IL8202322

WIS-82/33 June-PH

A study of charge symmetry breaking effects in elastic $\pi^{\pm}d$ scattering

A. S. Rinat and Y. Alexander

The Weizmann Institute of Science, Rehovot, Israel

Abstract

We computed external Coulomb and some strong charge symmetry breaking (CSB) effects in $\pi^{\pm}d \to \pi^{\pm}d$. These appear to account for charge asymmetry of differential cross sections, while approximate CSB spoils the agreement. We further report on a critical study of CSB effects extracted from $\pi^{\pm}d$ total cross section differences.

1. Introduction.

Consider reactions which apart from the charge of the incident pion are identical. If a) these reactions proceed through a unique isospin channel and b) electromagnetic effects are neglected, charge symmetry (CS) implies identical results for corresponding observables. Deviations from these predictions are then allocated to charge symmetry breaking (CSB).

Of course, neither in the analysis of experiments nor in calculations may one neglect electromagnetic effects. One thus attempts to remove these from data or - when finite - to correct strong predictions for those perturbations. In general neither can be done exactly. For instance for elastic scattering observables, one usually generates an elastic amplitude by a phenomenological optical potential to which V_{Coul} is added. Alternatively one sometimes starts from the eikonal (Glauber) limit of f_{el} and directly applies Coulomb corrections to it[1].

An example of a CSB test case is $\pi^{\pm}d \to \pi^{\pm}d$ for which I=1. Over the resonance region one disposes of precision data on $\sigma_{tot}^{\pm}$ [2] while for T_{π}=143 MeV, angular distributions have been taken with $\pi^{\pm}$ [3].

After removing Coulomb perturbations from σ_{tot} Pedroni et al. concluded the existence of CSB. Its effects were subsequently parametrized in terms of mass and width differences for the different Δ isobars, which are excited in the scattering of a pion from the target nucleons.[2)]

A different picture emerges from the $\pi^{\pm}d \to \pi^{\pm}d$ differential cross section measurements. From a first analysis these data appear to limit CSB effects as seen in the charge asymmetry

$$A(\theta)=\frac{d\sigma^{-}(\theta)-d\sigma^{+}(\theta)}{d\sigma^{-}(\theta)+d\sigma^{+}(\theta)} \qquad (1)$$

to $(0.4\pm0.5)\%$, i.e. consistent with no CSB at all.[3].

The πd case is actually unique in that realistic models for its description exist[4][5]. For instance a 3-body model for $\pi d \to \pi d$ may be formulated and solved exactly[6]. There exist modifications thereof[4] which depart from a strict 3-body model and which allow for π absorption (emission): these enable to include CSB effects in an accurate, albeit not exact fashion.

It is clearly of interest to sharpen as much as possible the arguments and to perform accurate calculations before reaching conclusions on CSB. Easiest to handle are the non-forward $\sigma(\theta)$ data. In section 2 we review the standard treatment of Coulomb effects on those strong amplitudes. We further present there an approximate treatment of CSB on strong amplitudes due to differences in πN scattering amplitudes in various charge states as well as to p,n mass differences.

No such procedure is possible for σ^{tot}, because of the forward Coulomb amplitude which remains overriding even after regularizing the Coulomb interaction by p,π formfactors. In general it requires a prescription which enables the extraction of finite numbers from the data. In general one cannot well argue about the choice of a suggested prescription which in the case of Pedroni et al.[2] is given in section 3. However, once defined, one may check the way the prescription is implemented numerically. Pedroni et al. thus used the impulse approximation and estimate multiple scattering and Fermi averaging corrections to be (20-25)%. Such a global estimate seems to be an overly optimistic one and cannot well hold for all energies throughout the resonance region. At least those parts can be checked in a realistic calculation, results of which are found in section 4. We conclude with a short discussion and some proposed experiments which bear on the establishment of possible charge symmetry breaking effects in πd interaction.

