

Hyperon Magnetic Moments and Total Cross Sections

Clues to QCD or the Generation Puzzle?

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ABSTRACT

The new data on both total cross sections and magnetic moments are simply described by beginning with the additive quark model in an $SU(3)$ limit where all quarks behave like strange quarks and breaking both additivity and $SU(3)$ simultaneously with an additional non-additive mechanism which affects only nonstrange quark contributions. The suggestion that strange quarks behave more simply than nonstrange may provide clues to underlying structure or dynamics. Small discrepancies in the moments are analyzed and shown to provide serious difficulties for most models if they are statistically significant.

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Recent measurements of hyperon magnetic moments [1,2] and hyperon-nucleon total cross sections [3] show disagreements with the predictions from the simple additive quark model (AQM) [4,5]. The small discrepancies can be dismissed if predictions are expected to hold only at the 20% level. But a clear signal in the noise shows a qualitatively different behavior in the contributions from quarks of different generations. The data can be characterized to a good approximation over a wide range as composed of two contributions:

1. A flavor-symmetric contribution satisfying the AQM;
2. An enhancement of the contributions from only first generation (nonstrange) quarks which simultaneously breaks flavor symmetry and additivity.

The suggestion that strange quarks behave more simply than nonstrange quarks may be a clue to the understanding of hadron structure and the generation puzzle.

The additive quark model[4], more properly called the universal additive valence quark model, predicts that many properties of hadrons are obtained by summing the contributions of the individual valence quarks, and that the contribution from a quark with a given set of quantum numbers is the same for all hadrons. The key words universal, additive and valence describe the essential assumptions and indicate possible causes of disagreement when the model fails. A breaking of universality appears as a variation of the contribution from a particular quark from one hadron to another; e.g. as a dependence of the quark magnetic moment on the hadron mass. A breaking of additivity appears as a necessity to describe the particular property with two-body or higher operators, rather than only with single particle

operators. A breaking of the valence quark assumption appears as a contribution from other constituents of the hadron, such as gluons, a sea of quark-antiquark pairs, or an external pion field. The validity of the AQM is itself a mystery, with all three basic assumptions open to question.

I. A Two-Component Model for Hadron-Nucleon Total Cross Sections

The conventional predictions of the AQM for hadron total cross sections [4] assume SU(3) symmetry breaking at the quark level by an ad hoc difference between strange and nonstrange quark contributions chosen to fit experiment. These AQM predictions are very successful within the meson and baryon sectors. Their breakdown at the 20% level in relations between mesons and baryons is interpreted as a failure of universality unrelated to SU(3) breaking.

A "Two-Component Pomeron" model which simultaneously broke quark model additivity and SU(3) symmetry [5] showed these two effects to be empirically related and described both by a single mechanism. The additive component of the total cross section due to Pomeron exchange was assumed to be universal and a pure SU(3) singlet with no symmetry breaking. All the strangeness dependence of the Pomeron component as well as the meson-baryon difference came from a single non-additive second component which enhanced contributions from nonstrange quarks by an amount depending upon the total number of quarks in the hadron. The SU(3) breaking appeared as an enhancement of the contribution from the nonstrange quarks, rather than as a suppression of the contribution of the strange quarks. The model has no simple dynamical basis, but its predictions have continued to fit new data better than expected, including the recently measured hyperon-nucleon total cross sections. Nonstrange enhancement suggests intuitive pictures involving pion clouds or

nonstrange quark-antiquark pairs, but no convincing derivation along these lines has been presented.

The most recent success of this model in the hyperon-nucleon cross sections is simply described by examining the strangeness dependence of the cross sections. In the AQM this dependence is universal in mesons and baryons and attributed to the difference at the quark level between the scattering amplitudes of the strange and nonstrange quarks. The difference between $\sigma(\Xi N)$, $\sigma(\Sigma N)$ and $\sigma(NN)$ must then be equal to the difference between $\sigma(KN)$ and $\sigma(\pi N)$ [5].

$$\sigma(pp) - \sigma(\Sigma p) = \sigma(\Sigma p) - \sigma(\Xi p) = \sigma(\pi^- p) - \sigma(K^- p) \quad (1a)$$

The two-component pomeron model, on the other hand, attributes the strangeness dependence to a second order effect which is a quadratic function of the quark numbers, rather than a linear function. The change in total cross section when a nonstrange quark is replaced by a strange quark is larger in baryons than in mesons by a factor of 3/2 [7,8].

$$\sigma(pp) - \sigma(\Sigma p) = \sigma(\Sigma p) - \sigma(\Xi p) = (3/2)\{\sigma(\pi^- p) - \sigma(K^- p)\} \quad (1b)$$

This difference by a factor of 3/2 between the predictions (1a) and (1b) of the AQM and of the two-component pomeron model has now been tested experimentally. The prediction (1b) from the two-component pomeron model agrees with experiment.

The general approach of this two-component pomeron model pinpoints certain features of the experimental data which have a simple physical interpretation. The failure of the AQM in relations like (1a) between the meson and baryon sectors can be attributed entirely to the contributions from nonstrange quarks. This is most clearly demonstrated by projecting out the contributions of strange and nonstrange quarks from the experimental baryon-nucleon and meson-nucleon cross sections.

