SU82074863

Institute for Nuclear Research
Academy of Sciences of the Ukrainian S S R

Preprint KINR-81-30

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On Nuclear Reaction Duration at the Range of Overlapping Resonances

Kiev - 1981

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On Nuclear Reaction Duration at the Range of Overlapping Resonances

Nuclear reaction duration above the threshold of overlapping resonances is investigated and its importance to obtain a new information on a collision mechanism is evidenced. It is shown also that the duration of resonant nuclear reactions is asymptotically decreasing according to the law $[E^2 n(E)]^{-1}$ when the energy E and the number of open channels $n(E)$ are increasing.

Исследуются длительности ядерных реакций выше порога области перекрывающихся резонансов, указывается на необходимость таких исследований для получения новой информации о механизме столкновений. Показано, что длительность резонансных ядерных реакций асимптотически убывает по закону $[E^2 n(E)]^{-1}$ с ростом энергии E и числа открытых каналов $n(E)$.

О ДЛИТЕЛЬНОСТИ ЯДЕРНЫХ РЕАКЦИЙ
В ОБЛАСТИ ПЕРЕКРЫВАЮЩИХСЯ РЕЗОНАНСОВ

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Printed by the decision of the Academic Council
of the Institute for Nuclear Research
of the Ukrainian Academy of Sciences

Institute for Nuclear Research
Academy of Sciences of the Ukrainian SSR

UDC 539.17;530.145

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Key words:
quantum mechanics, energy levels, energy-level density, energy dependence, compound-nucleus reactions, quantum operators, S matrix, cross sections;
квантовая механика, резонансные состояния, плотность энергетического уровня, энергетическая зависимость, компаунд-реакции, квантовые операторы, S матрица, сечения.

1. Although the collision duration is an observable in the quantum mechanics and a fundamental characteristics of the collision mechanism, up to now there is some inconsistency in the interpretation of the time and even in the definition of the collision duration*. This is understandable since during a long period the problems of the time operator and the expansion of the orthodox quantum mechanical axiomatics with including the time in the set of observables had not been resolved uniquely. The existance and properties of the time operator $\hat{t} = -i\hbar\,\partial/\partial E$ and the probabilistic properties of the duration of any quantum mechanical process are established in [3-5] In particular, the fundamental substantiation of the time-energy uncertainty relation which generalizes Mandelstam-Tamm's deriving [6] is presented in [3,4]. The critical analysis of the main previous approaches to the time problem is given in [3], where it is shown also that the consistent definition of the collision duration in the simplest cases is reduced to the Ohmura's prescription [7] and the expressions for the mean durations of practically all types of quantum processes are derived.

The direct and unique connection between cross sections and collision durations for the cases of a non-resonant scattering, isolated and weakly overlapping resonances is established in [1,3], where some results of the numerical calculations

* Lists of different approaches are for example in [1-3].

of the nuclear reaction durations on the basis of the experimental cross sections are presented too. As it was firstly mentioned in [8], the analysis of the durations can be used as a source of the information on the cross section hyperfine structure, which cannot be established in the measured cross sections with the real energy spread of incident beams.

At the range of overlapping resonances the reaction duration is shown in [2] to be proportional to the resonance density and inversely proportional to the number of channels. In the present work after defining some general expressions and relations for the partial and global durations the energy dependence of the partial durations in the same region is investigated and the law of the asymptotic duration decreasing with the energy increasing is established. Then the comparative analysis of cross sections and durations in respect to the information on the collision mechanism is discussed.

2. Using the results of [3-5] for the definition of mean collision durations $\langle\tau_{fi}\rangle$, we define the mean partial and global time delays* $\langle\Delta\tau_{fi}^{(J)}\rangle_{E_0}$ and $\langle\Delta\tau_{fi}^{(glob)}\rangle_{E_0}$ in the two-particle reactions $i\to f$ as

$$\langle\Delta\tau_{fi}^{(J)}\rangle_{E_0}=\hbar\langle|T_{fi}^{(J)}(E)|^2\partial\arg T_{fi}^{(J)}(E)/\partial E\rangle_{E_0}\Big/\langle|T_{fi}^{(J)}(E)|^2\rangle_{E_0}, \quad (1)$$

$$\langle\Delta\tau_{fi}^{(glob)}\rangle_{E_0}=\hbar\int_{[4\pi]}d\Omega_f\langle|T_{fi}(E,\theta_f)|^2\partial\arg T_{fi}(E,\theta_f)/\partial E\rangle_{E_0}\Big/\int_{[4\pi]}d\Omega_f\langle|T_{fi}|^2\rangle_{E_0} \quad (2)$$