2. Description.

Calculations based on charge symmetric models for the $NN\pi$ system[4)5)] produce elastic πd partial wave amplitudes and explicit charge effects on otherwise charge-symmetric (CS) strong amplitudes f^J, may thus be represented by[*)]

$$F^J_{LL'}=f^{Coul}_L\delta_{LL'}+e^{i(\sigma_L+\sigma_{L'})}f^J_{LL'} \qquad (2)$$

Here f^{Coul} and σ_L are partial wave amplitudes and phases belonging to the Coulomb potential of the $\pi^\pm d$ system.

Consider now strong CSB effects generated by πN rescattering in various charge states, i.e. through intermediate $N\Delta$ states (fig. 1a).

A precise calculation of these CSB effects to all orders requires a formidable effort and is actually unwarranted in the absence of precise $\pi^\pm n$ elastic data, which together with $\pi^\pm p$ provide the necessary input. We thus suggest an estimate of strong CSB effects by means of

$$f^{CSB}\approx f-f^{(2)}+f^{(2)CSB} \qquad (3)$$

The amplitude including strong CSB effects is represented by the full CS amplitude f corrected by lowest order CSB (shown in fig. 1b for $\pi^\pm d$.) Thus, although $f^{(2)}$ may be a rather poor approximation to f, we expect $f^{(2)CSB}-f^{(2)}$ to account well for the difference of the full amplitudes $f^{CSB}-f$.

Consider first the lowest order CS amplitude (fig. 1a)

$$f^{(2)}=B_{\pi d,N\Delta}G_\Delta B_{N\Delta,\pi d} \qquad (4)$$

---

*) See the appendix for remarks on (2)

Here

$$B_{\pi d,N\Delta}=\langle g_d|G_o^{(3)}|g_\Delta\rangle \qquad (5)$$

is the N exchange amplitude for $\pi d\to N\Delta$ in terms of deuteron and Δ form factors g_d, g_Δ and $G_o^{(3)}$ is the propagator of the free $NN\pi$ system. G_Δ in (4) is the dressed Δ propagator and appears in the representation of the 33 partial wave scattering matrix $t_\Delta=g_\Delta G_\Delta g_\Delta$, viz. ($M_o$ is some bare Δ_o mass; $E_p=(p^2+M_N^2)^{1/2}$, $\omega_p=(p^2+\mu_\pi^2)^{1/2}$; $\mu_p^{-1}=E_p^{-1}+\omega_p^{-1}$; $\Sigma_\Delta(s)$ is the Δ selfenergy)

$$G_\Delta^{-1}(s)=s-M_o^2-\Sigma_\Delta(s) \qquad (6a)$$
$$=s-M_o^2-\int\frac{g_\Delta^2(p)p^2dp}{16\pi^3\mu_p(s-(E_p+\omega_p)^2)}$$

A vertex of the form

$$g_\Delta=\frac{Np}{p^2+\beta^2} \qquad (7)$$

appears to provide an accurate fit to the π^+p data[7]. Lack of $\pi^\pm n$ data prevents a similar parameter extraction for g_Δ in other 33 charge states. One thus has to invoke approximations in line with the available resonance information and its accuracy. First one may wish to replace in G_i the self energies by their resonance values. Thus

$$G_i^{-1}(s)=s-M_{0,i}^2-N_i^2\mathcal{P}\int\frac{p^4dp}{16\pi^3\mu_p(M_i^2-(E_p+\omega_p)^2(p^2+p_i^2)^2}+iM_i\Gamma_i \qquad (6b)$$
$$s-M_i^2+iM_i\Gamma_i$$

Assuming all cut-off momenta to be equal to $\beta=\beta_{\Delta^{++}}$ (=.355 GeV[7]), ($\bar{p}_i$ is the πN CM resonance momentum)

$$\Gamma_i=\frac{N_i^2}{32\pi^2}\cdot\frac{\bar{p}_i^3}{M_i(\bar{p}_i^2+\beta^2)^2} \qquad (8)$$

For known masses and widths M_i, Γ_i, eq.(8) fixes N_i and through it, $f^{(2)}$ may be calculated using (4)(5)(7) and (6b).