$$\sigma(nN)_B = (1/6)\{\sigma(pp) + \sigma(pn)\} \quad (2a)$$

$$\sigma(nN)_M = (1/4)\{\sigma(\pi^-p) - \sigma(K^-p) + \sigma(\pi^+p) - \sigma(K^-n) + \sigma(K^+p) + \sigma(K^+n)\} \quad (2b)$$

$$\sigma(sN)_B = (1/6)\{\sigma(\Sigma^-p) + \sigma(\Sigma^-n) + \sigma(\Xi^-p) + \sigma(\Xi^-n) - \sigma(pp) - \sigma(pn)\} \quad (2c)$$

$$\sigma(sN)_M = (1/4)\{\sigma(K^-p) - \sigma(\pi^-p) + \sigma(K^-n) - \sigma(\pi^+p) + \sigma(K^+p) + \sigma(K^+n)\} \quad (2d)$$

where $\sigma(nN)_B$, $\sigma(nN)_M$, $\sigma(sN)_B$ and $\sigma(sN)_M$ denote the contributions from nonstrange and strange quarks to the isospin averaged baryon-nucleon and meson-nucleon scattering cross sections respectively as calculated from the AQM and the conventional duality assumption of equality of the contributions from strange quarks and antiquarks is used to eliminate antiquark contributions from eqs. (2b) and (2d),

$$\sigma(sN)_M = \sigma(\bar{s}N)_M \quad (2e)$$

The AQM predicts the equality of the corresponding quark-nucleon contributions to baryon and meson cross sections. Substituting the relations (2) gives two sum rules which can be tested against experimental data:

The nonstrange sum rule,

$$\sigma(nN)_B = \sigma(nN)_M \quad (3a)$$

$$(1/6)\{\sigma(pp) + \sigma(pn)\} =$$

$$= (1/4)\{\sigma(\pi^-p) - \sigma(K^-p) + \sigma(\pi^+p) - \sigma(K^-n) + \sigma(K^+p) + \sigma(K^+n)\} \quad (3b)$$

$$12.9 \pm 0.01 \text{ mb.} = 11.2 \pm 0.05 \text{ mb.} \quad (3c)$$

and the strange sum rule,

$$\sigma(sN)_B = \sigma(sN)_M \quad (4a)$$

$$(1/6)\{\sigma(\Sigma^-p) + \sigma(\Sigma^-n) + \sigma(\Xi^-p) + \sigma(\Xi^-n) - \sigma(pp) - \sigma(pn)\} =$$

$$= (1/4)\{\sigma(K^-p) - \sigma(\pi^-p) + \sigma(K^-n) - \sigma(\pi^+p) + \sigma(K^+p) + \sigma(K^+n)\} \quad (4b)$$

$$7.7 \pm 0.1 \text{ mb.} = 7.75 \pm 0.05 \text{ mb.} \quad (4c)$$

The experimental data quoted are taken at 100 GeV/c momentum, where there are both new data on hyperon-nucleon cross sections and previous data on the other hadronic cross sections available.

The strange sum rule (4) is seen to be in excellent agreement with experiment, while there is strong disagreement with the nonstrange sum rule (3). The 15% discrepancy is significant and shows that universality holds for the contribution from strange quarks to the hadron-nucleon cross sections, but that the contribution from nonstrange quarks is not universal; it is greater in baryons than in mesons. This indication that strange quark contributions are somehow simpler than nonstrange contributions is a significant and recurrent feature of the data which has no explanation from first principles.

A test of equation (4) by direct comparison with experiment was impossible before the recent measurements provided precise experimental values for the hyperon-nucleon total cross sections. Thus there was no direct evidence that the strange sum rule (4) was in much better agreement with experiment than the nonstrange sum rule (3). The 15% discrepancy was considered as acceptable for such a crude model. The large difference between the strange meson contribution (4) and the nonstrange contribution (3) was attributed in the AQM to the $SU(3)$ symmetry breaking in the additive one-body contribution. This approach, however, leads to the unsuccessful prediction (1a), which disagrees with the new hyperon total cross section data.

The non-additive "two-component Pomeron model" interpreted the discrepancy in eq. (3) as a breakdown of the additive model due to double exchange contributions. The strongest double-exchange expected, a combination of pomeron and f exchange which enhanced the contribution of the nonstrange quarks, was parametrized by adding an ad hoc non-additive two-body term to the pomeron contribution. This single contribution was found to

explain the SU(3) symmetry breaking which produces the difference between the strange and nonstrange contributions (3) and (4) as well as the breaking of additivity in the sum rule (3), without requiring any SU(3) breaking in the additive component. The assumption that this single mechanism explained both effects led to a new relation between these quantities, the two-component pomeron sum rule,

$$\sigma(nN)_B - \sigma(nN)_M = (1/2)\{\sigma(nN)_M - \sigma(sN)_M\} \quad (5a)$$

$$\begin{aligned} (1/6)\{\sigma(pp)+\sigma(pn)\} - (1/4)\{\sigma(\pi^-p)-\sigma(K^-p)+\sigma(\pi^+p)-\sigma(K^-n)+\sigma(K^+p)+\sigma(K^+n)\} = \\ = (1/4)\{\sigma(\pi^-p)-\sigma(K^-p)+\sigma(\pi^+p)-\sigma(K^-n)\} \end{aligned} \quad (5b)$$

$$1.69 \pm 0.05 \text{ mb.} = 1.73 \pm 0.04 \text{ mb.} \quad (5c)$$

The left hand side of the sum rule (5) vanishes in the AQM. The right hand side vanishes in the SU(3) symmetry limit. The equality (5) relates the breaking of additivity between the baryon and meson sectors with the SU(3) breaking in the meson sector. Since the same meson cross sections appear on both sides, a better comparison with experiment is obtained by rearrangement,

$$\begin{aligned} (1/6)\{\sigma(pp)+\sigma(pn)\} - (1/4)\{\sigma(K^+p)+\sigma(K^+n)\} = \\ = (1/2)\{\sigma(\pi^-p) - \sigma(K^-p) + \sigma(\pi^+p) - \sigma(K^-n)\} \end{aligned} \quad (5d)$$

$$3.42 \pm 0.03 \text{ mb.} = 3.47 \pm 0.08 \text{ mb.} \quad (5e)$$

The two-component pomeron model also predicted that the strange sum rule (4) relating the contributions of the strange quarks to baryon-nucleon and nucleon-nucleon scattering should hold, even though the corresponding sum rule (3) for nonstrange quarks was violated. This prediction has now been confirmed by the new data on hyperon-nucleon cross sections which enable a direct test of the sum rule (4).

