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Abstract

The effective nucleon mass of the Equidistant Multi-Layer Structure (EMULS) is discussed self-consistently. In the density region where the Fermi gas state in nuclear matter is unstable against the density fluctuation, the EMULS gives lower binding energy. It is, however, shown that such a structure with an ordinary nucleon mass collapses due to too strong attraction. We point out that such a collapse can be avoided by taking account of an effective nucleon mass affected by the localization of nucleons.

I. Introduction

Structures of the nuclear matter in which scalar mesons condense with momentum k is discussed. We assume that the nuclear matter consists of equal number of protons and neutrons interacting through neutral scalar mesons and neutral vector mesons. The Lagrangian density is given by

$$
\begin{aligned}
L = & \bar{\psi}(i\gamma^{\mu}\partial_{\mu}-M)\psi - \frac{1}{2}(\mu^2\phi^2-\partial_{\mu}\phi\partial^{\mu}\phi) \\
& - \frac{1}{4}(\partial_{\mu}V_{\nu}-\partial_{\nu}V_{\mu})\cdot(\partial^{\mu}V^{\nu}-\partial^{\nu}V^{\mu}) + \frac{1}{2}m_V^2 V_{\mu}V^{\mu} \\
& + g_S\bar{\psi}\phi\psi - g_V\bar{\psi}\gamma^{\mu}V_{\mu}\psi ,
\end{aligned}
\tag{1.1}
$$

where ψ is the nucleon field with mass $M = 940$ MeV, ϕ is the neutral scalar meson field with mass $\mu = 500$ MeV and V_{μ} is the neutral vector meson field with mass $m_V = 783$ MeV. The coupling constant between nucleons and scalar mesons is given by g_S, and that between nucleons and vector mesons is given by g_V.

Assuming the Fermi gas motion for nucleons in the nuclear matter, it has been shown that the system satisfies the saturation property[1)] with the effective nucleon mass

$$
M^* = M - g_S\langle\phi\rangle , \tag{1.2}
$$

where $\langle\phi\rangle$ is the expectation value of the neutral scalar meson sandwiched by the Fermi gas state and is called the classical field of the scalar meson. The non-relativistic Hamiltonian is given by[2)]

$$H = \sum_{\vec{p}} (M^* + \frac{\vec{p}^2}{2M^*}) C^\dagger_{\vec{p}} C_{\vec{p}} - \sum_{\vec{p},\vec{q},k\neq 0} \frac{1}{2V} (\frac{g_S^2}{\vec{k}^2+\mu^2} - \frac{g_V^2}{\vec{k}^2+m_V^2})$$

$$\times C^\dagger_{\vec{p}+\vec{k}} C^\dagger_{\vec{q}-\vec{k}} C_{\vec{q}} C_{\vec{p}} , \quad (1.3)$$

where $C^\dagger_{\vec{p}}$ and $C_{\vec{p}}$ are the creation and annihilation operators of nucleon with momentum $\vec{p}$. The binding energy per nucleon of the nuclear matter in the Fermi gas state is given by

$$E_{FG}/N = \frac{3}{10} \frac{p_F^2}{M^*} - \frac{1}{2} (\frac{g_S^2}{\mu^2} - \frac{g_V^2}{m_V^2})\rho$$

$$+ \frac{1}{2VN} \sum_{\vec{p},\vec{q}} \{ \frac{g_S^2}{\mu^2+(\vec{p}-\vec{q})^2} - \frac{g_V^2}{m_V^2+(\vec{p}-\vec{q})^2} \} , \quad (1.4)$$

where the nucleon density ρ is related with the Fermi momentum p_F as

$$\rho = \frac{2}{3\pi^2} p_F^3 . \quad (1.5)$$

The coupling constants are determined so as to make the binding energy -15.8 MeV at the normal density p_F = 275 MeV/c (see Fig. 1), i.e. we choose $g_S = 9.0$ and $g_V = 11.8$.

