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HIGH ENERGY MULTI-GLUON EXCHANGE AMPLITUDES

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HIGH ENERGY MULTI-GLUON EXCHANGE AMPLITUDES *

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ABSTRACT

We examine perturbative high energy n-gluon exchange amplitudes calculated in the Coulomb gauge. If n exceeds the minimum required by the t-channel quantum numbers, such amplitudes are non-leading in $\ln s$. We derive a closed system of coupled integral equations for the corresponding two-particle n-gluon vertices, obtained by summing the leading powers of $\ln(N_\mu p^\mu)$, where p^μ is the incident momentum and N^μ the gauge-defining vector. Our equations are infra-red finite, provided the external particles are colour singlets.

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I. Introduction

The problem of high energy small momentum transfer scattering in non-abelian gauge theories has received much attention during the recent years [1)-10)]. Essentially all this work has been done in the framework of the weak coupling perturbation theory, in the leading $\ln s$ approximation (LLA), where all powers of $g^2 \ln s$ are summed. In the deep inelastic scattering this corresponds to summing the powers of $g^2 \ln \frac{1}{x}$, where x is the Bjorken variable.

The main results obtained in the LLA and the motivation for the present work can easily be understood from the Feynman diagram calculations in the Coulomb gauge [5),10)]. Consider for definiteness (quasi) elastic scattering of two objects A,A' into B,B' in a reference frame such that the large momenta p_A and p_B are directed along the positive and negative z-axis respectively. In the LLA the "partons" (quarks and gluons) represented by internal lines in the Feynman diagrams can be meaningfully classified as "right-movers" and "left-movers" (i.e. with large momenta along p_A and p_B) or "wees" (with finite momenta). Then, in a massless unbroken gauge theory, we choose the Coulomb gauge with the gauge-defining vector $N^\mu = (1,0,0,0)$ in our reference frame. The most remarkable feature of this gauge is that, in contrast with the covariant gauges, the wee gluons exchanged between the right- and left-movers do not give any logarithms of energy. The n-gluon exchange amplitude is therefore proportional to sg^{2n} (function of $(g^2 \ln s)$). All the non-trivial $\ln s$-dependence comes from the integration over the longitudinal momenta of the right- and left-movers.

More precisely, in the LLA, the n-gluon exchange amplitude reads [10)]

$$T_n = -2is\frac{(-i)^n}{n!}(2\pi)^{-2(n-1)} \int \prod_{i=1}^{n} \frac{d^2 q_i}{\vec{q}_i^{\,2}}\, \delta^2\Big(\sum \vec{q}_i - \vec{\Delta}\Big) \cdot$$

$$\cdot f^{AA'(n)}_{a_1 \ldots a_n}(p_A^+; \vec{q}_1, \ldots, \vec{q}_n)\; f^{BB'(n)}_{a_1 \ldots a_n}(p_B^-; \vec{q}_1, \ldots, \vec{q}_n). \qquad (1)$$

Here $a_1, \ldots, a_n$ are the colour indices of the exchanged gluons, and $\vec{q}_1, \ldots, \vec{q}_n$ are their transverse momenta directed from the system A to B and ordered from the initial to the final state. The quantities f are integrals of the (n-1)-fold multiple discontinuities (in the mass variables) of the two-particle n-gluon vertex functions Γ, e.g.

$$f^{AA'(n)}_{a_1\ldots a_n}(p_A^+;\vec{q}_1,\ldots,\vec{q}_n) = -i(2\pi i)^{-(n-1)}(2p_A^+)^{-n}\cdot \int_0^{mp_A^+} d\sigma_1\ldots d\sigma_{n-1}\, \mathrm{disc}_{\sigma_1}\ldots \mathrm{disc}_{\sigma_{n-1}}\Gamma^{AA'+\ldots+}_{a_1\ldots a_n}(p_A;q_1,\ldots,q_n), \qquad (2)$$

where $\sigma_i = (p_A-q_1-\ldots-q_i)^2 \simeq -2p_A^+(q_1^- + \ldots + q_i^-)$ and the indices "+" in Γ are the Lorentz indices (we use the null-plane variables $p^{\pm} = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$). The upper cut-off on σ_i (m is an arbitrary mass scale, irrelevant in the LLA) is equivalent to the lower cut-off on the momenta k^+ of the right-moving partons.