The two sum rules (4) and (5) can be combined to give a new sum rule with a simple physical interpretation,

$$\sigma(nN)_B - \sigma(sN)_B = (3/2)\{\sigma(nN)_M - \sigma(sN)_M\} \quad (6a)$$

$$\begin{aligned}
 (1/3)\{\sigma(pp) + \sigma(pn)\} - (1/6)\{\sigma(\Sigma^+p) + \sigma(\Sigma^-n) + \sigma(\Xi^+p) + \sigma(\Xi^-n)\} = \\
 = (3/4)\{\sigma(\pi^+p) - \sigma(K^+p) + \sigma(\pi^+n) - \sigma(K^+n)\} \quad (6b) \\
 5.15 \pm 0.07 \text{ mb.} = 5.2 \pm 0.1 \text{ mb.} \quad (6c)
 \end{aligned}$$

Here the SU(3) symmetry breakings in the baryon and meson sectors are compared and shown to violate universality in the exact manner predicted by the two-component pomeron model. This prediction has now been strikingly confirmed by the new hyperon-nucleon data. It is just the factor 3/2 appearing in eqs. (6a) and (1b) that makes the difference between the two predictions (1a) and (1b). The analogous sum rule from the AQM differs from (6a) by not having the factor of 3/2 on the right hand side. The value of 3.45 ± 0.08 mb. obtained without the factor 3/2 is in strong disagreement with the value 5.15 ± 0.07 mb. on the left hand side.

The two-component pomeron model was extremely successful in fitting data available at the time and has successfully predicted a large quantity of data from subsequent experiments. The striking agreement between prediction and experiment shown in eqs. (4-6) is particularly impressive since no data was available at this energy and no hyperon-nucleon cross sections had been measured at all when the model was first proposed. The success of these sum rules seems to indicate that the breaking of SU(3) and additivity are mysteriously related and that the corrections to the simple SU(3)-symmetric AQM affect only the contributions of the nonstrange quarks.

The model has no convincing derivation from first principles. The original double exchange picture fails to explain the observed energy dependence of the particular combinations of cross sections appearing in the two-component pomeron sum rule (5b), which differs from predictions from pomeron-f double exchange. The success of the sum rules has motivated a

search for an alternative mechanism to give a contribution with the same dependence on quantum numbers as pomeron-f exchange but a different energy dependence. So far this search has been unsuccessful.

The observed energy dependence of the cross sections also suggests independently that the strange quark contribution to the scattering is simpler than the nonstrange contribution. The energy dependence of the nucleon-nucleon cross section is complicated; it first decreases to a minimum and then rises. The same is true for the expressions (2a-b) which separate out the contribution from nonstrange quarks. However, the energy dependence of the strange quark contribution (2d) is very simple; it rises monotonically with energy from a few GeV to the highest measured energies.

All the expressions (2) are constructed to project out the decreasing contributions associated with Reggeon exchange. This is easily done in a quark-duality picture where Reggeon exchange is described by Harari-Rosner duality diagrams which always involve the annihilation of a nonstrange antiquark on a quark in the target nucleon. Reggeon exchange is excluded by considering expressions which depend only upon nonstrange quark amplitudes and eliminating all antiquark amplitudes.

In the two-component pomeron model the complicated energy dependence of the hadron-nucleon cross sections is given by combining two components which each have simple energy dependence, a monotonically rising component and a monotonically falling component. The dominant rising component is assumed to be SU(3) symmetric and the decreasing component is assumed to be a non-additive enhancement of the nonstrange quark contribution which is proportional to the total quark number and therefore differs by a factor of $3/2$ between baryons and mesons. Only the rising component is present in the

strange quark contribution, thus explaining its simple energy dependence. The two components are isolated in the expressions (4) and (5). They were described in the model by simple power-law dependences which fit the data with a minimum number of free parameters,

$$\sigma(sN)_B = \sigma(sN)_M = 6.5 (P/20)^{0.13} \text{ mb.} = 8.0 \text{ mb.} \quad (7a)$$

$$\begin{aligned} \sigma(nN)_B - \sigma(nN)_M &= (1/2) \{ \sigma(nN)_M - \sigma(sN)_M \} = 2.2 (P/20)^{-0.2} \text{ mb.} \\ &= 1.6 \text{ mb.} \end{aligned} \quad (7b)$$

The numerical values at 100 GeV/c for these two empirical formulas are seen to be in good agreement with the experimental values (4c) and (5c).

A satisfactory dynamical description for the decreasing component (5) has not been given, nor any explanation for the equalities which holds over a wide energy range. But the new success of the predictions for hyperon-nucleon cross sections gives additional support to the indication that SU(3) symmetry breaking is a two-body effect and that it enhances the contribution of nonstrange quarks, rather than suppressing the strange quark contribution. Since the fundamental difference between strange and nonstrange quarks is not understood and is related to the unsolved generation puzzle, this qualitatively different behavior may have some deeper physical meaning. It is therefore of interest to look for similar effects in other areas of hadron physics where both SU(3) symmetry and quark model additivity break down.

II. A Two-Component Model for Baryon Magnetic Moments

Baryon magnetic moments have been treated with an additive quark model where SU(3) breaking is introduced by suppressing the additive contribution of the strange quarks {9},{10}. However, it now appears that additivity is also broken {11}. It may be that here also the SU(3) breaking mechanism is

better described as a non-additive enhancement of the nonstrange quark contribution, rather than a suppression of the additive strange quark contribution. We therefore examine the present status of baryon magnetic moments from this point of view.