$$=\hbar\sum_J\langle\sigma_{fi}^{(J)}\partial\arg T_{fi}^{(J)}/\partial E\rangle_{E_0}\Big/\sum_J\langle\sigma_{fi}^{(J)}\rangle_{E_0},$$

where $T_{fi}^{(J)}$ and $\sigma_{fi}^{(J)}$ are defined with usual simplifications (central interactions, neglecting spins) from the following expansions of the T-matrix and the cross section σ_{fi}:

$$T_{fi}(E,\theta_f)=\sum_J T_{fi}^{(J)}P_J(\cos\theta_f)=(-\hbar^2/8i\pi^2\mu_f[k_fk_i]^{1/2})\sum_J(2J+1)[S_{fi}^{(J)}-\delta_{fi}]P_J(\cos\theta_f), \quad (3)$$

$$\sigma_{fi}=\sum_J\sigma_{fi}^{(J)}=(\pi/k_i^2)\sum_J(2J+1)|S_{fi}^{(J)}-\delta_{fi}|^2, \quad (4)$$

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* The time delay is defined as the mean collision duration $\langle\tau_{fi}\rangle$ minus the sum of two durations of the initial and final particle free flights along the distances, which are equal to the correspondent interaction radius

E and θ_f being the energy of the bombarding particles and the emitting angle of the final particles in the center-of-mass system. The symbol $\langle\ \rangle_{E_0}$ marks averaging with some spectral density g in the interval δE near the mean energy E_0. The function g is defined by the weight function of the wave packets and also the density matrix for the macroscopic ensemble of pairs "one particle in the incident beam + one nucleus in the target", whose energies are distributed in the interval δE and whose initial phases are uncorrelated.

When $\delta E \ll E_0$ and $\delta E \ll \Delta E$, ΔE being the interval of the noticeable changing of $\sigma_{fi}^{(J)}$ and $\partial \arg T_{fi}^{(J)}/\partial E$, the symbol $\langle\ \rangle_{E_0}$ in (1) and (2) one can omit and if the sum in (4) converges and $|\partial \arg T_{fi}^{(J)}/\partial E|$ is limited one can see that

$$\Delta\tau_{fi}^{(\ell)}(E) \leq \Delta\tau_{fi}^{(glob)}(E) \leq \Delta\tau_{fi}^{(\mathcal{L})}(E), \tag{5}$$

ℓ and $\mathcal{L}$ being the values of the angular momentum J for which the values of $\Delta\tau_{fi}^{(J)}$ are minimal and maximal in the set of all the values factually considered in (1)-(4). The relation (5) points out that it is sufficient to limit oneself by the examination of $\Delta\tau_{fi}^{(J)}(E)$.

3. Let us use the following representation of the S-matrix [9] :

$$\hat{S}^{(J)}(E) = \hat{U}\prod_{\nu=1}^{N}\left(1 - \frac{i\Gamma_\nu \hat{P}_\nu}{E - E_\nu + i\Gamma_\nu/2}\right)\hat{U}^T, \tag{6}$$

where $\hat{U}$ is a unitary matrix which does not depend on the energy, $(\hat{U}^T)_{ij} = U_{ji}$, $\hat{U}\hat{U}^T$ is a symmetrical "background" (nonresonant) S-matrix, $\hat{P}_\nu$ are proection matrices which do not commute with each other ($\hat{P}_\nu = \hat{P}_\nu^+ = \hat{P}_\nu^2$, $\mathrm{Trace}\,\hat{P}_\nu = 1$). Evidently,

$$\Delta\tau_{fi}^{(J)}(E) = \hbar\,\partial \arg N_{fi}^{(J)}/\partial E + \sum_\nu \frac{\Gamma_\nu/2}{(E-E_\nu)^2 + \Gamma_\nu^2/4} \tag{7}$$

with

$$N_{fi}^{(J)} = \left\{\hat{U}\prod_\nu [E - E_\nu - i\Gamma_\nu(\hat{P}_\nu - 1/2)\hat{U}^T\right\}_{fi} - \delta_{fi}\prod_\nu (E - E_\nu + i\Gamma_\nu/2).$$