For the Hamiltonian (1.3) we have pointed out that the Fermi gas state becomes unstable against the density fluctuation,[2] and the condition of instability is geven by[3]

$$1 \leq \frac{2M^* p_F}{\pi^2} (\frac{g_S^2}{k^2+\mu^2} - \frac{g_V^2}{k^2+m_V^2}) \{ \frac{1}{2} + \frac{4-(k/p_F)^2}{8k/p_F} \ln \left| \frac{2p_F+k}{2p_F-k} \right| \} . \quad (1.6)$$

With the above coupling constants the Fermi gas state is unstable in the wide range of density including the normal density (Fig. 2). In such an unstable region various structures of the nuclear matter are proposed instead of the Fermi gas state. The Equidistant Multi-Layer Structure (EMULS) is known as the most favorable structure from the comparison of the binding energy.[2] In this structure mesons condense with momentum k related with the interval of layers. Also in case of pion condensation some resembling equidistant structures were proposed.[4),5)]

It is shown in Sec. II that arranging nucleons equidistantly leads to a catastrophe, i.e. such a structure with the ordinary nucleon mass collapses due to too strong attraction. However, we will prove in Sec.III that the difficulty can be overcome by introducing the effective nucleon mass taking the space dependence of meson fields into consideration.

II. Collapse of Equidistant Structure

In the region where the Fermi gas state is unstable against the density fluctuation, it is favorable to arrange nucleons equidistantly.[2] Therefore, we start our discussion with the wave function of the EMULS

$$\Psi_{n\vec{p}}(\vec{r}) = \frac{1}{(\sqrt{\pi}aL^2)^{1/2}} e^{i\vec{p}\cdot\vec{r}} e^{-\frac{(z-nD)^2}{2a^2}} , \qquad (2.1)$$

where $\vec{p}$ and $\vec{r}$ are two-dimensional vectors perpendicular to the z-direction. Nucleons are localized equidistantly with the

Gaussian distribution along the z-direction, and move freely as the Fermi gas in the x-y plane. The interval of layers along the z-direction is D, the half width of the Gaussian distribution is a and $n = 0, \pm 1, \pm 2, \cdots\cdots$. The momentum of condensed meson k is related with the layer interval D as

$$kD = 2\pi. \tag{2.2}$$

The binding energy per nucleon of the system can be calculated with this wave function.

$$\begin{aligned} E/N = & \frac{p_{Fr}^2}{4M} + \frac{1}{4Ma^2} \\ & - \frac{1}{2} \sum_{n=-\infty}^{+\infty} \left\{ \frac{g_S^2}{\mu^2 + (nk)^2} - \frac{g_V^2}{m_V^2 + (nk)^2} \right\} \rho e^{-\frac{a^2}{2}(nk)^2} \\ & + \frac{N_Z^2}{2VN} \sum_{n=-\infty}^{\infty} \sum_{\vec{p},\vec{q}} \left\{ \frac{g_S^2}{\mu^2 + (\vec{p}-\vec{q})^2 + (nk)^2} - \frac{g_V^2}{m_V^2 + (\vec{p}-\vec{q}) + (nk)^2} \right\} \\ & \times e^{-\frac{a^2}{2}(nk)^2} \end{aligned} \tag{2.3}$$

where

$$\rho = \frac{p_{Fr}^2}{\pi D} \tag{2.4}$$

with the Fermi momentum p_{Fr} in the x-y plane. The last term of the right-hand side of Eq. (2.3) is so-called exchange energy, and the magnitude of this term is smaller than one-fourth of the direct energy (the third term) because of the saturation of spin and isospin degrees of freedom. Thus we neglect the exchange term in the following discussion.

For a given density, variational parameters are a and k, while D and p_{Fr} are written with the condensation momentum k. If we assume that ak is not so small, the infinite sum in the third term of Eq. (2.3) is well approximated only by a few terms around n = 0. Then it is easy to calculate the minimum energy about a and k.

It seems, however, that very small ak is more favorable. To see this we must estimate the complete sum of the third term. We replace the summation by the integration approximately as

$$\sum_{n=-\infty}^{+\infty} \frac{e^{-\frac{a^2}{2}(nk)^2}}{\mu^2+(nk)^2} = \frac{1}{k}\int_{-\infty}^{+\infty} dx\, \frac{e^{-\frac{a^2x^2}{2}}}{\mu^2+x^2}$$

$$= \frac{2\sqrt{\pi}}{k}\, e^{\frac{a^2\mu^2}{2}}\, \mathrm{Erfc}\left(\frac{a\mu}{\sqrt{2}}\right) , \qquad (2.5)$$

where Erfc(x) is the Gaussian error function;

$$\mathrm{Erfc}\left(\frac{a\mu}{\sqrt{2}}\right) = \frac{\sqrt{\pi}}{2} - \frac{a\mu}{\sqrt{2}}\left\{1 - \frac{1}{3}\,\frac{a^2\mu^2}{2} + \frac{1}{5\cdot 2}\left(\frac{a^2\mu^2}{2}\right)^2 - \cdots\right\} . \qquad (2.6)$$