In a recent paper [10] we derived integral equations for the two-particle n-gluon vertices $f^{(n)}$, valid when n is the minimum number of gluons required by the quantum numbers in the t-channel (e.g. for the SU(2) gauge group such is the case of "isospin" I=n exchange). We found that the amplitudes T_n are Regge-behaved as a result of exponentiation of the leading IR divergences due to the exchange of colour.

In the vacuum quantum number exchange (Pomeron) channel the leading contribution comes from n = 2 . It is well known [2,4,6-8] that in this approximation the total cross section grows as a power of energy, in violation of unitarity. In our physical picture the reason for that is clear: the two-gluon exchange contribution is roughly (exactly when the gluons' transverse momenta $\vec{q}_1,\vec{q}_2$ are very large) proportional to the product of the average multiplicities of (virtual) right-moving and left-moving partons; and in the LLA these multiplicities grow as powers of the energy (they are in fact the multiplicities of gluon jets in the double-logarithmic approximation). From our point of view, if the number of partons, i.e. "constituents" in the systems A and B is very large, the two-gluon exchange approximation becomes inadequate. A step towards restoring unitarity would be thus to include multiple scattering, i.e. multiple gluon exchange.

A partial solution to this problem is given in the present paper: we study the amplitudes T_n with n exceeding the minimum required by the t-channel quantum numbers. We calculate, however, the vertices $f^{(n)}$ keeping only terms $f^{AA'(n)} \sim g^n$ (function of $(g^2 \ell n p_A^+)$); it is in this sense that we shall use the term LLA in the following. We derive a closed system of coupled integral equations for the vertices $f^{(2)}, f^{(3)}, \ldots, f^{(n)}$,

and then discuss their IR properties.

II. The equations

Let us consider for definiteness the simplest non-trivial example of the n=3 vertex in a channel where also the 2-gluon exchange is allowed. Using the methods of ref. 10 we obtain the equation for $f^{AA'(3)} \equiv f^{(3)}$ represented in Fig. 1, and interpreted according to the diagrammatic rules given in [10]. The equation applies to the Mellin transform

$$f^{(n)}(E;\vec{q}_1,\ldots,\vec{q}_n) = -\int_m^{\infty} dp_A^+ (p_A^+)^{E-1} f^{(n)}(p_A^+;\vec{q}_1,\ldots,\vec{q}_n)$$

(E is related to the angular momentum by j = 1-E) and can be written, schematically, as

$$E f^{(3)} = \Phi^{(3)} + \sum_i^3 \omega(i) f^{(3)} + \sum_{i<j}^3 V^{(2,2)}(i,j) f^{(3)} + V^{(3,2)}(1,2,3) f^{(2)} \equiv$$
$$\equiv \Phi^{(3)} + \Omega f^{(3)} + V^{(2,2)} f^{(3)} + V^{(3,2)} f^{(2)}. \qquad (3)$$

The terms explicitely drawn in Fig. 1 involve $\Phi^{(3)}, \omega(1), V^{(2,2)}(1,2)$ and $V^{(3,2)}(1,2,3)$. The term $\Phi^{(3)}$ (Fig. 1 a) is the lowest order inhomogeneous term; $\omega(1)$ (Fig. 1 b,c) is due to the virtual gluon loop insertions on the line $\vec{q}_1$ and can be interpreted as the lowest order Reggeon (= reggeized gluon) energy; $V^{(2,2)}(1,2)$ (Fig. 1 d,e,f,g) is the $2 \to 2$ gluon vertex involving the gluons $\vec{q}_1$ and $\vec{q}_2$; $V^{(3,2)}(1,2,3)$ (Fig. 1 h,i,j,k) is the $2 \to 3$ gluon vertex involving the lines $\vec{q}_1, \vec{q}_2, \vec{q}_3$. The terms including V should be understood as convolutions with gluon propagators; thus eq.(3) written in full is