Let us make the following approximations : (a) $\Gamma \gg D$ and $\overline{|(P_\nu)_{ij}|}\,\Gamma \gg D$ (Γ, D and $\overline{(P_\nu)_{ij}}$ are the mean resonance width, spacing and $(P_\nu)_{ij}$) ; (b) $\delta E \gg D$; (c) the phases of S_{fi} and N_{fi} are chaotically distributed and then the [redacted] values of $|S_{fi} - \delta_{fi}|$ and $|N_{fi}|$ are practically constant in the interval δE and the values of the terms with $\sin \arg S_{fi}$, $\sin \arg N_{fi}$ ecc. are practically zeros after averaging ; (d) all $\hat{P}_\nu$ are equal to

each other ($\hat{\rho}_\nu = \hat{\rho}$) ; (e) the number $n(E)$ of the open channels is sufficiently large: $n \gg 1, |\alpha_{fi}^{(\gamma)}| < |\langle S_{fi}^{(\gamma)}\rangle| < 1$ with $\alpha_{fi} = (\hat{U}\hat{\rho}\hat{U}^T)_{fi}$.
Hence

$$\langle \Delta\tau_{fi}^{(\gamma)} \rangle_{E_0} \approx \beta_{fi}^{(\gamma)} \mathcal{T}^{(\gamma)}, \qquad (8)$$

with $\mathcal{T}^{(\gamma)} = \hbar \langle \sum_\nu \frac{\Gamma_\nu}{(E-E_\nu)^2 + \Gamma_\nu^2/4} \rangle$, $\beta_{fi}^{(\gamma)} = \overline{|\alpha_{fi}^{(\gamma)}|^2} / \overline{|S_{fi}^{(\gamma)}|^2}$ for $f \neq i$ and $\beta_{ii}^{(\gamma)} = \overline{|\alpha_{ii}^{(\gamma)}|^2}$. Evidently, $0 < |\alpha_{fi}^{(\gamma)}|^2 \leqslant |(\hat{U}\hat{U}^T)_{fi}|^2$ $(i,f = 1,2,\dots,n)$ and $0 < \beta_{fi}^{(\gamma)} \leqslant 1$. If one consider all the channels as equivalent in that sense that all $|S_{fi}^{(\gamma)}|^2$ are the same and, consequently, equal to* $1/n(E)$ and all $|\alpha_{fi}^{(\gamma)}|^2$ are equal to each other too, then for $f \neq i$ the values of $\langle \Delta\tau_{fi}^{(\gamma)} \rangle_{E_0}$ must be the same ($\langle \Delta\tau_{fi}^{(\gamma)} \rangle_{E_0} = \langle \Delta\tau^{(\gamma)} \rangle_{E_0}$) and for the elastic scattering $\langle \Delta\tau_{ii}^{(\gamma)} \rangle_{E_0} \approx \langle \Delta\tau^{(\gamma)} \rangle_{E_0} / [n(E)]$, being the least between all the reactions. Apparently, it can be explained by the inevitable presence of the diffractional (shadow) scattering. The estimation of $\mathcal{T}^{(\gamma)}$ in the case of the sufficiently narrow resonances ($\mathcal{D} \ll \Gamma \ll \delta E$) yields [2] :

$$\mathcal{T}^{(\gamma)} = 2\pi\hbar / \mathcal{D}. \qquad (9)$$

If in the same case one would suppose the equal durations in all the channels including the elastic scattering and use the equalities

$$\mathrm{Trace} \langle Q^{(\gamma)}(E) \rangle_{E_0} \approx \mathcal{T}^{(\gamma)}(E_0) \approx \sum_{ij} \langle \Delta\tau^{(\gamma)} \rangle_{E_0} [\delta_{ij} + \langle |S_{ji}^{(\gamma)}|^2 \rangle_{E_0}] = 2n \langle \Delta\tau^{(\gamma)} \rangle_{E_0}, \qquad (10)$$

where $Q_{ij}^{(\gamma)} = -i\hbar \sum_k S_{ki}^{(\gamma)*} dS_{kj}^{(\gamma)}/dE$ and the approximations $\langle S_{ii}^{(\gamma)} \rangle_{E_0} \approx 0$, $\mathrm{Im} \langle dS_{ii}^{(\gamma)}/dE \rangle_{E_0} \approx 0$ are supposed relative to [2], then the known result [2]

$$\langle \Delta\tau^{(\gamma)} \rangle \approx \pi\hbar / n\mathcal{D} \qquad (11)$$

will be obtained.

Thus, the nuclear reaction mean durations in the different channels can have the various values in the large interval from 0 to $2\pi\hbar/\mathcal{D}$, becoming equal to (11) only in the approximation of the channel equivalency.