When wishing to use energy-dependent self energies Σ_i (and have a unitary t_Δ as a consequence) one needs in practice an additional assumption. Suppose that M_i^2 under the integral in (6b) may be replaced by $\langle M_i^2 \rangle$. Then

$$G_\Delta^{-1}(s)=s-M_{o,i}^2-N_i^2A-iM_i\Gamma_i \tag{6c}$$

The constant A in (6c) can be calculated since M_o, N, M are known for Δ_{++} [7]:

$$\left(M_{o\Delta^{++}}\right)^2+\left(N_{\Delta^{++}}\right)A=M^2_{\Delta^{++}} \tag{9}$$

Eq.(9) can then be used to determine all bare masses $M_{o,i}$ and allows use of (an approximation to) $G_i(s)$, eq.(6a).

The parameters needed in (6b) are assembled in table I. In the second column there, one finds directly (Δ^{++}, Δ^0) and indirectly measured ($\Delta^- - \Delta^{++}$) masses[8], while M_{Δ^+} is taken from the broken mass relation $M_{\Delta^-}-M_{\Delta^{++}}=3(M_{\Delta^0}-M_{\Delta^+})$. The widths are as in ref.(8). All are used without testing the error regions.

Finally we remark that an additional CSS effect is generated by differentiating between masses for the p,n spectators of the πN scattering (cf fig.(1b)).

With the thus defined input we applied the theory of refs. 4,5 in order to determine CSS on the dominant amplitudes $J^\pi L(=L')$: 2^+1; $0^+,1$; $1^-,0$; 1^+1

3. Data handling.

Since total cross section differences with point-Coulomb effects are infinite, a procedure is necessary to define finite numbers and to correlate these with data. Rewriting (2) as

$$F^J_{LL'}=\left(f^{Coul}_L\delta_{LL'}+f^J_{LL'}\right)+\left(e^{i(\sigma_L+\sigma_{L'})}-1\right)f^J_{LL'} \qquad (2')$$

$$\simeq f^{Coul}_L\delta_{LL'}+\left(f^J_{LL'}\right)^{imp}+\Delta_{dist}(f^{Coul};f^{str}) \qquad (2'')$$

one has in (2') a division which leads to, respectively, Coulomb-nuclear interference (CNI), and Coulomb distortion (CD) differences:

$$\Delta\sigma^{tot}=\Delta\int\left[\left(\frac{d\sigma}{d\Omega}\right)^{el}+\left(\frac{d\sigma}{d\Omega}\right)^{inel}\right]d\Omega$$

$$=\Delta\int\left[\left(\frac{d\sigma}{d\Omega}\right)^{CNI}+\left(\frac{d\sigma}{d\Omega}\right)^{CD}+\left(\frac{d\sigma}{d\Omega}\right)^{str}\right]^{el+inel} \qquad (10)$$

Pedroni et al. suggested the following handling of the terms in (10):[2)]

a) Instead of (2'), eq.(2") is used where the strong amplitude is replaced by its impulse approximation.

b) A finite part of CNI is defined by

$$\Delta^{CNI}_{fin}=\frac{\mathrm{Re}\ f_{\pi d}(0^o)}{\mathrm{Re}(f_{\pi\phi}(0^o)+f_{\pi n}(0^o))}\ \Delta\int_{\Omega_{min}}^{4\pi}\left(\frac{d\sigma}{d\Omega}\right)^{CNI,el}_{imp}d\Omega \qquad (11)$$

with Ω_{min} some small angle. The normalizing multiplier of the integral contains ratio's of forward scattering amplitudes.

c) The Coulomb distortion part

$$\Delta^{CD}=\Delta\int\left(\frac{d\sigma}{d\Omega}\right)^{CD,el}d\Omega \qquad (12)$$

is the one calculated by the Auvil procedure[9)].

d) Finally, finite strong total cross section differences are <u>defined</u> as

$$\Delta\sigma^{tot}_{str}\equiv\Delta\sigma^{tot}_{extrap}-\Delta^{CNI}_{f}-\Delta^{CD} \qquad (13)$$

The first term on the rhs of (13) is the contribution replacing in the infinite part

$$\Delta_{infin}^{CNI}=\Delta\int_{0}^{\Omega_{min}}\left(\frac{d\sigma}{d\Omega}\right)^{CNI,el}d\Omega \qquad (11')$$

the actual differential cross section by the one extrapolated from given $\Omega_{max}\to\Omega_{min}\to 0$. If the procedure were immaculate, charge symmetry implies 0 for the lhs of (13).