$$E f^{(3)}_{a_1a_2a_3}(E;\vec{q}_1,\vec{q}_2,\vec{q}_3) = \Phi^{(3)}_{a_1a_2a_3}(\vec{q}_1,\vec{q}_2,\vec{q}_3) + \omega(\vec{q}_1) f^{(3)}_{a_1a_2a_3}(E;\vec{q}_1,\vec{q}_2,\vec{q}_3) + \ldots$$
$$+ (2\pi)^{-3}\int \frac{d^2k_1}{\vec{k}_1^2}\frac{d^2k_2}{\vec{k}_2^2}\,\delta^2(\vec{q}_1+\vec{q}_2-\vec{k}_1-\vec{k}_2)\, V^{(2,2)b_1b_2}_{a_1a_2}(\vec{q}_1,\vec{q}_2;\vec{k}_1,\vec{k}_2)\, f^{(3)}_{b_1b_2a_3}(E;\vec{k}_1,\vec{k}_2,\vec{q}_3) + \ldots$$
$$+ (2\pi)^{-3}\int \frac{d^2k_1}{\vec{k}_1^2}\frac{d^2k_2}{\vec{k}_2^2}\,\delta^2(\vec{q}_1+\vec{q}_2+\vec{q}_3-\vec{k}_1-\vec{k}_2)\, V^{(3,2)b_1b_2}_{a_1a_2a_3}(\vec{q}_1,\vec{q}_2,\vec{q}_3;\vec{k}_1,\vec{k}_2)\, f^{(2)}_{b_1b_2}(E;\vec{k}_1,\vec{k}_2). \qquad (3')$$

The explicit expressions for ω and V are given below.

The novel feature of eq.(3) is the appearance of the transition from the 2-gluon state to the 3-gluon state. Using the well-known equation for the 2-gluon system [4],

$$E f^{(2)} = \Phi^{(2)} + \Omega f^{(2)} + V^{(2,2)} f^{(2)},$$

we obtain the solution for $f^{(3)}$ in the form

$$f^{(3)} = \left[E-\Omega-V^{(2,2)}\right]^{-1} \left\{\Phi^{(3)} + V^{(3,2)} \left[E-\Omega-V^{(2,2)}\right]^{-1} \Phi^{(2)}\right\}.$$

The first contribution is $\Phi^{(3)} \sim g^3$ followed by $2 \rightarrow 2$ gluon interactions $V^{(2,2)} \sim g^2$. The second is $\Phi^{(2)} \sim g^2$ followed by $V^{(2,2)}$ interactions, then the transition $V^{(3,2)} \sim g^3$ to the 3-gluon state, and then again $V^{(2,2)}$ interactions. If we expand in powers of g^2/E, both contributions are of order g^3/E ·(function of (g^2/E)).

In the general case of the n-gluon system we obtain the integral equation

$$\begin{aligned} E f^{(n)} &= \Phi^{(n)} + \sum_i^m \omega(i) f^{(n)} + \sum_{i_1<i_2}^m V^{(2,2)}(i_1,i_2) f^{(n)} + \\ &+ \sum_{i_1<i_2<i_3}^m V^{(3,2)}(i_1,i_2,i_3) f^{(n-1)} + \ldots + V^{(n,2)}(1,2,\ldots,n) f^{(2)} \equiv \\ &\equiv \Phi^{(n)} + \Omega f^{(n)} + V^{(2,2)} f^{(n)} + V^{(3,2)} f^{(n-1)} + \ldots + V^{(n,2)} f^{(2)}. \end{aligned} \quad (4)$$