If $a\mu \lesssim \sqrt{2}$, the binding energy is approximately written by

$$E/N = \frac{\pi^2}{2M}\,\frac{\rho}{k} + \frac{1}{4Ma^2}$$
$$- \frac{\pi}{2}\left(\frac{g_S^2}{\mu} - \frac{g_V^2}{m_V}\right)\frac{\rho}{k} + \sqrt{\frac{\pi}{2}}\, a(g_S^2-g_V^2)\frac{\rho}{k}$$
$$+ \cdots\cdots . \qquad (2.7)$$

Using the previous values as the coupling constants, for 2.8×10^{-3} MeV^{-1} > a > 4.8×10^{-4} MeV^{-1} we obtain

$$\frac{\pi^2}{2M} - \frac{\pi}{2}\left(\frac{g_S^2}{\mu} - \frac{g_V^2}{m_V}\right) + \sqrt{\frac{\pi}{2}}\, a(g_S^2 - g_V^2) < 0 \tag{2.8}$$

This condition will be satisfied whenever Eq. (2.7) is a good approximation and the system is in a bound state. In this case it is clear that the smaller k is the smaller the binding energy is, and for $k \to 0$ E/N approaches negative infinity. In other words, the distance between layers becomes infinitely large, and the system encounters a catastrophe.

Since the parameters a and k are determined to make the binding energy minimum, the above mentioned catastrophe is sure to occur. In general when we use attractive interactions with finite magnitude for the limiting momentum $k \to 0$, such a catastrophe must occur for equidistant structures.

In the nuclear or neutron star matter interacting through pions, the equidistant alternating spin or isospin layer (ALS) structure is proposed to give lower binding energy than the Fermi gas state.[5] The p-wave pion-nucleon interaction vanishes for $k \to 0$, thus in the ALS structure induced by the pion-nucleon interaction the above catastrophe does not occur.

III. Effective Nucleon Mass in the EMULS

Ordinary Fermi gas calculations with attractive interactions do not satisfy the saturation property and have a catastrophe (collapse) that the binding energy approaches negative infinity as the density ρ (or the Fermi momentum p_F) becomes large.

This catastrophe is avoided by introducing the effective nucleon mass involving contributions of scalar meson fields with $k = 0$.[1)]

The catastrophe of the EMULS occurs for $k \to 0$, i.e. $p_{Fr} \to \infty$. Namely we can say that this catastrophe is the collapse of the two-dimensional Fermi gas state. Thus it is expected that the catastrophe can be avoided by introducing a new effective mass. In the case where the Fermi gas state is assumed for nucleons, mesons are distributed uniformly, but the new effective nucleon mass must include contributions of meson fields affected by localized nucleon distribution.

We start with the *non-relativistic equations of motion:*

$$(-\frac{\nabla^2}{2M} - g_S\phi)\psi = \varepsilon\psi \tag{3.1a}$$

$$(-\nabla^2 + \mu^2)\phi = g_S\psi^+\psi \tag{3.1b}$$

Time dependence of scalar meson fields is neglected, because we are interested only in effects of space distribution of meson fields. The vector meson field is also neglected because it contributes to energy of the system but not to effective nucleon mass.

Substituting the wave function of the EMULS (2.1) for ψ of Eq. (3.1b) and making Fourier transformation we obtain from eq. (3.1b)

$$(k^2+\mu^2)\phi_k = g_S\rho e^{-\frac{a^2k^2}{4}} \sum_{n=-\infty}^{+\infty} \delta(k - \frac{2n\pi}{D}) \ . \tag{3.2}$$

Then the meson field is given by

$$\phi(\vec{r}) = g_S\rho \sum_{n=-\infty}^{+\infty} \frac{e^{-\left(\frac{n\pi a}{D}\right)^2}}{\mu^2+(2n\pi/D)^2} \cos \frac{2n\pi}{D} z \ . \tag{3.3}$$

The meson field is uniform in the x-y plane, but along the z-direction it has the distribution corresponding to the localization of nucleons as described in Eq. (3.3).