4. Numerical results.

4a. Differential charge asymmetries.

In figs. 2 we show non-forward charge asymmetries $A(\theta)$, eq.(1) for T_π=32,116,142,180,256 MeV. The drawn lines correspond to charge symmetric strong amplitudes, while the dashed lines are results for CSB strong amplitudes, eq.(3). Those marked 'G^f' were computed with the full isobar propagator (6a), while 'G^e' corresponds to the constant self energy approximation (6b) to it.

Notice first that $A(\theta)$ tends to 0 with increasing angles reflecting a faster decrease of f^c than of f^{str}. Next $A(\theta, T_\pi)$ may apparently become negative as would be the case for f^{str} corresponding to a repulsive πd interaction. Indeed, like for genuine resonances Re $f^J_{\pi d}$ changes sign around $T_\pi \sim 165$ MeV[10,11] approximately marking the energy beyond which the strong attraction turns into an effective repulsion.

From fig. 2b one sees that a CS strong amplitude reproduces the T_π=143 MeV data, much the same as concluded by Thomas (cited in [3]). In view of different data points for the same θ it is not clear whether there is a genuine discrepancy for $\theta_\gtrsim 30^o$. These one would anyhow give less weight than the $\theta_\lesssim 80^o_A$ points, since individual cross sections are far larger than for $\theta_\lesssim 80^o$.

Angular asymmetries when calculated with CSB strong amplitudes appear to decrease faster than the CS counterparts, and disagree with the available T_π=143 MeV data. We shall return later to this point.

4b. Total cross section differences.

Theory can produce values for σ^{tot} when the Coulomb potential is somehow regularized by p,π form factors. However, the available data relate to $\Delta\sigma^{tot}$ after the manipulations described in section 3. Those involve the determination of a finite part of the total CNI cross section differences and the total Coulomb distortion part, eqs.(11),(12).

As argued in the introduction, we tend to distrust an impulse approximation around a resonance as used for CNI[2]. By way of example we give in table II the full dominant 2^+1 amplitude and its lowest order term $f^{(2)}$ (essentially the impulse approximation with Fermi motion effects included). Whenever $\mathrm{Im} f^{2^+} \gg |\mathrm{Re}\, f^{2^+}|$, a reasonable representation of Im f alone will about reproduce cross sections, but not polarizations, etc.

The mentioned sensitivity of Re f_L^J around the resonance region of course also affects Re $f_{\pi d}(0^o)$. The authors of ref.(2) used a) results from dispersion relations with uncertainties discussed there[12], b) a calculation by Butterworth using a fixed scatterers model[13].

In fig 3 we show $\mathrm{Re} f_{\pi d}(0^o)$ computed with the realistic model of ref[4] together with the abovementioned result. The expected sensitivity causes a shift of the Re $f_{\pi d}(0^o)$ curve, changes in particular the position of the sign change in $\mathrm{Re} f_{\pi d}(0^o)$ and thus significantly affects the multiplier of the integral in (11) around the resonance energy.

Next we computed the integral in (11) when instead of the impulse approximation (for either the elastic or the closure approximation for elastic + inelastic scattering) we used the full elastic amplitude. For T_π=116,142,180,256 MeV we entered in columns 2,3,4 of tableIII $\Delta\sigma_1$, the CNI corrected differences from two data sets as cited in ref[2], the correction to it for our $\mathrm{Re}f_{\pi d}(0^{\circ})$ and the differences of the same when the full amplitude and $f^{(2)}$ are used.

Coulomb distortion parts have been calculated using CS and CSB strong amplitudes, and in columns 5,6 of table III a comparison is made with the results of ref[2]. It seems that at least some CSB effects entered the results of Column 5 in which case there would be reasonable agreement for CD effects except for 256 MeV[*)].