The $2 \rightarrow n$ vertex $V^{(n,2)}$ is represented in Fig. 2 and, according to the rules of ref. 10, reads

$$\begin{aligned} \frac{1}{\vec{k}_1^2}\frac{1}{\vec{k}_2^2} V^{(m,2)\,b_1 b_2}_{a_1\ldots a_m}(\vec{q}_1,\ldots,\vec{q}_m;\vec{k}_1,\vec{k}_2) &= R^{b_1 b_2}_{a_1\ldots a_m} \cdot \\ \cdot g^m \Bigg\{ 2\frac{\vec{k}_1\cdot\vec{k}_2}{\vec{k}_1^2\vec{k}_2^2} - 2\frac{\vec{k}_1\cdot(\vec{k}_2-\vec{q}_m)}{\vec{k}_1^2(\vec{k}_2-\vec{q}_m)^2} - 2\frac{(\vec{k}_1-\vec{q}_1)\cdot\vec{k}_2}{(\vec{k}_1-\vec{q}_1)^2\vec{k}_2^2} &+ 2\frac{(\vec{k}_1-\vec{q}_1)\cdot(\vec{k}_2-\vec{q}_m)}{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_2-\vec{q}_m)^2}\Bigg\} \end{aligned} \quad (5)$$

where R is the group factor built from the structure constants of the colour group

$$R^{b_1 b_2}_{a_1\ldots a_m} = (-i)^m C_{b_1 a_1 c_1} C_{c_1 a_2 c_2} \cdots C_{c_{m-1} a_m b_2}.$$

The four terms in eq.(5) correspond to the four terms in Fig. 2. We shall write, accordingly,

$$V = V_\alpha + V_\beta + V_\gamma + V_\delta .$$

The Reggeon energy is given by

$$\omega(\vec{q}) = N_c g^2 (2\pi)^{-3} \int d^2k \left\{ \frac{\vec{k}\cdot(\vec{q}-\vec{k})}{\vec{k}^2(\vec{q}-\vec{k})^2} + \frac{1}{\vec{k}^2} \right\} \quad (6)$$

where N_c is the number of colours. The two terms in the brackets correspond to Fig. 1 b and c. We shall distinguish these two contributions by writing

$$\omega = \omega_\alpha + \omega_\beta .$$

III. IR finiteness

In general there can be in eq.(4) infra-red (IR) divergences caused by vanishing of the denominators $\vec{k}_1^{\,2}$ or $(\vec{k}_1-\vec{q}_1)^2$; (we can regularise eq. (4) by going to $d>2$ transverse dimensions). In the case of $n=2$ [6] and $n=3$ in the odd charge conjugation ($C=-1$) state [9] it was shown that the divergences are absent, provided the external particles are colour singlets, e.g. quark-antiquark bound states. In these cases each coupling of a gluon $\vec{q}_i$ changes the group representation to which the fast-moving system A belongs; e.g. for $n=2$ A changes from a singlet to the adjoint representation and back to a singlet. From the orthogonality (in the colour space) of these states it follows that the lowest order term Φ vanishes when any of the momenta $\vec{q}_i$ vanishes; this is sufficient to show the IR finiteness.

The case of non-leading contributions with larger n is more complicated. Not every gluon emission changes the representation and in general $\Phi^{(n)}$ does not vanish when $\vec{q}_i \rightarrow 0$. Instead, as a consequence of the Ward identities, $\Phi^{(n)}$ can be expressed in terms of $\Phi^{(n-1)}$:

$$\Phi^{(n)}_{a_1\ldots a_n}(\vec{q}_1,\ldots,\vec{q}_n) \xrightarrow[(\vec{q}_i\rightarrow 0)]{} -ig\sum_{l=1}^{i-1} C_{a_l a_i b_l}\, \Phi^{(n-1)}_{a_1\ldots b_l\ldots \not{a}_i\ldots a_n}(\vec{q}_1,\ldots \not{\vec{q}}_i,\ldots,\vec{q}_n), \quad (7)$$

if A is a colour singlet and

$$\Phi^{(n)}_{a_1\ldots a_n}(\vec q_1,\ldots,\vec q_n) \xrightarrow[(\vec q_i\to 0)]{} -ig\sum_{r=i+1}^{n} C_{a_i a_r b_r}\,\Phi^{(n-1)}_{a_1\ldots \not{a}_i\ldots b_r\ldots a_n}(\vec q_1,\ldots,\not{\vec q}_i,\ldots,\vec q_n), \tag{7'}$$

if A' is a colour singlet. Here the slash indicates that a given index or argument is missing. In terms of diagrams, these equations say that a soft line can be removed from $\Phi^{(n)}$ and attached to the lines $\vec q_\ell\,(\ell<i)$ to the left of its original position (eq. (7)) or to the lines $\vec q_r\,(r>i)$ to the right of it (eq. (7')). In particular, the RHS's of eqs. (7) and (7') vanish for $n=2$, and for $n=3$ after projecting onto the $C=-1$ state, i.e. contracting the colour indices with the symmetric $d_{a_1a_2a_3}$ symbol.