A simple static quark model with SU(3) symmetry breaking determined from hadron masses gives excellent agreement with the magnetic moments of the nucleon and Σ , but gives a much poorer fit to the Ξ and Λ moments. On the other hand a similar model with no SU(3) symmetry breaking fits the Ξ and Λ moments quite well, but not the Σ and nucleon. Furthermore, a more precise examination of the fit to the Σ moments shows a small discrepancy which is not easily explained by any model.

The essential features of the discrepancies are summarized as follows:

- (1). The difference between the proton and Σ^+ moments is too large.
- (2). The difference between the Ξ^- and Λ moments has the wrong sign.

In the standard static model and most other models the proton and Σ^+ moments are dominated by the contributions from the two u quarks which are the same in both baryons. It is difficult to explain the large difference without requiring the contributions from the two u quarks to be different in the two baryons. The Ξ^- moment is predicted to be less than the Λ moment and the observed opposite sign of this difference suggests that the contributions from the strange quarks are different in the two baryons. These two effects cannot be simply explained by a dependence on the baryon mass because the nonstrange quark contribution appears to decrease with increasing baryon mass and the strange quark contribution appears to increase. These inconsistencies plague all models which attempt to obtain better agreement by introducing new effects.

The essential details of the problem are shown in Table 1.

TABLE I

Theoretical and Experimental Values of Baryon Magnetic Moments

Baryon	SU(3) Symmetric Model	Experimental Value	Standard Broken SU(3)	Two Comp. Broken SU(3)
proton	1.83	2.793	2.79	2.75
neutron	-1.22	-1.913	-1.86	-1.83
Λ	-0.61	-0.614 ± 0.005	-0.61	-0.61
Σ^+	1.83	2.33 ± 0.13	2.67	2.24
Σ^-	-0.61	-0.89 ± 0.14	-1.09	-0.81
Ξ^-	-0.61	-0.75 ± 0.06	-0.50	-0.61
Ξ^0	-1.22	-1.25 ± 0.014	-1.44	-1.22
$R(p, \Sigma^+, \Xi^-)$	0	2.76 ± 0.85	0.38	2.50
$R(\Xi, \Lambda)$	1.0	1.09 ± 0.03	1.06	1.0
$R'(\Xi, \Lambda)$	1.0	1.12 ± 0.02	1.0	1.0
$R''(\Xi, \Lambda)$	1.0	1.22 ± 0.1	0.82	1.0

Included in the table are four functions of the magnetic moments which project out certain physically interesting features of the data,

$$R(p, \Sigma^+, \Xi^-) = -3\{\mu(p) - \mu(\Sigma^+)\} / \{\mu(\Xi^0) - \mu(\Xi^-)\} \quad (8a)$$

$$R(\Xi, \Lambda) = \{\mu(\Xi^0) + \mu(\Xi^-)\} / 3\mu(\Lambda) \quad (8b)$$

$$R'(\Xi, \Lambda) = \{\mu(\Xi^0) + 2\mu(\Xi^-)\} / 4\mu(\Lambda) \quad (8c)$$

$$R''(\Xi, \Lambda) = \mu(\Xi^-) / \mu(\Lambda) \quad (8d)$$

The expressions (8a) and (8b) were defined in ref. [11]. The expressions (8c) and (8d) are modified versions of (8b) which are shown below to be somewhat more sensitive to disagreements with the model predictions.

Motivated by the suggestion that the AQM works better for strange quarks we begin our analysis from the unorthodox SU(3) symmetry limit in which all

quarks have the mass of the strange quark and use the Λ magnetic moment as input. The magnetic moment of a baryon with a configuration denoted by $(1,2;3)$, where quarks 1 and 2 have the same flavor, is then given in the static model with $L=0$ $SU(6)$ wave functions by

$$\tilde{\mu}(1,2;3) = (2q_1 + 2q_2 - q_3) \mu(\Lambda) = (4q_1 - q_3) \mu(\Lambda), \quad (9a)$$

where $\tilde{\mu}$ denotes the magnetic moment in the $SU(3)$ limit where all quarks have the mass of the strange quark, q_i denotes the electric charge of quark i , and $q_1 = q_2$. This $SU(3)$ -symmetric form is scaled to give the correct Λ moment.

The predictions of the standard broken- $SU(3)$ model are obtained from this limit by enhancing the contributions of the nonstrange quarks by a factor which fits the proton moment, while leaving the strange quark contributions unchanged.

$$\begin{aligned} \mu(1,2;3) &= (2q_1 + 2q_2 - q_3) \mu(\Lambda) + (2q_1 x_1 + 2q_2 x_2 - q_3 x_3) \{ (1/3) \mu(p) - \mu(\Lambda) \} \\ &= (4q_1 - q_3) \mu(\Lambda) + (4q_1 x_1 - q_3 x_3) \{ (1/3) \mu(p) - \mu(\Lambda) \}; \end{aligned} \quad (9b)$$

where x_i is defined to be 1 if quark i is a nonstrange quark and zero if it is a strange quark and $x_1 = x_2$. The symmetry breaking terms are all proportional to x_i which vanishes for strange quarks. Thus the scaling of the strange quark contribution to the moment remains unchanged. Equation (9b) can be rewritten

$$\begin{aligned} \mu(1,2;3) &= (2q_1 + 2q_2 - q_3) (1/3) \mu(p) - \\ &\quad - [2q_1(1-x_1) + 2q_2(1-x_2) - q_3(1-x_3)] \{ (1/3) \mu(p) - \mu(\Lambda) \} \\ &= (4q_1 - q_3) (1/3) \mu(p) - [4q_1(1-x_1) - q_3(1-x_3)] \{ (1/3) \mu(p) - \mu(\Lambda) \} \end{aligned} \quad (9c)$$

This is the conventional form in which the $SU(3)$ symmetry breaking appears as a suppression of the strange quark contribution rather than an enhancement of the nonstrange contribution. The $SU(3)$ -symmetric term is scaled to give the correct proton moment and the symmetry breaking terms are all proportional to $(1-x_i)$ which vanishes for nonstrange quarks.