---

*) We remark that contrary to Pedroni et al. we Coulomb perturb the πd and not the πp relative motion.

5. Discussion.

We analyzed above the way Pedroni et al. extracted an apparent CSB total cross section differences The fact that a presoription is needed to extract finite numbers for $\Delta\sigma^{total}$ as well as the use of the imprecise impulse approximation to correct for Coulomb-nuclear interference, causes us to believe that from $\Delta\sigma^{total}$ one can as yet not conclude the existence of CSB effects.

As a better tool apear charge asymmetries of differential cross ections. The T_{π}=143 MeV data are well described with charge symmetric strong amplitudes, while discrepancies appear when CSB effects are included. Amongst possible causes we mention the proposed parametrization of Δ propagators, the CSB treatment to second order and the neglect of Coulomb distortions in intermediate states.

It is clear that additional A(θ) data are needed, but obviously maximal differential charge effects are expected for polarization observables which depend on interferences of various partial wave amplitudes. A difficulty is the fact that theory[4)5)] not always reproduces the data[14)15)], but for a case where it does, viz. the vector polarization $it_{11}(\theta)$ for T_{π}=142 MeV one would learn from measurements with pions in both charge states.

Acknowledgement. We thank Tony Thomas for drawing our attention to the topic discussed and for the stimulus to undertake the analysis.

Appendix.

Eq.(2) for the Coulomb-perturbed strong amplitude is only an approximation to the exact two-potential result for the t matrix ($\omega=1+G_o t$, $\omega_c=1+G_o t_c$)

$$t=t_c+\omega_c^+ v_{str}\omega \tag{A1}$$

Iteration of (A1) gives

$$t=t_c+\omega_c^+ v_{str}\sum_{n=0}^{\infty}(G_o\omega_c v_{str})^n\omega_c^+ v_{str}\omega_c, \tag{A2}$$

and neglect of Coulomb rescattering in intermediate states (i.e. $\omega_c\to 1$ inside the bracket in (A2)) leads to (A1). Defining

$$\tilde{t}_{str}=v_{str}(1+\tilde{G}\tilde{t}_{str})$$
$$\tilde{G}^{-1}=G_o^{-1}-v_c \tag{A3}$$

one may rewrite (A2) as

$$t=t_c+\omega_c\tilde{t}_{str}\omega \tag{A4}$$

which in an alternative way shows that waves in intermediate states are Coulomb distorted. The distortion effect may be estimated by replacing in (A3) v_c (for $\Delta^{\pm}$p intermediate states) by an average of (0.7-1.0)MeV, i.e. a correction well below the uncertainty of the masses entering the Δ propagators (6a),(6b).

No doubt (A4) ought to be used instead of (2) when knowledge of more accurate Δ_1 resonance parameters warrants a parallel, more precise calculation.

References.

1) J.-F. Germond and C. Wilkin, Ann. of Phys. (N.Y.) 121 (1978) 285.

2) E. Pedroni, K. Gabathuler, J. J. Domingo, W. Hirt, P. Schwaller, J. Arvieux, C. H. Q. Ingram, P. Gretillat, J. Piffaretti, N. W. Tanner and C. Wilkin, Nucl. Phys. A300 (1978) 321.

3) T. G. Masterson, E. F. Gibson, J. J. Kraushaar, R. J. Peterson, R. S. Raymond, R. A. Ristmin and R. L. Boudrie, Phys. Rev. Lett. 47 (1981) 220.

4) See for instance A. Thomas and A. S. Rinat, Phys. Rev. C20 (1979) 216; Y. Avishai and T. Mizutani, Nucl. Physics A326 (1979) 352; A338 (1980) 377; I. R. Afnan and B. Blankleider, Phys. Rev. C22 (1980) 377; C. Fayard, G. H. Lamot and T. Mizutani, Phys. Rev. Lett. 45 (1980) 524.

5) A. S. Rinat and Y. Starkand, submitted for publication.