Another relation for $\Phi^{(n)}$, valid independently of whether or not the external particles are colour singlets, is

$$-iC_{a_i a a_{i+1}}\,\Phi^{(n)}_{a_1\ldots a_n}(\vec q_1,\ldots,\vec q_n) \xrightarrow[(\vec q_i,\vec q_{i+1}\to 0)]{} \frac{1}{2}N_c g\,\Phi^{(n-1)}_{a_1\ldots a_{i-1}a a_{i+2}\ldots a_n}(\vec q_1,\ldots,\vec q_{i-1},\vec 0,\vec q_{i+2},\ldots,\vec q_n). \tag{8}$$

Indeed, it follows from the antisymmetry of C_{abc} that both soft lines, $\vec q_i$ and $\vec q_{i+1}$, must couple to the same quark or gluon line. Then the use of the triangle identity reduces $\Phi^{(n)}$ to $\Phi^{(n-1)}$ on the RHS of eq.(8).

Imagine now solving eq.(4) iteratively, and suppose that its solution to a certain order in g^2/E, say $\varphi^{(n)}$, satisfies eqs. (7), (7') and (8). It can be shown that the next order correction

$$\tilde\varphi^{(n)} = \frac{1}{E}\left[\Omega\varphi^{(n)} + V^{(2,2)}\varphi^{(n)} + V^{(3,2)}\varphi^{(n-1)} + \ldots + V^{(n,2)}\varphi^{(2)}\right]$$

(I) is IR-finite and

(II) satisfies eqs. (7), (7') and (8).

This is sufficient to prove inductively the properties (I) and (II) for the full solution $f^{(n)}$ of eq. (4).

Now the proof of the property (I) goes as follows: It is useful to distinguish between the "non-exceptional" configurations, when all the external momenta $\vec q_i$ and sums of all subsets of $\vec q_i$'s are non-zero, and "exceptional" configurations, if otherwise.

(i) For non-exceptional momenta the divergences may come only from ω_β and $V_\delta^{(2,2)}$. Eqs. (7) and (7') express the fact that the external particles are colour singlets. Therefore $\varphi^{(n)}_{a_1\ldots a_n}$ transforms as an invariant tensor in the colour space and satisfies

$$\sum_j C_{c a_j b_j}\,\varphi^{(n)}_{a_1\ldots a_{j-1} b_j a_{j+1}\ldots a_n} = 0.$$

Hence, we can write [10]

$$\left\{\sum_i^n \omega_\beta(i)\,\varphi^{(n)}\right\}_{a_1\ldots a_n} = 2\sum_{i<j}^{n} G^{b_i b_j}_{a_i a_j}\, g^2 (2\pi)^{-3}\int\frac{d^2k}{k^2}\,\varphi^{(n)}_{a_1\ldots b_i\ldots b_j\ldots a_n}(\vec q_1,\ldots,\vec q_n),$$

which shows that

$$\left[\sum_i \omega_\beta(i) + \sum_{i<j} V_\delta^{(2,2)}(i,j)\right]\varphi^{(n)} = \text{finite}, \tag{9}$$

because the singularities in the integrand of $\omega_\beta\,\varphi^{(n)}$ cancel those in the integrand of $V_\delta^{(2,2)}\,\varphi^{(n)}$.

(ii) For $\vec q_i = 0$ there are additional cancelling divergences in

$$\left[\omega_\alpha(i) + \sum_{r>i} V_\beta^{(2,2)}(i,r) + \sum_{\ell<i} V_\gamma^{(2,2)}(\ell,i)\right]\varphi^{(n)} + \sum_{\ell<i}\sum_{r>i} V_\delta^{(3,2)}(\ell,i,r)\,\varphi^{(n-1)} = \text{finite}; \tag{10}$$

they cancel because, due to eqs. (7), (7') and (8), the integrand of eq. (10) is finite when $\vec K_i \equiv \vec k_i - \vec q_i = \vec q_r - \vec k_r = \vec q_\ell - \vec k_\ell \to 0$.