The results (9)-(11) are true when the contributions from the remoted resonances can be neglected. However that is justified only for the sufficiently narrow resonances. The estimate of $\mathcal{T}^{(\gamma)}$ in the case, when $\mathcal{D} \ll \delta E \ll \Gamma$, gives

*That follows from the unitarity condition $\sum_f |S_{fi}^{(\gamma)}|^2 = 1$.

$$\mathcal{T}^{(1)}(E) \sim \hbar \int_{E_{min}}^{\infty} \rho(E') \frac{\Gamma}{(E-E')^2+\Gamma^2/4} dE', \qquad (12)$$

where $\rho(E)$ is the resonance dencity in the continuum approximation, the lower limit E_{min} of which must satisfy the evident conditions: $E_{min} \leqslant E-\Gamma$ and $\rho(E_{min}) < \rho_{max}$. For a rough estimate of (12) one can limit oneself by the approximation $\Gamma = \gamma (E/E_0)^{1/2}$ or even $\Gamma = const\,(E > E_{min})$. For the last case the integral in (12) is defined by $\rho(E)$. The expression like

$$\rho(E) \sim \rho_{min} \exp(\alpha \sqrt{E^*}) \qquad (13)$$

($E^* = E + B$ is the compound nucleus excitation energy, B is the bound energy for the nucleon in the nuclear ground state) is often used in various compound nucleus models. But (13) is siutable only for the small excitations. In a realistic model the one-particle discrete energy spectrum has to be limited above the contimuum. Therefore the increasing of the density $\rho(E)$ of the compound states which are formed from such one-particle states must deccelerate, then be stopped at some energy $\mathcal{E}$ and be changed into decreasing [10,11] according to the law

$$\rho(E) \sim \rho_{max} \cdot \exp(-\alpha' \sqrt{E^*}), \quad E > \mathcal{E} \qquad (14)$$

In accordance with [11], the possible values of $\mathcal{E}$ must be in the interval between 0 and the energy of the nucleon knock-out from the deepest one-particle level. The elementary estimate of (12) gives the result

$$\mathcal{T}^{(1)} \xrightarrow[E\to\infty]{} \hbar\Gamma \left[\frac{2\rho_{min} \cdot x}{(E-\mathcal{E})^2+\Gamma^2/4} + \frac{\rho_{max}}{E^2(\alpha')^2} \right], \qquad (15)$$

$$x \sim [\sqrt{\mathcal{E}} \exp(\alpha\sqrt{\mathcal{E}}) - \sqrt{E_{min}} \exp(\alpha\sqrt{E_{min}})]/\alpha - [\exp(\alpha\sqrt{\mathcal{E}}) - \exp(\alpha\sqrt{E_{min}})]/\alpha^2 \quad \text{for } E \gg B.$$

Thus, for $E \to \infty$ the resonant nuclear reaction duration decreases asymptotically according to the law $[E^2 n(E)]^{-1}$.

4. The results obtained here generalize the results of [2] and contain the law of the decreasing of nuclear reaction durations with the energy increasing. Besides, one can see the following important circumstance. At the range of the strongly overlapping resonances the smooth and even fluctuational dependence $\sigma_{fi}(E)$ averaged on δE can be practically fitted by the essentially different functions $\mathcal{S}_{fi}(E)$ with the different resonance densities. But $\langle \Delta\tau_{fi} \rangle$ is essentially sensible to $\mathcal{D}$ according to (8)-(15) and therefore gives a certain in-

formation on $\mathcal{D}$. And the method for the duration calculations, developed in [1] , and also the Hauser-Feshbach method for the cross-section calculations become invalid with the energy increasing. Consequently, at the range above the threshold of the overlapping resonances the development of experiment on the measurement of the nuclear reaction durations becomes especially important.

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Manuscript was received to the Division of Scientific and Technical Information of the Institute for Nuclear Research on July,16,1981.

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(препринт КИЯИ-81-30)

Редакторы: Е.Б.Серман,
Н.А.Солдатенко

БФ 33746	Бумага офсетная	Усл.-печ.л. 0,5
Изд.№КИЯИ-81-30	Печать офсетная	Уч.-изд.л. 0,39
Тип.заказ № 264	Формат бумаги 60x90/16	Тираж 180 экз.
Подписано к печати	20.10.81 г.	Цена 3 коп.

Институт ядерных исследований АН УССР, ОНТИ.
252650, ГСП, Киев-28, проспект Науки, 119

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