6) E. O. Alt, W. Sandhas, H. Zankel and H. Ziegelman, Phys. Rev. Lett. 37 (1976) 1537; D. Eyre, A. C. Phillips and F. Roig, Nucl. Phys. A275 (1977) 13.

7) R. M. Woloshyn, E. J. Moniz and R. Aaron, Phys. Rev. C12 (1975) 909.

8) F. Myrher and H. Pilkuhn, Z. Physik A276 (1976) 29; L. G. Dakhno et al., Sov. J. Nucl. Phys. 33 (1981) 59.

9) P. R. Auvil, Phys. Rev. 168 (1968) 1568, D4 (1971) 240.

10) M. M. Hoenig and A. S. Rinat, Phys. Rev. C10 (1974) 2102.

11) J. Arvieux and A. S. Rinat, Nucl. Phys. A350 (1980) 205.

12) T. E. O. Ericson and M. P. Locher, Nucl. Phys. A148 (1970).

13) D. S. Butterworth, London Ph.D. thesis 1977, cited in ref.(1).

14) J. Bolger, E. Boschitz, G. Proebstle, G. R. Smith, S. Mango, F. Vogler, R. R. Johnson and J. Arvieux, Phys. Rev. Lett. 46 (1981) 169.

15) R. J. Holt, J. R. Specht, K. Stephenson, B. Zeidman, J. Frank, M. J. Leitch, J. D. Moses, E. J. Stephenson and R. M. Laszewski, Phys. Rev. Lett. 47 (1981) 472; J. Ulbricht, V. Koenig, W. Gruebler, P. A. Schmelzbach, B. Jenny, F. Sperisen, K. Elsener, C. Schweizer and A. Chisholm, Phys. Rev. Lett. 48 (1982) 311.

Table I

	M_i (GeV)	Γ_i (GeV)
Δ^{++}	1.2311	0.115
Δ^{+}	1.2305	0.1135
Δ^{0}	1.2325	0.1159
Δ^{-}	1.2370	0.1160

Δ masses and widths[7] without error estimates

Table II

T_π (MeV)	116	142	180	256
$f^{2^+}_{11}$ (in fm) full	.28916+.23420i	.21829+.37008i	-.04945+0.23542i	-.13068+.19102i
$f^{2^+}_{11}$ (in fm) lowest order	.36410+.17104i	.36249+.33086i	+.06315+0.20264i	-.07395+.26437i

Full and lowest order $f^{(2)}$ πd amplitudes (latter as measure for Fermi averaged impulse approximation).

Table III

T_π (MeV)	$\Delta\sigma_1$ *) (mb)	$(\Delta\sigma_1)_{corr}$ (mb)	$\Delta\sigma_1^f-\Delta\sigma_1^{(2)}$ (mb)	$(\Delta^{CD})^{Auvil}$ (mb)	$(\Delta^{CD})^{th}$ (mb)
116	0.9,2.6	0.9,2.6	-3.73	-8.1	-3.0 -8.1,-6.1
142	-1.9,-2.5	-1.9,-2.5	-4.80	-9.2	-2.9 -10.9,-7.9
180	0.7,-3.5	2.8,-14.	-2.89	-2.7	-1.2 -5.7,-4.6
256	5.2,4.9	4.0,3.7	-0.44	-0.7	1.7 4.2,4.5

Coulomb corrected π^-d,π^+d total cross section differences. Columns 2-4 are CNI, respectively calculated in impulse approximation using two data sets, the same corrected for our value of Re $f_{\pi d}(0^\circ)$ and the difference of $\Delta\sigma_1$, computed with the full and second order (impulse) amplitude. Columns 5 give the CD contribution from ref 2 and ours calculated with CS and the two CSB strong amplitudes discussed in section 3.