(iii) For $\vec q_i + \vec q_j = 0$ we have, due to eqs. (7) and (7'), a cancellation of divergences in

$$V_\alpha^{(2,2)}(i,j)\,\varphi^{(n)} + \left[\sum_{r>i} V_\beta^{(3,2)}(i,j,r) + \sum_{\ell<i} V_\gamma^{(3,2)}(\ell,i,j)\right]\varphi^{(n-1)} + \sum_{\ell<i}\sum_{r>i} V_\delta^{(4,2)}(\ell,i,j,r)\,\varphi^{(n-2)} = \text{finite},$$

etc. In general, there is a cancellation of purely virtual IR singularities (i.e. uncut gluon lines) in $V_\alpha^{(m,2)}$ with real singularities (i.e. cut gluon lines) in $V_{\beta,\gamma}^{(m+1,2)}$, and virtual singularities in $V_{\beta,\gamma}^{(m+1,2)}$ with real singularities in $V_\delta^{(m+2,2)}$.

Let us note here that for <u>non-exceptional</u> momenta the colour neutrality of the external particles is not a necessary condition for the cancellation of IR singularities; in this case it is enough to project onto a colour-singlet state in the t-channel, i.e. contract $f^{(n)}_{a_1\ldots a_n}$

with any invariant tensor $P_{a_1 \dots a_n}$.

The proof of (II) employs the properties of the vertices V , following from their group structure, analogous to the properties (7), (7') and (8). A crucial point is to show that, when $\vec{q}_i \to 0$, the terms involving $\omega(i)$, $V^{(m,2)}(\ell_1, \ell_2, \dots, i)$ and $V^{(m,2)}(i, r_1, r_2, \dots)$ vanish. Indeed, these terms do vanish in the limit $\vec{q}_i \to 0$ with the integration variables $\vec{k}_i$ fixed. The convergence is, however, non-uniform and the integrals could in general acquire finite contributions from $\vec{k}_i \sim \vec{q}_i$. In our case this does not happen because of cancellations at small $\vec{k}_i$, mentioned in (i) and (ii) above.

IV. Discussion

The IR cancellations in our case have a very simple physical interpretation. Consider for definiteness the IR divergence discussed in (ii) above, when one of the external momenta vanishes, $\vec{q}_i \simeq 0$. To visualise the space-time structure of the process we can draw the relevant diagrams in the time-ordered form. It is easy to see (cf. ref. (5)) that they have the structure shown in Fig. 3 : in the LLA the gluon coupling to the external line q_i has the longitudinal momentum ℓ_z much smaller than all the other "partons" in the fast-moving system, depicted collectively as the shaded stripe. The IR divergence could arise when $\vec{\ell}' \simeq \vec{\ell} - \vec{q}_i \to 0$. Now, the crucial point is that the external lines $q_1, \dots, q_n$ couple to the fast-moving system after the emission of the line ℓ , and prior to its re-absorption. Consequently, the soft line ℓ is emitted/absorbed by a system having the same total colour quantum numbers as the incident/outgoing particles, which by assumption are colour singlets. Therefore, when $\vec{\ell} \to 0$, the emission/absorption amplitudes vanish, thus preventing the IR divergence (cf. ref. 11).

Summarizing, we have derived a closed system of coupled integral equations for (the multiple discontinuities of) the two-particle n-gluon vertices, $f^{(n)}$, in the Coulomb gauge. The approximation we used was to keep only the lowest order terms in g and sum to all orders in $g^2 \ell n N_\mu p^\mu$ where N^μ is the gauge-defining vector and p^μ the incident particle momentum. In this approximation $f^{(n)}$ is a sum of all Reggeon-calculus-like diagrams involving only effective $2 \to m$ $(m \geqslant 2)$ gluon vertices. The relation between gluons and Reggeons (Reggeized gluons) is not immediate: in general several

gluons may combine into one Reggeon. Our equations, however, can be shown [12] to be equivalent to the Reggeon calculus equations for two-particle n-Reggeon amplitudes, involving $1 \to 2m+1$ and $2 \to 2m$ Reggeon vertices, so that the number of Reggeons can only grow with every step in rapidity.