If the comparatively small discrepancy in the Ξ^- moment is neglected, the predictions in the SU(3) symmetry limit shown in Table 1 are seen to fit the Ξ moments reasonably well and the nucleon moments very badly. The introduction of symmetry breaking fits the nucleons well, but at the price of a much stronger disagreement in the Ξ sector. The Σ^+ and Σ^- moments are midway between the two predictions, but the large error on the Σ^- moment allows a fit to either within two standard deviations. Furthermore, the discrepancy in the difference between the proton and Σ^+ moment remains and is nearly unaffected by the symmetry breaking.

This paradox suggests that SU(3) symmetry breaking is not a simple phenomenon and goes beyond enhancing the magnetic moment of the nonstrange quarks relative to that of the strange quarks by the same factor in all hadrons. Nonstrange quark moments seem to be quenched in strange hadrons [11] (or equivalently enhanced in nonstrange hadrons), perhaps by pion exchange [12,13]. However, the results tabulated in Table 1 show a very large enhancement of about 50% needed to fit the nucleon data, with no enhancement required for the Ξ 's. This suggests that all SU(3) breaking comes from a new dynamical mechanism like pion exchange, described by a two-body operator enhancing the nonstrange contributions mainly in the nucleon, with no additional breaking from the quark mass difference. This is difficult to believe; yet it corresponds exactly to the "Two-Component Pomeron" model described above for high energy scattering. The common feature in the total cross section and magnetic moment data that the nonstrange quarks break the SU(3) symmetry rather than the strange quarks may be significant.

This point motivated a "two-component" model with one fully SU(3)-symmetric component given by eq. (9a) and the second component breaking

SU(3) with a non-additive enhancement of the contributions of the nonstrange quarks by an ad hoc factor chosen to fit the nucleon moments $\{1+(1/4)N\}$, where N is the number of additional nonstrange quarks in the baryon; i.e. N=0, 1 and 2 in the Ξ , Σ and nucleon respectively.

$$\begin{aligned} \mu(1,2;3) &= (2q_1+2q_2-q_3)\mu(\Lambda) + \\ &+ \{2q_1x_1(x_2+x_3) + 2q_2x_2(x_3+x_1) - q_3x_3(x_1+x_2)\}[\mu(\Lambda)/4] \\ &= (4q_1-q_3)\mu(\Lambda) + \{2q_1x_1(x_1+x_3) - q_3x_3x_1\}[\mu(\Lambda)/2] \end{aligned} \quad (10)$$

The simultaneous breaking of SU(3) symmetry and additivity is evident in this relation, since the symmetry-breaking terms all contain two-quark products $x_i x_j$. This contrasts with eq. (9b) where all symmetry breaking terms are linear in the x_i 's. The predictions of this model are listed in Table I as "Two Comp. Broken SU(3)".

The empirical formula (10) has no theoretical basis. Its superiority over the standard model which also has only one SU(3)-breaking parameter shows that the data are parametrized better by non-additive rather than additive enhancement of the nonstrange quark contribution. The factor (1/4) is chosen to fit the well-known approximate enhancement factor of 3/2 for the nucleon. This corresponds in the standard model to a quark mass ratio of 3/2. With parameters fixed in both models by fitting the nucleon and Λ , the significant test comes in the Σ moments. The additive standard model (9b) predicts the same nonstrange enhancement in all baryons and gives the same enhancement in the Σ as in the nucleon in disagreement with experiment. The data show that the nonstrange enhancement needed for the Σ is roughly half that needed for the nucleon, which agrees with the prediction of the non-additive two-component model, eq.(10).

If better data on the Σ^- and Ξ^- agree with this simple model (10), dynamical models which give non-additive enhancement of the contributions of nonstrange quarks should be seriously considered. Pion exchange is one example, but there may be others. However, without better data it is difficult to test such fine tuning effects by overall fits.

III. Implications of the Experimental Ξ^- Moment

We now examine the smaller discrepancies in the fits of the experimental moments to the data. One disturbing qualitative feature is seen at this level. All deviations of the experimental moments from the symmetry limit are in the same direction; i.e. an increase in the absolute magnitude of the moment. Any symmetry breaking mechanism which enhances the nonstrange quark contribution without changing the strange quark contribution works in different directions in different hadrons. The magnitude of the total moment is increased by enhancing the contribution of the nonstrange quarks only if the nonstrange contribution has the same sign as the total moment. But the seven baryons in Table I include many different angular momentum couplings of three quarks with different electric charges. It is impossible that all should have the nonstrange contribution parallel to the total moment. This is immediately seen in the Ξ doublet, where the two experimental moments have the same sign, but the nonstrange quarks in the Ξ^0 and Ξ^- have electric charges of opposite sign and are required by isospin invariance to have the same wave functions. The two nonstrange contributions thus have opposite signs and one must be opposite to the total moment. In the standard broken-SU(3) model it is the Ξ^- moment that has a nonstrange contribution of opposite sign to the total moment. The Ξ^- moment is seen to be the guilty party in Table I; the direction of the experimentally observed SU(3) breaking is opposite to the prediction of standard broken SU(3).