Figure Captions

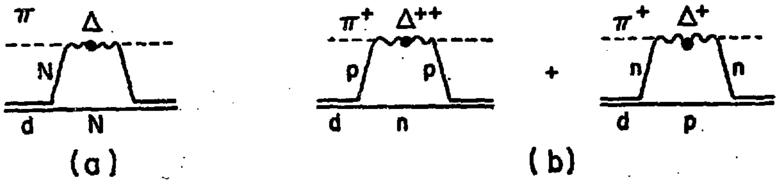
Fig. 1. a) Lowest order πN rescattering in dominant 33 state in a CS picture for πd.

b) Lowest order, CSB π^+p, π^+n rescattering in 33 states for π^+d.

Fig. 2. a) Charge asymmetry A, eq.(1) for the 82, 116, 180, 256 MeV differential πd cross sections.

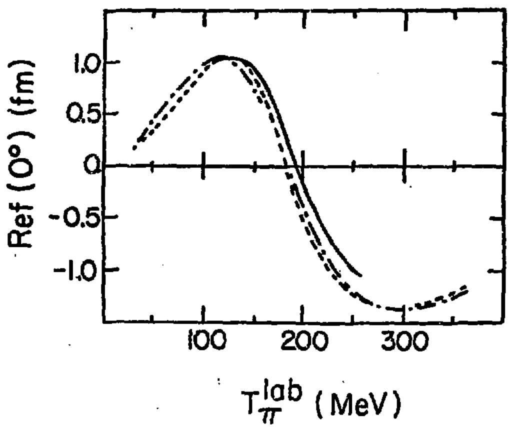
b) Same for T =143 MeV. Drawn lines correspond to CS, dashed lines to CSB strong amplitudes, G^f and G^e correspond to isober propagators (6a), resp. (6b).

Fig. 3. Re $f_{\pi d}(0^o)$ from the theory of ref.(4) (drawn line), dispersion calculation[12] (dashed line) and a fixed scatterers theory[13] (dashed dots).



π
Δ
N
d
N
(a)
π⁺
Δ⁺⁺
p
p
d
n
+
π⁺
Δ⁺
n
n
d
p
(b)

Fig 1



1.0
0.5
0
-0.5
-1.0
Ref (0°) (fm)
100
200
300
T_π^{lab} (MeV)

Fig 2

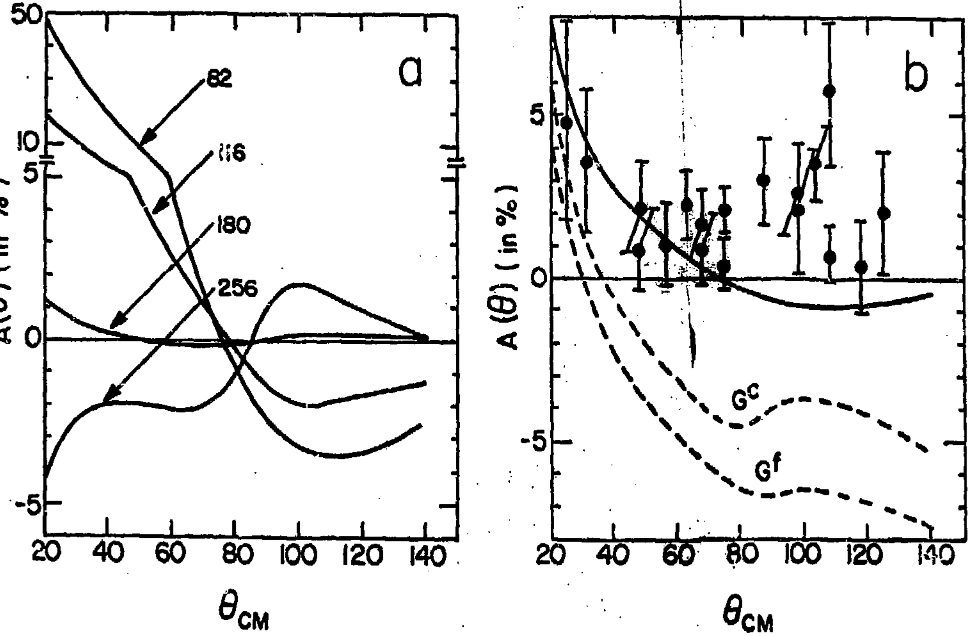


Fig 3