comparison of diagonal two-body matrix elements of empirical and the Kuo interactions. The particle orbits are indicated by $5(d_{5/2})$, $1(s_{1/2})$ and $3(d_{3/2})$. The triplet of points for each matrix element show, from left to right, the values of the Chung-Wildenthal(△), the Kuo(•), and the NOALS(⊙) interactions respectively.

Fig. 1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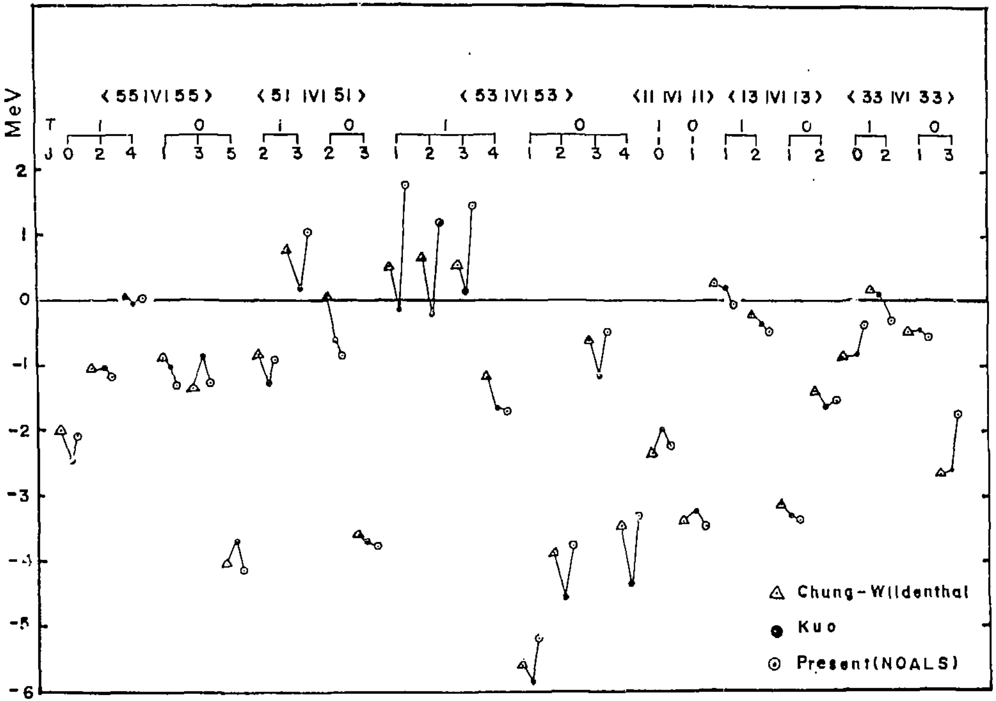
Exp.
Kuo
Modified Kuo
Chung- Wildenthal
Present (NOALS)
a
b
c
d
^{24}Mg

Fig. 2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T 1 0 1 0 1 0 1 0 1 0 1 0
J 0 2 4 1 3 5 2 3 2 3 1 2 3 4 1 2 3 4 0 1 1 2 1 2 0 2 1 3
△ Chung-Wildenthal
● Kuo
⊙ Present(NOALS)