We now analyze this difficulty quantitatively to spell out its essential ingredients and its implications for all models. The $\Xi^0 - \Xi^-$ difference is entirely due to nonstrange quark contributions. SU(3) symmetry breaking increases this difference since it enhances the contribution of the nonstrange quarks. We can then write the inequality

$$1 - \frac{\{\bar{\mu}(\Xi^-) - \bar{\mu}(\Xi^0)\}}{\{\mu(\Xi^-) - \mu(\Xi^0)\}} > 0 \quad (11a)$$

This can be rewritten

$$\frac{\{\mu(\Xi^-) - \bar{\mu}(\Xi^-)\}}{\{\mu(\Xi^-) - \mu(\Xi^0)\}} + \frac{\{\mu(\Xi^0) - \bar{\mu}(\Xi^0)\}}{\{\mu(\Xi^0) - \mu(\Xi^-)\}} > 0 \quad (11b)$$

A similar relation is obtainable for the Σ moments,

$$\frac{\{\mu(\Sigma^-) - \bar{\mu}(\Sigma^-)\}}{\{\mu(\Sigma^-) - \mu(\Sigma^+)\}} + \frac{\{\mu(\Sigma^+) - \bar{\mu}(\Sigma^+)\}}{\{\mu(\Sigma^+) - \mu(\Sigma^-)\}} > 0 \quad (11c)$$

All proposed models satisfy these conditions.

From the experimental values in Table 1,

$$\{\mu(\Xi^-) - \mu(\Xi^0)\}/\{\mu(\Xi^-)\} < 0. \quad (12a)$$

$$\{\mu(\Sigma^-) - \mu(\Sigma^+)\}/\{\mu(\Sigma^-)\} > 0. \quad (12b)$$

Both terms on the left hand side of (11b) are seen to have the same sign since the charges of the u quark in the Ξ^0 and the d quark in the Ξ^- have opposite signs and similarly for the Σ moments in eq. (11c). Thus each term is positive. From the positivity of the first term and the experimental relations (12a) and (12b) we obtain the inequality

$$\{\mu(\Xi^-)/\bar{\mu}(\Xi^-)\} < 1 < \{\mu(\Sigma^-)/\bar{\mu}(\Sigma^-)\} \quad (13)$$

These results follow very naturally in the standard models with conventional spin couplings. The spin of the nonstrange quark in the Ξ is antiparallel to the total spin, whereas the spins of the nonstrange quarks in

the Σ are parallel to the total spin. The enhancement of the nonstrange quark moments by $SU(3)$ symmetry breaking thus works in opposite directions in the Σ^- and Ξ^- . In the Σ^- the magnitude of the moment is increased by symmetry breaking; in the Ξ^- it is decreased.

In the standard model the nonstrange quarks in the Λ do not contribute to the magnetic moment because they are coupled to spin zero, and

$$\mu(\Lambda) = \mu(s). \quad (14a)$$

where $\mu(s)$ denotes the magnetic moment of the strange quark. The standard model also predicts

$$\{\bar{\mu}(\Xi^-) = \bar{\mu}(\Sigma^-) = \mu(s) \quad (14b)$$

Combining (9d), (13), (14a) and (14b) gives

$$R^{\mu}(\Xi, \Lambda) = \{\mu(\Xi^-)/\mu(\Lambda)\} < 1 < \{\mu(\Sigma^-)/\mu(\Lambda)\} \quad (14c)$$

All simple models predict that the magnitude of the Ξ^- moment should be less than or equal to the Λ moment, as required by the condition (14c); i.e. that the quantity $R^{\mu}(\Xi, \Lambda) \leq 1$. However the experimental data show a Ξ^- moment which is larger than the Λ moment by 0.14 n.m. and $R^{\mu}(\Xi, \Lambda) = 1.22 \pm 0.1$, discrepancies just a bit more than two standard deviations. If this discrepancy persists, it is back to the drawing board for all theorists.

We now examine more carefully the assumptions underlying the relations (13), (14a) and (14b) which lead to the prediction (14c) that seems to disagree with experiment. The inequality (13) follows from very general model-independent conditions. The relations (14b) and (14c) between the moments in the $SU(3)$ symmetry limit can be broken if there is configuration mixing, but we now show that such mixing does not appreciably affect the relation (14b) and that the magnetic moment of the Ξ^- cannot be very much larger in magnitude than that of the strange quark.

The values of $\bar{\mu}(\Xi^-)$ and $\bar{\mu}(\Sigma^-)$ are easily calculated in any three-quark model with arbitrary configuration mixing. Since \underline{d} and \underline{s} quarks have the same magnetic properties in this limit all the quarks in the Ξ^- and Σ^- are equivalent. Thus the two magnetic moments are equal and depend only upon the total orbital and spin angular momenta \underline{L} and S of the three quark system.

$$\{\bar{\mu}(\Xi^-)/\mu(s)\} = \{\bar{\mu}(\Sigma^-)/\mu(s)\} = \langle 2S_z + L_z \rangle = (1 - \langle L_z \rangle). \quad (15a)$$

Multiplying eq.(15a) by the inequality (13) gives the inequality,

$$\{\mu(\Xi^-)/\mu(s)\} < \{\bar{\mu}(\Sigma^-)/\mu(s)\} = (1 - \langle L_z \rangle). \quad (15b)$$

From angular momentum algebra for a state with definite values of \underline{L} and S ,

$$1 - \{\bar{\mu}(\Xi^-)/\mu(s)\} = \langle L_z \rangle = (1/3)\{-(S - \frac{1}{2})(S + \frac{3}{2}) + L(L + 1)\} \quad (15c)$$

The maximum value of the right hand side of eqs. (15a) and (15b) cannot be very much greater than unity, since the dominant configuration in the wave function is certainly the $L=0$ state of the standard model and nearly all possible admixtures have positive values of $\langle L_z \rangle$. Eq. (15c) shows that only configurations with $S=3/2$, $L=1$ have a negative expectation value for L_z . A very extreme upper limit for the ratio (15b) is given by considering only the admixture of $S=3/2$, $L=1$ configurations. Eq. (15c) then gives the inequality,

$$\{\bar{\mu}(\Xi^-)/\mu(s)\} - 1 \leq P_1/3 \quad (15d)$$

where P_1 denotes the magnitude of the $S=3/2$, $L=1$ component in the wave function. An admixture of 30% of $L=1$, $S=3/2$ gives only a 10% increase in $\bar{\mu}(\Xi^-)$ over $\mu(s)$. But such an admixture is highly improbable.