In order to check the self-consistency, let us substitute the meson field (3.3) in Eq. (3.1a). Then it is shown that if we can approximate $\cos \frac{2n\pi}{D} z$ in Eq. (3.3) by $\left(1-2\frac{n^2\pi^2}{D^2} z^2\right)$ the nucleon wave function is obtained with the Gaussian type, and the self-consistency relation is given by

$$a = \left\{M^* g_S^2 \rho \sum_{n=-\infty}^{+\infty} \frac{\left(2\frac{n\pi}{D}\right)^2}{\mu^2+(2n\pi/D)^2} e^{-\left(\frac{n\pi a}{D}\right)^2}\right\}^{-1/4} . \tag{3.4}$$

We consider the effects of the space dependence of the meson field to the effective nucleon mass. Nucleons have the Gaussian distribution, and near the center of the distribution meson fields have large magnitude and nucleons are much affected by meson fields. Thus we average over the scalar meson field weighted with nucleon density distribution, i.e. the mean meson field is estimated by

$$\begin{aligned}\langle\phi\rangle &\equiv \int d^3 r \phi(\vec{r}) \psi^+(\vec{r}) \psi(\vec{r}) \\ &= g_S\rho \sum_{n=-\infty}^{+\infty} \frac{e^{-2\left(\frac{n\pi a}{D}\right)^2}}{\mu^2+(2n\pi/D)^2} \ .\end{aligned} \tag{3.5}$$

Using the $\langle\phi\rangle$, the effective nucleon mass is defined by

$$M^* \equiv M - g_S\langle\phi\rangle \qquad (3.6)$$

The nucleon mass M in Eq. (2.3) must be replaced by the new effective mass M^*.

In the case that a is very large or D is very small, only the term of $n = 0$ contributes to the mean field (3.5). This contribution, of course, corresponds to that of the Fermi gas state.

The catastrophe we consider occurs for $D \to \infty$. For $D \to \infty$,

$$\begin{aligned}\langle\phi\rangle &\sim g_S\rho \lim_{N\to\infty} \frac{2N+1}{\mu^2} \\ &\to \infty\ . \end{aligned} \qquad (3.7)$$

Thus the effective mass M^* becomes naught:

$$\begin{aligned} M^* &= M - g\langle\phi\rangle \\ &\to 0 \end{aligned} \qquad (3.8)$$

It is clear that this effect makes the kinetic energy terms infinitely large so that D cannot become so large. That is, the catastrophe due to $D \to \infty$ might be avoided by the effective mass.

Actually, numerical calculations show that finite values of a and k (or D) give the binding energy minimum with this effective nucleon mass M* and the catastrophe discussed in the last section does not occur.

Though it seems that M* becomes negative if $\langle\phi\rangle \to \infty$, it is because of the inadequacy of non-relativistic approximation.[3)]

IV. Conclusion

Walecka showed that it is necessary to introduce the effective nucleon mass including classical field of scalar mesons in order to have the saturation property of the nuclear matter.[1)] In this paper it was pointed out that considering meson fields affected by nucleon distribution is very important. Though it seems that equidistant structures like the EMULS which are suggested to be most favorable for the nuclear matter consisting of meson condensation state would collapse, this catastrophe can be avoided by considering the space dependence of the scalar meson field. As nucleons are sources of mesons, we must consider such space dependence of meson fields whenever nucleons have space dependence.

Numerical calculations including contributions of vector mesons and exchange energies are shown in Fig. 1. We see that at low density the EMULS has lower binding energy than that of the Fermi gas state at the normal density (p_F = 275 MeV). At near the normal density though the Fermi gas state is unstable, the EMULS does not give lower energy. In this region it is shown that the two-wave approach on the basis of the Fermi sphere (TWAS) gives lower binding energy (See ref. 2). That is, since large localization of nucleons gives too large kinetic energy, the EMULS is not favorable in such a region.

Similarly we can imagine two-dimensional or three-dimensional localization of nucleons, but it is shown that each of them does not give lower binding energy than the EMULS or the Fermi gas state. We must notice that there exists same dangerous catastrophe if we treat them without considering the space dependence of scalar meson field.