A physical situation to which this approximation would apply, could be e.g. elastic hadron scattering on a very large nucleus composed of $\mathcal{N} \gg 1$ nucleons. The dominant diagrams would be then those with a large number of "Pomerons" (pairs of Reggeized gluons) coupled to <u>different</u> nucleons (cf. ref. 13) . The "Pomeron" exchange interaction would be enhanced by a factor $\mathcal{N}$, and our approximation would become exact in the limit $g^2 \to 0$ with $g^2 \ell ns$ and $g^4 \mathcal{N}$ fixed. Let us also mention another feature of this limit. In general, the IR-finiteness of our approximate 2-particle n-gluon vertices $f^{(n)}$ does not guarantee the IR-finiteness of their convolution in eq. (1). The divergences can come from the propagators $1/\vec{q}_i^{\,2}$. In the limit considered this does not happen because then only two gluons can couple to each colour singlet particle (nucleon) and therefore the coupling vanishes as $\vec{q}_i \to 0$.

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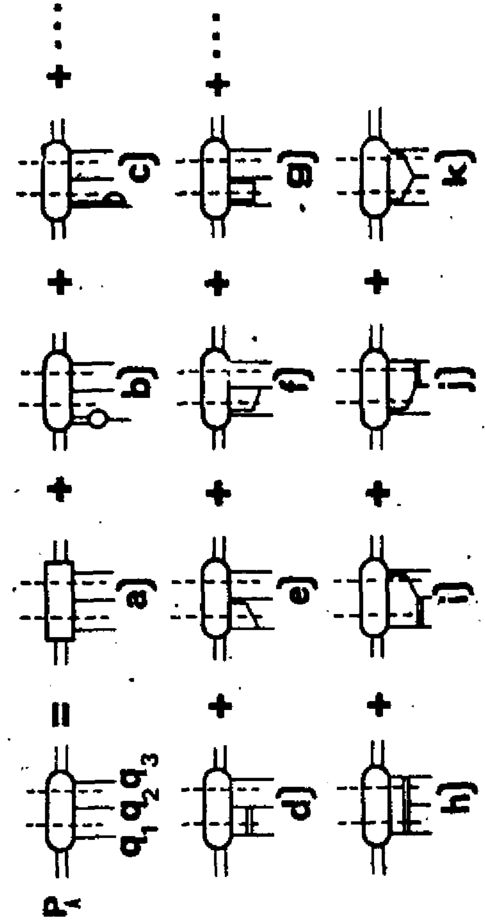

FIG 1

The integral equation for $f^{(3)}$

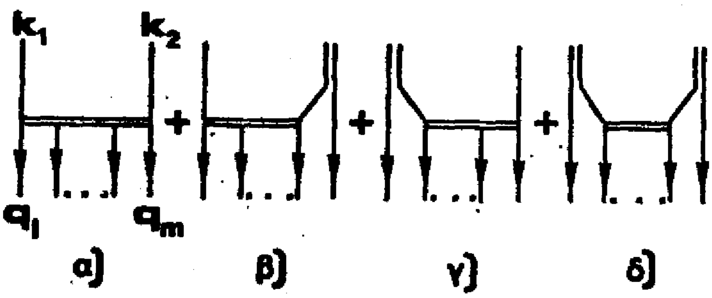

FIG 2

The 2 → m gluon vertex $V^{(m,2)}$.

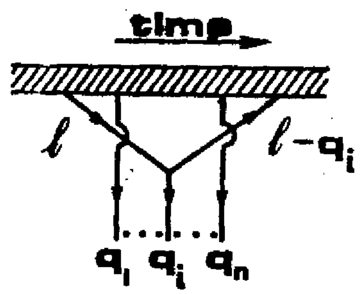

FIG 3

The structure of time-ordered diagrams contributing to Eq.(10).

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