The four lowest configurations have $\{|L_1, L_0\rangle |L_1, S_0\rangle |S\rangle = \{|1,1\rangle 0; \{0,1/2\} 1/2\rangle, \quad \{|1,1\rangle 1; \{0,1/2\} 1/2\rangle, \quad \{|2,0\rangle 2; \{1,1/2\} 3/2\rangle, \quad \text{and} \quad \{|0,2\rangle 2; \{1,1/2\} 3/2\rangle$, where S_1 and L_1 denote the spin and the relative orbital

angular momentum of the two identical quarks and S_0 and L_0 denote the spin of the odd quark and its angular momentum relative to the center of mass of the two identical quarks. All these configurations have positive values of $\langle L_z \rangle$, and similarly for the vast majority of higher configurations. Note that even parity for the baryon requires the values of L_1 and L_0 to be either both even or both odd and that the Pauli principle for colored quarks requires $S_1 = 0$ for odd values of L_1 and $S_1 = 1$ for even values. Configurations with $S=3/2$, $L=1$ are thus obtainable only with equal even values of L_1 and L_0 , beginning with $L_1 = L_0 = 2$ coupled to a total $L = 1$, rather than 0 or 2, and with S_1 and S_0 coupled to a total $S = 3/2$ rather than $1/2$. The lowest configuration of this type has two relative orbital d-waves and would be expected to be mixed into the baryon wave functions much less than configurations listed above with one d-wave and one s-wave or with two p-waves. Thus $\bar{\mu}(\Xi^-)$ cannot be much larger than $\mu(s)$.

The inequality (15d) can be sharpened further by noting that the inequality (11b), which follows only from the assumption that the SU(3) symmetry breaking enhances the contribution of the nonstrange quarks, can also be written

$$\mu(\Xi^-) - \bar{\mu}(\Xi^-) = K\{\mu(\Xi^-) - \mu(\Xi^0)\}, \quad (16a)$$

where K is a positive constant which can be evaluated using the formalism developed in ref. {7}.

$$K = [1 - \{\mu(s)/\mu(d)\}]/(3 - \epsilon) \quad (16b)$$

where ϵ is a small number defined in ref. {7} to include the possibility of an additional isoscalar component in the magnetic moments of the nonstrange quarks,

$$\mu(u) = (-2 + \epsilon) \mu(d) \quad (16c)$$

substituting eq.(16b) and eq.(15a) into eq. (16a) gives

$$\begin{aligned}
\{\mu(\Xi^-)/\mu(s)\} - 1 &= \\
&= -\langle \mu_z \rangle - [\{\mu(\Xi^0) - \mu(\Xi^-)\}/\mu(s)]\{1 - \{\mu(s)/\mu(d)\}\}/(3-\epsilon) \ll \\
&\ll (P_1/3) - [\{\mu(\Xi^0) - \mu(\Xi^-)\}/\mu(s)]\{1 - \{\mu(s)/\mu(d)\}\}/(3-\epsilon) \quad (16d)
\end{aligned}$$

Since the relations (15d) and (16d) hold even in the presence of configuration mixing the relation (14b) is justified. The most obvious explanation for the disagreement with experiment is that the strange quark magnetic moment in the Ξ is larger in magnitude than the Λ moment. This cannot be explained by mass dependent factors, which would work in the opposite direction and make the moment smaller in heavier baryons.

One possible explanation is that eq. (14a) may not hold and the moment may not be simply equal to the strange quark moment; i.e. that there is configuration mixing and that the nonstrange quark contribution does not vanish as in the lowest configuration where the u and d quarks are coupled to angular momentum zero.

Another possibility is that there is also a nonadditive enhancement of the strange quark contribution to magnetic moments, which increases with the number of strange quarks. This would explain why the magnetic moment of the strange quark seems to be larger in the Ξ^- than in the Λ .

It is also possible to invoke other constituents in addition to valence quarks. The inequality (15d) does not hold for configurations with gluon components in the wave function. Large values of $\mu(\Xi^-)/\mu(s)$ are obtainable by coupling the quark spins to $S=3/2$ and having the gluons carry $L=1$. However, many such gluonic configurations are possible and including them in calculations at this stage introduces too many additional free parameters to enable a significant test against experiment. This possibility must be kept in mind and investigated when either more and better data become available or

QCD reaches the stage where unambiguous predictions can be made without too many parameters.

IV. Conclusions

The conclusions from this analysis are most easily stated in terms of the two discrepancies mentioned above and defined quantitatively by the expressions (8a) and (8c). The quantity $R(p, \Sigma^+, \Xi^-)$ defined by eq. (8a) exhibits the problem of the proton - Σ^+ difference. It vanishes in the SU(3) symmetry limit and the standard broken SU(3) prediction is very much smaller than the experimental value. It is shown in ref. [11] that the large experimental value cannot be fit by the universal additive model, and indicates that the contribution of the nonstrange quarks in the proton is considerably larger than in the Σ^+ . The empirical non-additive model gives a good fit.

The quantities $R'(\Xi, \Lambda)$ and $R''(\Xi, \Lambda)$ defined by eqs. (8c) and (8d) exhibit the problem of the $\Lambda - \Xi^-$ difference. The experimental value 1.12 for $R'(\Xi, \Lambda)$ is significantly larger than the prediction of unity from all models considered. The difference between the experimental value 1.22 for $R''(\Xi, \Lambda)$ and the predictions of unity or less is even larger. The discrepancies are still only a bit more than two standard deviations. If the disagreement persists with better data there is no obvious simple way to fix up the models.