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References

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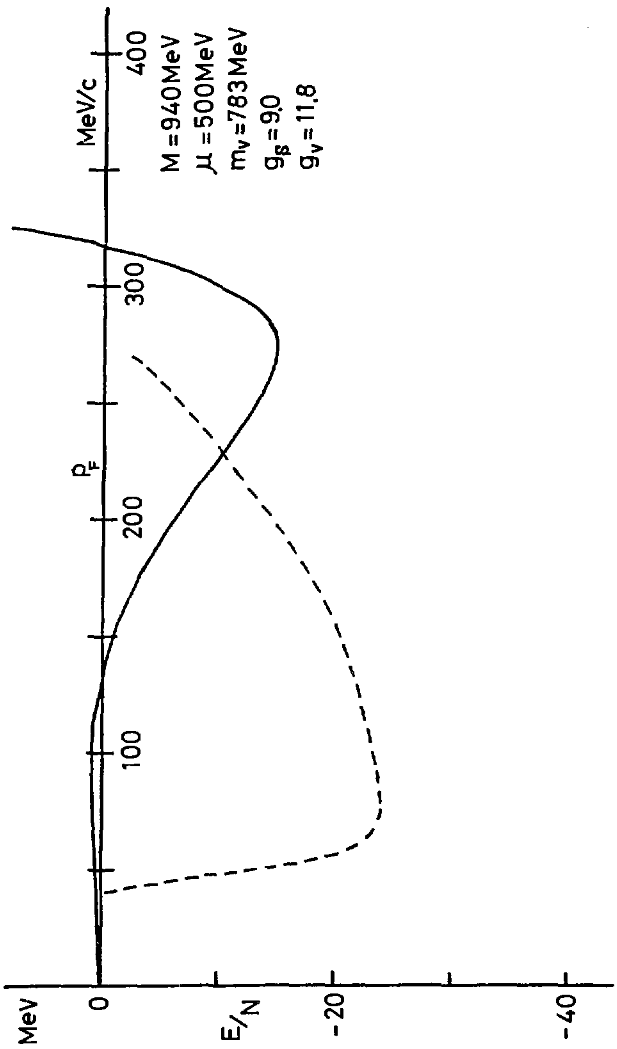
Figure Captions

Fig. 1. Binding energies per nucleon in the nuclear matter. The solid line represents the estimation for the Fermi gas state, and the dashed line represents one for the EMULS.

Fig. 2. Instability of the Fermi gas state. The Fermi gas state is unstable in the shadowed region.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E/N
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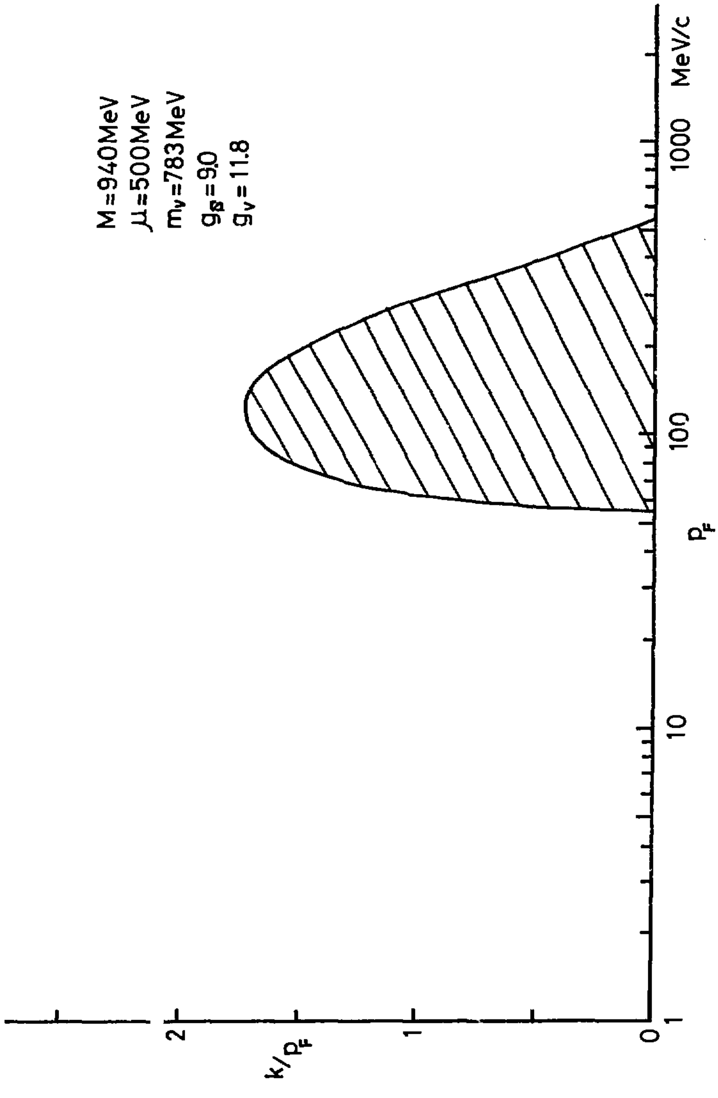
-40
100
200
300
400
P_F
MeV/c
M = 940 MeV
μ = 500 MeV
m_v = 783 MeV
g_s = 9.0
g_v = 11.8

Fig.1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m_v=783MeV
g_s=9.0
g_v=11.8
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1
2
1
10
100
1000
MeV/c
P_F

Fig. 2