The discrepancies in the quantities $R(p, \Sigma^+, \Xi^-)$, $R'(\Xi, \Lambda)$ and $R''(\Xi, \Lambda)$ cannot be fitted simply by mass parameters, because they depend on the baryon mass in opposite directions. Scaling the quark contribution to the baryon moment with baryon mass factors to improve the value of $R(p, \Sigma^+, \Xi^-)$, will increase the disagreements with the values of $R'(\Xi, \Lambda)$ and $R''(\Xi, \Lambda)$. Either

configuration mixing or a new nonadditive contribution can help, but both approaches involve introducing an additional free parameter to fit one experimental number. New experimental input from the Ω^- moment can help.

Since the baryon octet contains only states with one or two strange quarks, additional information of great interest can be obtained from the measurement of the magnetic moment of the Ω^- . The simple prediction is that this moment is just three times the magnetic moment of the strange quark. We thus obtain two very different predictions, depending upon whether $\mu(s)$ is obtained from the Λ moment via eq. (14a) or constrained by the Ξ^- moment via the inequality (16d). Equation (14a) gives

$$\mu(\Omega^-) = 3 \mu(s) = 3 \mu(\Lambda) = -1.9 \text{ n.m.}, \quad (17a)$$

Equation (16d) gives

$$\mu(\Omega^-) = 3 \mu(s) < 3 \mu(\Xi^-) < -2.2 \text{ n.m.}, \quad (17b)$$

These results are insensitive to effects of configuration mixing. The analogs of eqs. (15) for the $J=3/2$ state of three identical quarks gives

$$\{\mu(\Omega^-)/\mu(s)\} = \{\bar{\mu}(\Omega^-)/\mu(s)\} = \langle 2S_z + L_z \rangle = 3\{1 - (1/3)\langle L_z \rangle\} \quad (18a)$$

where angular momentum algebra now gives

$$1 - \{\mu(\Omega^-)/3\mu(s)\} = (1/3)\langle L_z \rangle = (1/15)\left[\frac{3}{2} - S\right]\left(\frac{5}{2} + S\right) + L(L+1) \geq 0 \quad (18b)$$

Thus

$$\{\mu(\Omega^-)/\mu(s)\} = \{\bar{\mu}(\Omega^-)/\mu(s)\} \leq 3 \quad (18c)$$

In any case the effects of configuration mixing are seen to be much smaller than for the Ω^- case because of the factor 1/15 in the correction term from configuration mixing in eq.(18b) in comparison with the factor of 1/3 in eq. (15c) for the Ξ^- case.

The quantity $R(\Xi, \Lambda)$ defined by eq. (8b) is not useful in this context, because it contains contributions from both strange and nonstrange quarks in

a way that compensates for the disagreements found in the quantities $R(p, \Sigma^+, \Xi^-)$ and $R'(\Xi, \Lambda)$. Thus the agreement found in ref. [11] for this quantity does not really test the strange quark contribution to the moments. The quantities $R'(\Xi, \Lambda)$ and $R''(\Xi, \Lambda)$ are much better. Which of the two is a more sensitive test depends upon the relative magnitudes of experimental errors.

Models for hadron structure and dynamics must eventually describe all properties including masses, magnetic moments and scattering cross sections. The standard broken-SU(3) model predicts the Λ magnetic moment from the proton moment and hadron mass differences. This clue to hadron structure should not be easily discarded in correcting the model to fit other hyperon moments. It is interesting and puzzling that both the magnetic moments and the total cross sections are fit reasonably well by a two-component description in which both additivity and SU(3) symmetry are broken only by a second component which enhances nonstrange contributions. Unfortunately there is no simple relation between the two "second components" in eqs. (6) and (10). Further experimental work in hyperon physics may help to clarify these paradoxes and provide new insight into hadron structure and the generation puzzle. A model which explains why strange quarks seem to have a simpler behaviour than nonstrange quarks would be very interesting.

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ii) Observables largely dependent on $|f|$ or $\text{Im } f$ like $d\sigma_{\pi d}/d\Omega$ or $\sigma_{\pi d}^{\text{tot}}$ are predicted to be relatively insensitive to input parametrization. In contradistinction, those resulting from interferences between $\text{Re } f$ and $\text{Im } f$ (typically polarizations like t_{11}) will not.

7. Specific sensitivities

7.1 P_{11} background t_b

As is manifest from the $P_{11} \pi N$ phase, the P_{11} background nearly cancels the pole contribution in eq (5b) for $T_{\pi} < 200$ MeV (cf. fig. 2) but this need not also be the case for the off-shell t_{11} . The extent to which a partial cancellation may occur is undoubtedly governed by the parametrization and since t_p is presumably best known, t_b in (5b) appears most uncertain.

We illustrate this point in table III. There we entered various calculations of, amongst others, low J^{π} amplitudes which are most sensitive to π absorption (and thus dependent on $(t_{11})_b$ parametrization). From previous comparisons we know that in the absence of π absorption there is satisfactory agreement amongst various predictions, i.e. relatively little influence of phase equivalent input. Inclusion of the pole term causes drastic changes for the $0^+, 1^-$ amplitudes which are reduced but not annihilated by the P_{11} background. This indirectly demonstrates that off-shell there is only partial cancellation between \tilde{t}_p, t_b in t_{11} , eq (5). Moreover, the differences in amplitudes containing absorption effects and computed by various groups all using phase equivalent P_{11} parametrization, eloquently demonstrates this sensitivity to $(t_{11})_b$. There is very little chance to make an a priori discrimination and we may have to live with this indeterminacy. We further recall the accurate quenching of ω, ρ exchange through a small ρ , eq (11) which value is also determined by the P_{11} parametrization.

7.2 $NN\pi$ cut-off. There is little sensitivity in the $\beta_{NN\pi}$ explored (0.9-1.0 GeV). We already commented that $\beta \sim 1.2$ GeV would cause $\sigma(\bar{d}, \pi d \rightarrow NN)$ to agree with experiment, but these values are not allowed for the chosen P_{11} background parametrization.

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